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The quantum nonthermal effect of a nonstationary Kerr–Newman black hole and the average range of the effective particles*

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We have found that the nonthermal radiation of a nonstationary Kerr–Newman black hole is affected by interstellar materials. In particular, the interstellar gas deeply influences the average range of nonthermal radiation particles, while the average range depends on the maximum energy of the radiation and the energy extent of the radiation.

Keywords: radiation, particle, average range
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1. Introduction

Black holes can be classified as stationary and nonstationary. Stationary and nonstationary black holes can also be divided into various kinds: charged, magnetized, or both charged and magnetized. Since Hawking proposed in 1974 that black holes could produce thermal radiation, much research has been carried out on black holes. These theoretical works have increased our knowledge of stationary and nonstationary black holes, especially the thermal and nonthermal radiation of nonstationary black holes, In the fields of observation and measurement, scientists have examined the existence of black holes in radio, visible light, x-rays, γ-rays, etc, by various possible means. They have used the energetic gamma ray experiment telescope on the gamma ray observatory satellites to explore some γ bursts successfully. In recent years, studies on TeV γ-ray bursts and even higher energy γ rays have been conducted both at home and abroad. However, the relationship between the interstellar gas and black hole radiation has not been considered. This relationship may provide some clues for exploring black holes. We take the nonstationary Kerr–Newman black hole as an example in the following to study the interstellar gas influence on nonthermal radiation.

2. Nonthermal radiation of nonstationary Kerr–Newman black holes

The line element of nonstationary Kerr–Newman black hole spacetime, according to the advanced Eddington coordinate, is expressed as[1,2]

$$\begin{aligned} \mathrm{d}s^2 = &-\left(1-\frac{2M_1 r - Q^2}{\rho^2}\right)\mathrm{d}\nu^2 + 2d\nu\mathrm{d}r \\ &- 2\frac{(2M_1 r - Q^2)a\sin^2\theta}{\rho^2} \\ &\times \mathrm{d}\nu\mathrm{d}\varphi - 2a\sin^2\theta\mathrm{d}r\mathrm{d}\varphi + \rho^2\mathrm{d}\theta^2 \\ &- \left[(r^2+a^2) + \frac{(2M_1 r - Q^2)a\sin^2\theta}{\rho^2}\right]\sin^2\theta\mathrm{d}\varphi^2, \end{aligned} \tag{1}$$

where $\rho = r^2 + a^2\cos^2\theta$, $a = a(\nu)$, $Q = Q(\nu)$, $M_1 = M_1(\nu)$.

According to Eq.(1), the metric determinant and the nonzero component of the contra-variant metric can be written respectively as

$$g = -\rho^4\sin^2\theta, \tag{2}$$

$$g^{00} = \frac{a^2\sin^2\theta}{\rho^2}, \qquad g^{01} = \frac{(r^2+a^2)}{\rho^2},$$

$$g^{03} = \frac{a}{\rho^2}, \qquad g^{13} = \frac{a}{\rho^2},$$

$$g^{22} = \frac{1}{\rho^2}, \qquad g^{33} = -\frac{\sin^2\theta}{\rho^2},$$

$$g^{11} = \frac{(r^2+a^2+Q^2-2M_1 r)}{\rho^2}. \tag{3}$$

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From Eqs.(1) and (3) and the null-hypersurface equation $g^{\mu\nu}\frac{\partial f}{\partial x^\mu}\frac{\partial f}{\partial x^\nu}=0$, we have

$$\begin{aligned}&a^2\sin^2\theta\left(\frac{\partial f}{\partial\nu}\right)^2+2(r^2+a^2)\frac{\partial f}{\partial\nu}\frac{\partial f}{\partial r}\\&+2a\frac{\partial f}{\partial\nu}\frac{\partial f}{\partial\varphi}+(r^2+a^2+Q^2-2M_1r)\left(\frac{\partial f}{\partial r}\right)^2\\&+2a\frac{\partial f}{\partial r}\frac{\partial f}{\partial\varphi}+\left(\frac{\partial f}{\partial\theta}\right)^2+\frac{1}{\sin^2\theta}\left(\frac{\partial f}{\partial\varphi}\right)^2=0,\end{aligned}\tag{4}$$

where

$$f(\nu,r,\theta)=0\tag{5}$$

and

$$\begin{aligned}&\frac{\partial f}{\partial r}\frac{\partial r}{\partial\theta}+\frac{\partial f}{\partial\theta}=0,\\&\frac{\partial f}{\partial r}\frac{\partial r}{\partial\nu}+\frac{\partial f}{\partial\nu}=0,\\&\frac{\partial f}{\partial\varphi}=0.\end{aligned}\tag{6}$$

