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Parameter identification and synchronization of an uncertain Chen chaotic system via adaptive control*

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A systematic design process of adaptive synchronization and parameter identification of an uncertain Chen chaotic system is provided. With this new and effective method, parameter identification and synchronization of the Chen system, with all the system parameters unknown, can be achieved simultaneously. Theoretical proof and numerical simulation demonstrate the effectiveness and feasibility of the proposed method.

Keywords: synchronization, parameters identification, uncertain Chen chaotic system
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1. Introduction

Starting with the original work of Pecora and Carroll[1] about synchronizing two chaotic systems in the so-called drive–response configuration, increasing interest has been generated in chaotic synchronization and its practical application in different fields, such as secure communication, chemical and biological systems, information science, biotic science and so on.[2,3]

In order to achieve chaotic synchronization, different approaches have been proposed. The preliminary type of chaos synchronization consists of a drive–response (or master–slave) system investigated by Pecora and Carroll. They showed that a necessary condition for chaotic synchronization is that all the conditional Lyapunov exponents for the response system should be negative. Another type of technique for synchronization was suggested by Pyragas[4] to synchronize two chaotic systems A and B using an external feedback $s(A)$-$s(B)$, where $s(A)$ is the output of A and $s(B)$ is that of B. The third type of method is the parameter perturbation method which is based on the Ott–Grebogi–Yorke (OGY)[5] scheme for controlling unstable periodic orbits.

However, the above-mentioned methods and many other synchronization methods are valid for a chaotic system only when the system parameters are known. Achieving synchronization between two chaotic systems with unknown parameters is far from being straightforward. There has been little work on this challenging problem because it involves both the identification of the unknown parameters and the design of the controller. Guan *et al*[6] applied an observer to identification of the unknown parameter of the Lorenz system. Lü *et al*[7] studied the same problem for the Chen chaotic system using the same method. But since the design of an observer is a task full of technique and sensitively depends on the considered dynamical system, the method can be used in the system with only one unknown parameter. For the system with many unknown parameters, this method no longer works.

In this paper, a novel parameter identification and synchronization method is proposed for the Chen chaotic system with all the system parameters unknown based upon the adaptive control. Using this method, one can achieve chaos synchronization and identify the unknown parameters simultaneously. Furthermore, this method provides a detailed design process for chaos synchronization of uncertain systems and has wide practical applications in many other complex dynamical systems with unknown parameters.

2. Problem formulation

The Chen chaotic system was founded by Chen recently in anti-controlling chaos[8] and is relatively difficult to control as compared to the Lorenz and the Chua systems, due to its complicated topological structure, especially its rapid change in the z-direction.[9] Furthermore, it has been implemented by

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circuit and has a wide applicable prospect in secret communication. The nonlinear differential equations for describing the Chen system are

$$\begin{cases} \dot{x}_d = a(y_d - x_d), \\ \dot{y}_d = (c-a)x_d - x_d z_d + c y_d, \\ \dot{z}_d = x_d y_d - b z_d, \end{cases} \tag{1}$$

having a chaotic attractor when $a = 35$, $b = 3$ and $c = 28$. We denote it as

$$\dot{X}_d = f(X_d) \tag{2}$$

for simplicity.

In practice, the state signals of system (1) could be transmitted to the response system, but the system parameters are unknown. Therefore, the goal of identification and synchronization is to find an appropriate controller $U = (u_1, u_2, u_3)$ and parameter adaptive laws of $\hat{a}, \hat{b}, \hat{c}$, such that the controlled Chen chaotic system

$$\begin{cases} \dot{x}_r = \hat{a}(y_r - x_r) + u_1, \\ \dot{y}_r = (\hat{c} - \hat{a})x_r - x_r z_r + u_2, \\ \dot{z}_r = x_r y_r - \hat{b} z_r + u_3, \end{cases} \tag{3}$$

could be synchronous with the drive system (1) and

$$\begin{aligned} \lim_{t\to\infty} \tilde{a} &= \lim_{t\to\infty} (\hat{a} - a) = 0, \\ \lim_{t\to\infty} \tilde{b} &= \lim_{t\to\infty} (\hat{b} - b) = 0, \\ \lim_{t\to\infty} \tilde{c} &= \lim_{t\to\infty} (\hat{c} - c) = 0. \end{aligned} \tag{4}$$