From Eqs.(4) and (6), we obtain the equation of the event horizon of the black hole

$$g^{00}\dot r^2-2g^{01}\dot r+g^{11}+g^{22}(r')^2=0\tag{7}$$

on the event horizon surface $r=r_{\rm H}$. So the event horizon of a nonstationary Kerr–Newman black hole is

$$r_{\rm H}=\frac{m+\{m^2-(1-2\dot r_{\rm H})[a^2(1-2\dot r_{\rm H}+\dot r_{\rm H}^2\sin^2\theta)+Q^2+(r'_{\rm H})^2]\}^{1/2}}{1-2\dot r_{\rm H}},\tag{8}$$

where

$$r_{\rm H}=r_{\rm H}(\nu,\theta)\quad \dot r_{\rm H}=\frac{\partial r_{\rm H}}{\partial\nu},\quad r'_{\rm H}=\frac{\partial r_{\rm H}}{\partial\theta}.$$

In the curved spacetime expressed by Eq.(1), the equation of motion for a particle with electric charge q and mass m is[3]

$$g^{\mu\nu}\left(\frac{\partial s}{\partial x^\mu}+qA_\mu\right)\left(\frac{\partial s}{\partial x^\nu}+qA_\nu\right)+m^2=0,\tag{9}$$

where s is a Hamilton master function, A_μ is an electromagnetic four-potential, and $A_\mu=(A_0,0,0,A_3)$. Substituting Eqs.(3) and (9) into Eq.(8), we have

$$\begin{aligned}&g^{00}\left(\frac{\partial s}{\partial\nu}+qA_0\right)^2+2g^{01}\left(\frac{\partial s}{\partial\nu}+qA_0\right)\left(\frac{\partial s}{\partial r}\right)\\&+2g^{03}\left(\frac{\partial s}{\partial\nu}+qA_0\right)\left(\frac{\partial s}{\partial\varphi}+qA_3\right)\\&+g^{11}\left(\frac{\partial s}{\partial r}\right)^2+2g^{13}\left(\frac{\partial s}{\partial r}\right)\left(\frac{\partial s}{\partial\varphi}+qA_3\right)\\&+g^{22}\left(\frac{\partial s}{\partial\theta}\right)^2+2g^{33}\left(\frac{\partial s}{\partial\varphi}+qA_3\right)^2+m^2=0.\end{aligned}\tag{10}$$

By using the general tortoise coordinate transformation[4–9]

$$\begin{aligned}&r_*=r+\frac{1}{2k}\ln(r-r_{\rm H}),\\&\theta_*=\theta-\theta_0,\quad \nu_*=\nu-\nu_0,\end{aligned}\tag{11}$$

formula (10) can be written as

$$\begin{aligned}&g^{00}\left[\frac{\partial s}{\partial\nu_*}-\frac{\dot r_{\rm H}}{2k(r-r_{\rm H})}\frac{\partial s}{\partial r_*}+qA_0\right]^2\\&+2g^{01}\frac{2k(r-r_{\rm H})+1}{2k(r-r_{\rm H})}\frac{\partial r}{\partial r_*}\\&\times\left[\frac{\partial s}{\partial\nu_*}-\frac{\dot r_{\rm H}}{2k(r-r_{\rm H})}\frac{\partial s}{\partial r_*}+qA_0\right]\\&+2g^{03}\left[\frac{\partial s}{\partial\nu_*}-\frac{\dot r_{\rm H}}{2k(r-r_{\rm H})}\frac{\partial s}{\partial r_*}+qA_0\right]\\&\times\left(\frac{\partial s}{\partial\varphi}+qA_3\right)+g^{11}\left[\frac{2k(r-r_{\rm H})+1}{2k(r-r_{\rm H})}\right]^2\left(\frac{\partial s}{\partial r_*}\right)^2\\&+g^{33}\left(\frac{\partial s}{\partial\varphi}+qA_3\right)^2+g^{22}\left[\frac{\partial s}{\partial\theta_*}-\frac{\dot r_{\rm H}}{2k(r-r_{\rm H})}\frac{\partial s}{\partial r_*}\right]^2\\&+2g^{13}\frac{2k(r-r_{\rm H})+1}{2k(r-r_{\rm H})}\left(\frac{\partial s}{\partial r_*}\right)\left(\frac{\partial s}{\partial\varphi}+qA_3\right)+m^2=0.\end{aligned}\tag{12}$$

As there is the Killing vector $\left(\frac{\partial s}{\partial\varphi}\right)^a$ in the curved spacetime represented by Eq.(1), we have $\frac{\partial s}{\partial\varphi}=M$(constant).

We let

$$s=T(\nu_*,r_*,\theta_*)+M\varphi\tag{13}$$

and

$$\frac{\partial s}{\partial\nu_*}=-\varepsilon,\quad \frac{\partial s}{\partial\theta_*}=p_2,\tag{14}$$

where ε is the energy of a particle and p_2 is a component of the general momentum. Substituting Eqs.(13) and (14) into Eq.(12), the component of general mo-

mentum $\dfrac{\partial s}{\partial r_*}$ is

$$\frac{\partial s}{\partial r_*}=\frac{\partial T}{\partial r_*}=2\kappa(r-r_{\rm H})\frac{B\pm(B^2-AC)^{1/2}}{A}, \quad (15)$$

where

$$\begin{aligned}A=&g^{00}\dot r_{\rm H}^2-2g^{01}\dot r_{\rm H}[2k(r-r_{\rm H})+1]\\&+g^{11}[2k(r-r_{\rm H})+1]^2+g^{22}(r'_{\rm H})^2,\\B=&g^{00}(-\varepsilon+qA_0)\dot r_{\rm H}-g^{01}(-\varepsilon+qA_0)\\&\times[2k(r-r_{\rm H})+1]+g^{03}\dot r_{\rm H}(M+qA_0)\\&-g^{13}(M+qA_3)[2k(r-r_{\rm H})+1]+g^{22}p_2r'_{\rm H},\\C=&g^{00}(-\varepsilon+qA_0)^2+2g^{03}(-\varepsilon+qA_0)(M+qA_0)\\&+g^{22}p_2^2+g^{33}(M+qA_0)^2+m^2. \quad (16)\end{aligned}$$

Since s and $\dfrac{\partial s}{\partial r_*}$ cannot be imaginary numbers, (B^2-AC) in Eq.(15) can be written as

$$B^2-AC\geq 0. \quad (17)$$

Solving Eq.(17), we obtain

$$\begin{aligned}\varepsilon^{\pm}=&qA_0\\&+\frac{b\pm(AH)^{1/2}}{[g^{00}\dot r_{\rm H}-g^{01}[2k(r-r_{\rm H})+1]]^2-Ag^{00}}. \quad (18)\end{aligned}$$

Considering the sign of inequality in Eq.(17), we have

$$\varepsilon>\varepsilon^+ \ \text{ and } \ \varepsilon<\varepsilon^-. \quad (19)$$

In Eq.(18),

$$\begin{aligned}b=&\{g^{00}\dot r_{\rm H}-g^{01}[2k(r-r_{\rm H})+1]\}\\&\times\{g^{03}\dot r_{\rm H}(M+qA_3)-g^{13}(M+qA_3)\\&\times[2k(r-r_{\rm H})+1]+g^{22}p_2r'_{\rm H}\}\\&-Ag^{03}(M+qA_3),\end{aligned}$$