3. Identification and synchronization of an uncertain Chen system

In this section, a systematic design process of synchronization and identification of an uncertain Chen system is provided. It should be pointed out that, although the process is focused on the Chen chaotic system with unknown parameters, we believe that the complexity of the Chen system also justifies the practical application of the proposed method to many other complex dynamical systems with unknown parameters.

3.1. Nonadaptive design based upon a nonlinear observer

In the case that the state signals and system parameters are available, there are many synchronization methods. We use the method based upon a nonlinear observer to synchronize the Chen chaotic system. Informally, an observer is a dynamical system designed to be driven by the output of another dynamical system. More precisely, for a given dynamical system

$$\dot{X}_d = F(X_d) \tag{5}$$

with an output $Z = S(X_d) \in R^n$, the dynamical system

$$\dot{X}_r = F(X_r) + G(Z - S(X_r)) \tag{6}$$

is said to be a nonlinear observer of system (5), if X_r converges to X_d as $t \to \infty$, where $G : R^n \to R^n$ is a suitably chosen function.[10] Moreover, system (6) is said to be a global observer of system (5) if X_r converges to X_d as $t \to \infty$ for any initial condition $X_r(0)$, X_d (0).[10,11]

Obviously, system (6) is a (global) observer of system (3) if the error system

$$\begin{aligned} \dot{e} &= F(X_r) - F(X_d) + G(S(X_d) - S(X_r)) \\ &= F(X_d + e) - F(X_d) + G(S(X_d) - S(X_d + e)) \\ &= H(e, t) \end{aligned} \tag{7}$$

has a (global) asymptotically stable equilibrium point e =0.

It is known that the control theory offers us no general method to choose a function $G(Z - S(X_r))$ such that the nonlinear and non-autonomous system (7) has a (global) asymptotically stable equilibrium point e =0. But for the Chen chaotic system, we have the following theorem.

Theorem 1. *For the Chen chaotic system* (2), *if* $Z = S(X_d) = ((a-1)x_d - c y_d, x_d z_d - (c+1)y_d, (b-1)z_d - x_d y_d)^T$, $G(Z - S(X_r)) = -Z + S(X_r)$, *then the dynamical system*

$$\dot{X}_r = f(X_r) + G(Z - S(X_r)) \tag{8}$$

is a global observer of the Chen chaotic system (2).

Proof Subtracting system (2) from system (8) yields the error system

$$\begin{cases} \dot{e}_1 = -e_1 + (a-c)e_2, \\ \dot{e}_2 = (c-a)e_1 - e_2, \\ \dot{e}_3 = -e_3, \end{cases}$$

where $e_1 = x_r - x_d, e_2 = y_r - y_d, e_3 = z_r - z_d$. Selecting a Lyapunov function as

$$V(e) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2),$$

its derivative along the solution of (7) is

$$\frac{\mathrm{d}V(e)}{\mathrm{d}t} = -2V(e),$$

which implies $V(e) = V(0)\mathrm{e}^{-2t}$. Therefore, for any initial condition $X_r(0), X_d(0)$, $\lim\limits_{t\to\infty} V(e) = 0$, and this proves the theorem as desired.

From theorem 1, we choose $U = G(Z - S(X_r))$ as above, and the controlled Chen chaotic system

$$\dot{X}_r = f(X_r) + U \tag{9}$$

is synchronous with the Chen system (2).

3.2. Adaptive synchronization and identification

The controller designed in section 3.1 requires knowledge of the system parameters. However, in many real applications it is difficult to exactly determine the values of the system parameters in advance. The effects of these uncertainties will destroy the synchronization and even break it. Therefore, the adaptive synchronization of the chaotic system in the presence of an unknown parameter is essential. To this end, we have the following theorem.

Theorem 2. *With the controller* $U = (u_1, u_2, u_3) = -Z + S(X_r)$, *where* $Z = S(X_d) = ((\hat{a}-1)x_d - \hat{c}y_d,\ x_d z_d - (\hat{c}+1)y_d,\ (\hat{b}-1)z_d - x_d y_d)$, *and the parameter adaptive laws of* $\hat{a}, \hat{b}, \hat{c}$ *as below, the controlled Chen chaotic system*

$$\begin{cases} \dot{x}_r = \hat{a}(y_r - x_r) + u_1, \\ \dot{y}_r = (\hat{c} - \hat{a})x_r - x_x z_x + \hat{c}y_r + u_2, \\ \dot{z}_r = x_r y_r - \hat{b}z_r + u_3 \end{cases} \tag{10}$$

is synchronous with the drive system (1) *and satisfies Eq.*(4), *where*

$$\begin{cases} \dot{\hat{a}} = (x_d - y_d)e_1 + x_d e_2, \\ \dot{\hat{b}} = z_d e_3, \\ \dot{\hat{c}} = -(x_d + y_d)e_2. \end{cases} \tag{11}$$

Proof According to the drive system (2) and the controlled response system (10), we have the error dynamical system

$$\begin{cases} \dot{e}_1 = -e_1 + (\hat{a} - \hat{c})e_2 + \tilde{a}(y_d - x_d), \\ \dot{e}_2 = (\hat{c} - \hat{a})e_1 - e_2 + (\tilde{c} - \tilde{a})x_d + \tilde{c}y_d, \\ \dot{e}_3 = -e_3 - \tilde{b}z_d. \end{cases} \tag{12}$$

Consider a Lyapunov function as

$$V_1(e, \tilde{a}, \tilde{b}, \tilde{c}) = V(e) + \frac{1}{2}(\tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2),$$

where $V(e) = \dfrac{1}{2}(e_1^2 + e_2^2 + e_3^2)$. Taking the time derivative of $V_1(e, \tilde{a}, \tilde{b}, \tilde{c})$ along the trajectories of the error dynamical system (12) leads to

$$\begin{aligned} \frac{\mathrm{d}V_1}{\mathrm{d}t} &= -2V(e) + \tilde{a}[(y_d - x_d)e_1 - x_d e_2 + \dot{\hat{a}}] \\ &\quad + \tilde{b}(-z_d e_3 + \dot{\hat{b}}) + \tilde{c}((x_d + y_d)e_2 + \dot{\hat{c}}) \\ &= -2V(e). \end{aligned} \tag{13}$$

Since $V_1(e, \tilde{a}, \tilde{b}, \tilde{c})$ is a positive definite function and $\dfrac{\mathrm{d}V_1}{\mathrm{d}t}$ is a negative semi-definite function, it follows that the equilibrium points $e = 0$, $\hat{a} = a$, $\hat{b} = b$, $\hat{c} = c$ of the systems (11) and (12) are uniformly stable, i.e. $e(t) \in L_\infty$, $\hat{a} \in L_\infty$, $\hat{b} \in L_\infty$, $\hat{c} \in L_\infty$. From Eq.(13) we can easily show that the square of $e(t)$ is integrable with respect to time, i.e. $e(t) \in L_2$. Next, system (12) implies $\dot{e}(t) \in L_\infty$ for any initial conditions, which in turn implies $e(t) \to 0$ as $t \to \infty$.[12] Furthermore, from the error system, one can easily see that $\ddot{e}(t)$ is bounded, thus using a basic calculus method one can arrive at the result $\lim\limits_{t\to\infty} \dot{e}(t) = 0$, which, together with the above result, implies the conclusion of theorem 2. The proof is completed.