$$\begin{aligned}H=&g^{00}\{g^{03}\dot r_{\rm H}(M+qA_3)-g^{13}(M+qA_3)\\&\times[2k(r-r_{\rm H})+1]+g^{22}p_2r'_{\rm H}\}^2\\&+A[g^{03}(M+qA_3)]^2-2\{g^{03}\dot r_{\rm H}(M+qA_3)\\&-g^{13}(M+qA_3)[2k(r-r_{\rm H})+1]\\&+g^{22}p_2r'_{\rm H}\}\{g^{03}g^{00}\dot r_{\rm H}-g^{03}g^{01}\\&\times[2k(r-r_{\rm H})+1](M+qA_3)\\&+[g^{22}p_2^2+g^{33}(M+qA_3)^2+m^2]\}\\&\times\{[g^{00}\dot r_{\rm H}-g^{01}[2k(r-r_{\rm H})+1]]^2-Ag^{00}\}. \quad (20)\end{aligned}$$

In the nonstationary Kerr–Newman spacetime the Dirac energy levels distribution is described exactly in Eqs.(18)–(20), but the energy states which satisfy the extent $\varepsilon^-<\varepsilon<\varepsilon^+$ are in the forbidden region. The width of the forbidden region is

$$\begin{aligned}D=&\varepsilon^+-\varepsilon^-\\=&\frac{2(AH)^{1/2}}{\{g^{00}\dot r_{\rm H}-g^{01}[2k(r-r_{\rm H})+1]\}^2-Ag^{00}}, \quad (21)\end{aligned}$$

when $r\to r_{\rm H}$, because of

$$A|_{r\to r_{\rm H}}=g^{00}\dot r_{\rm H}^2-2g^{01}\dot r_{\rm H}^2+g^{11}+g^{22}(r'_{\rm H})^2=0. \quad (22)$$

This formula is the same as Eq.(7). Then in the event horizon location of a nonstationary Kerr–Newman black hole, from formulae (18) and (22) we obtain[3]

$$\lim_{r\to r_{\rm H}}\varepsilon^{\pm}=\varepsilon_0=\frac{a(1-\dot r_{\rm H})(M+qA_3)-p_2r'_{\rm H}}{r_{\rm H}^2+a^2-\dot r_{\rm H}a^2\sin^2\theta}+qA_0. \quad (23)$$

This means that the width of the forbidden region vanishes at the event horizon, and there is a crossing of positive and negative energy levels near the event horizon. When $\varepsilon_0>m$, the Starobinsky–Unruh process (spontaneous radiation) must occur. This means that there is a nonthermal radiation from the region near the event horizon. The maximum energy of a particle in this nonthermal quantum effect is ε_0.[10–13] While the energy extent of the radiation particle is[14–16]

$$m<\varepsilon\leq\varepsilon_0=\frac{a(1-\dot r_{\rm H})(M+qA_3)-p_2r'_{\rm H}}{r_{\rm H}^2+a^2-\dot r_{\rm H}a^2\sin^2\theta}+qA_0. \quad (24)$$

This radiation is irrelevant with temperature and it is a nonthermal radiation which is different from Hawking's radiation.[17–21] Formula (24) indicates that the energy extent of a radiation particle varies with radiation direction, which differs from that in a stationary Kerr–Newman black hole.[22–25].

In a stationary black hole, $r'_{\rm H}=0$, $\dot r_{\rm H}=0$, therefore Eq.(24) can be expressed as

$$m<\varepsilon\leq\varepsilon_0=\frac{aM+qQr_{\rm H}}{r_{\rm H}^2+a^2}=\Omega_{\rm H}M+qV, \quad (25)$$

where

$$\Omega_{\rm H}=\frac{a}{r_{\rm H}^2+a^2} \quad (26)$$

is the dragging velocity of the event horizon of the black hole, and

$$V=\frac{Qr_{\rm H}}{r_{\rm H}^2+a^2} \quad (27)$$

is the static electric potential at the two pole points of the black hole. It is clear that the nonthermal radiation of a stationary Kerr–Newman black hole is independent of angle θ.[26–28]

3. The average range of nonthermal radiation particles

In recent years, much research has been carried out on black hole radiation, but none has been done on the spreading of radiation in interstellar materials.