It should be noted that unlike most existing adaptive synchronization methods, our adaptive synchronization method can not only achieve synchronization but also identify the system parameters.

4. Numerical simulation

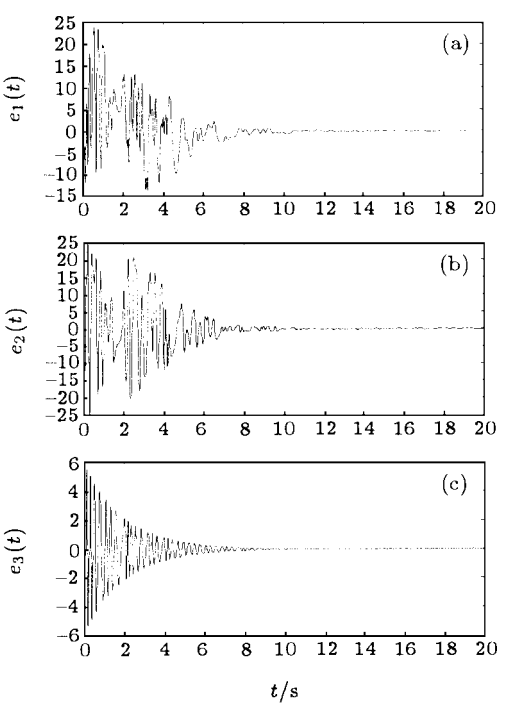


Fig.1. Graphs of synchronization errors varying with time: (a) $e_1(t) = x_r(t) - x_d(t)$; (b) $e_2(t) = y_r(t) - y_d(t)$; (c) $e_3(t) = z_r(t) - z_d(t)$.

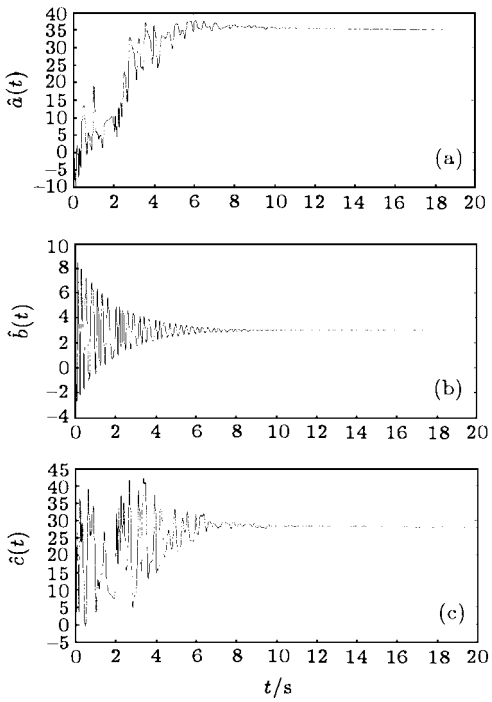


Fig.2. Graphs of parameters identification results: (a) $\hat{a}(t)$; (b) $\hat{b}(t)$; (c) $\hat{c}(t)$.

In order to verify the effectiveness of the proposed method, we suppose that the drive signals are from the Chen system (1) with system parameters a =35, b =3, c =28, and the initial condition (8.0, 9.0, 10.0). We suppose the initial condition of the controlled Chen system (10) is (3.0, 4.0, 5.0) and the unknown parameters have zero initial condition. The numerical simulation shows that parameter identification and adaptive synchronization are achieved successfully. Figures 1 and 2 display the results.

5. Conclusion

In this paper we introduce an adaptive synchronization and parameter identification method for the Chen chaotic system with all the system parameters unknown. With this method one can achieve synchronization and parameter identification simultaneously. The Lyapunov direct method is used to prove the stability of the method. The numerical experiment shows the effectiveness of the proposed method.

It should be noted that, although this paper is focused on the Chen chaotic system, the systematic design process could be used for many other complex dynamical systems with unknown parameters.

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