Interstellar materials include fixed stars, planets, interstellar gases and so on. The main component of interstellar gases is hydrogen and its density is about one hydrogen-atom per cubic centimetre. When the particles in the nonthermal radiation ot the black hole spread in such interstellar materials, the interstellar gas will play an important part in this influence. As to the interstellar gas, the density is represented by ρ_0 which is usually very small, while q/m is usually very large. Accordingly, the scattering of the nonthermal radiation particles in the process of spreading is very serious. When the particles with a certain energy spread through the interstellar gas, its range is related with its rate of energy loss in the interstellar gas, and the energy loss in the collision is random, which is a statistic, fluctuated and noncontinuous process. So the distribution of the range has a statistical mean. An important quantity is the average range $\overline{R}$, which can be expressed as follows[29,30]

$$\overline{R} = \int \left(- \frac{\mathrm{d}\varepsilon'}{\mathrm{d}r} \right)^{-1} \mathrm{d}\varepsilon'. \tag{28}$$

For a particle with an electric charge q, the rate of energy loss can be expressed as[29,30]

$$\begin{aligned} \left(- \frac{\mathrm{d}\varepsilon'}{\mathrm{d}r} \right) =& \frac{4\pi q^4 N_{\mathrm{A}} \rho_0 Z}{m\nu_0^2 A_0} \left[\ln \left(\frac{m\nu_0^2}{2I^2(1-\beta^2)} \right) \right. \\ & - (2\sqrt{1-\beta^2} - 1 + \beta^2) \ln 2 + 1 - \beta^2 \\ & \left. + \frac{1}{8} \left(1 - \sqrt{1-\beta^2} \right)^2 \right], \end{aligned} \tag{29}$$

in which N_{A} is the Avogadro constant, $\beta = \nu_0/c$, I is the average excitation energy, for hydrogen atom I=18eV, $Z = 1$, A_0=1.007979g.

According to Eqs.(28), (29), (23) and (24), the average range of the outgoing particles from a nonstationary Kerr–Newman black hole can be expressed as

$$\begin{aligned} \overline{R} =& \int_m^{\varepsilon_0} \left(- \frac{\mathrm{d}\varepsilon'}{\mathrm{d}r} \right) \mathrm{d}\varepsilon' \\ =& \frac{A_0 (2I^2)^{1/3} (mc^2)^{4/3}}{12\pi q^4 N_0 \rho_0 Z e^{1/24}} \left[\ln \frac{\ln(\varepsilon_0^2/2I^2mc^2) + 1/8}{\ln(m/2I^2c^2) + 1/8} \right. \\ & + \frac{1}{1 \cdot 1!} (\ln \varepsilon_0 - \ln m) \\ & + \frac{1}{2 \cdot 2!} (\ln \varepsilon_0 - \ln m)^2 + \cdots \\ & \left. + \frac{1}{n \cdot n!} (\ln \varepsilon_0 - \ln m)^n + \cdots \right]. \end{aligned} \tag{30}$$

In the square brackets, there is a converging series about $\ln(\varepsilon_0/m)$. So the average range of the radiation particles is inversely proportional to ρ_0, while it is directly proportional to $\ln(\varepsilon_0/m)$. Obviously, the less the ρ_0 is, the larger the average range is, and the larger the ε_0/m is, the larger the average range is. In a stationary black hole, we have

$$\Delta\varepsilon = \varepsilon_0/m = (\Omega_{\mathrm{H}} M + qV)/m, \tag{31}$$

from Eqs.(30) and (31), and the average range of nonthermal radiation particles from a nonstationary Kerr–Newman black hole is different from a stationary Kerr–Newman black hole. From Eqs.(30) and (31) we can conclude that the average range of the nonthermal radiation particles of a stationary Kerr–Newman back hole is irrelevant with direction, while in the nonstationary case, the average range is relevant with direction. In the nonstationary black hole, along the two pole points of the event horizon, the average range is minimum, while the average range of the radiating particles from the equator is maximum. In other directions the average ranges lie between these two values.

4. Conclusion

Nonstationary and stationary Kerr–Newman black holes have nonthermal radiation. Interstellar materials have influences on different kinds of radiation. We have shown that the interstellar material, especially interstellar gas, deeply influences the average range of particles of nonthermal radiation. The method and the conclusion of this paper may have some significance to the study of black holes.

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