Vol 11 No 6, June 2002
1009-1963/2002/11(06)/0537-06
Chinese Physics
© 2002 Chin. Phys. Soc.
and IOP Publishing Ltd

Teleportation of a three-particle entangled W state*

Zheng Yi-Zhuang(郑亦庄)[a)b)†], Gu Yong-Jian(顾永建)[b)c)], and Guo Guang-Can(郭光灿)[b)‡]

[a)] *Department of Physics, Wenzhou Normal College, Wenzhou 325027, China*
[b)] *Laboratory of Quantum Communication and Quantum Computation, University of Science and Technology of China, Hefei 230026, China*
[c)] *Department of Physics, Weifang University, Weifang 261043, China*

(Received 17 November 2001; revised manuscript received 6 January 2002)

We have investigated the problem of teleporting a three-particle entangled W state and we propose a scheme based on entanglement swapping to complete the teleportation. We also put forward a scheme for the teleportation of a general W state by using nonmaximally entangled quantum channels. The probability of success of the latter scheme is obtained.

Keywords: teleportation, three-particle entanglement, W state
PACC: 0365, 4250

1. Introduction

Quantum entanglement is one of the most striking features of quantum mechanics and has been used as an essential resource for many applications in quantum information processing, such as teleportation,[1] superdense coding,[2] quantum cryptographic key distribution,[3] entanglement-enhanced communication complexity,[4] and quantum computation.[5] Much attention has been paid to generating entanglement,[6] but for some applications entangled states have to be transported between spatially separated spots; for example, distributed quantum computation[7] needs to exchange data encoded in multiparticle entangled states between different dots of a network. So teleportation of entangled states, especially multiparticle entangled states, is demanded in many schemes of quantum information processing.

As pointed out by Bennett *et al*[1] in their original proposal for quantum teleportation, entanglement can be transferred through teleportation of the state of one of the entangled pairs. This is known as entanglement swapping.[8] Based on this, several schemes for teleportation of entangled states have been presented. Teleportation of a two-particle entangled state in various conditions has been studied by Gorbachev *et al*,[9] Shi *et al*,[10] Ikram *et al*,[11] Lee *et al*[12] and Ye *et al*.[13] Yang and Guo[14] and Lu[15] have proposed schemes for the teleportation of a three-particle entangled state by using three Einstein–Podolsky–Rosen (EPR) pairs and one EPR pair and one Greenberger–Home–Zeilinger (GHZ) triplet, respectively. These schemes can be generated to N-particle entangled states, but all of these are restricted within the simplest form

$$|\Psi\rangle = x\prod_{i=1}^{N}|u_i\rangle \pm y\prod_{i=1}^{N}|u_i^c\rangle\,, \qquad (1)$$

where u_i represents the binary variable, and $u_i^c = 1 - u_i$, x and y are complex numbers with $|x|^2 + |y|^2 = 1$.

For a two-particle system, schemes for teleportation of the entangled state in the form of Eq.(1) can be regarded as general since any other two-particle entangled state can be converted to this form under asymptotic local operations assisted by classical communication (LOCC).[16] For a system of three or more particles, unfortunately recent contributions have shown that there are several inequivalent forms of entanglement under asymptotic LOCC.[17] Schemes presented in Refs.[9]–[15] are not general for teleportation of entangled states of three or more particles. Fortunately Dür *et al*[18] have found that if we just demand that the conversion of states is through stochastic local operations and classical communication (SLOCC),[17]

*Project supported by the National Natural Science Foundation of China (Grant No 19874056), and by the Natural Science Foundation of Zhejiang Province, China (Grant No 100036).
†E-mail: yzzheng@wznc.zj.cn
‡E-mail: gcguo@ustc.edu.cn

that is, through LOCC but without imposing that it has to be achieved with certainty, then any (nontrivial) three-particle entangled state can be converted, by means of SLOCC, into one of two standard forms, namely either the GHZ state[19]

$$|\Psi_{\rm GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \tag{2}$$

or else a second state (the so-called W state)

$$|\Psi_W\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle). \tag{3}$$

This surprising result allows us to teleport an arbitrary three-particle entangled state if only we can carry out teleportation of states $|\Psi_{\rm GHZ}\rangle$ and $|\Psi_W\rangle$. That is, for an arbitrary three-particle entangled state we can first convert it into one of the two states $|\Psi_{\rm GHZ}\rangle$ and $|\Psi_W\rangle$ by means of SLOCC. Then the teleportation operation will be carried out on $|\Psi_{\rm GHZ}\rangle$ or $|\Psi_W\rangle$, after which the states $|\Psi_{\rm GHZ}\rangle$ or $|\Psi_W\rangle$ can be reconverted into the initial entangled state if we wish. As we know that previous work in Refs.[14] and [15] has presented schemes for teleportation of the state $|\Psi_{\rm GHZ}\rangle$, the remaining work is to find a method to teleport the state $|\Psi_W\rangle$. In this paper, we propose a scheme for the teleportation of a three-particle entangled W state $|\Psi_W\rangle$. We show that, if a GHZ triplet and an EPR pair are taken as quantum channels, after two Bell measurements and a Hadamard operation on Alice's side and some unitary operations on Bob's side, the state $|\Psi_W\rangle$ can be teleported completely. Then we show that if the quantum channels are constructed by nonmaximally entangled states and the W state takes a general form $\left|\Psi'_W\right\rangle = (x|001\rangle + y|010\rangle + z|100\rangle)$, where x, y and z are complex numbers and $|x|^2 + |y|^2 + |z|^2 = 1$, the teleportation can still be implemented with certain probability by introducing a collective unitary transformation in Bob's laboratory.

2. Teleportation of a three-particle entangled W state

Suppose Alice want to teleport a three-particle entangled W state

$$|\Psi_W\rangle_{123} = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)_{123} \tag{4}$$

to Bob, and the two other entangled states they share and take as quantum channels are

$$|\Psi_{\rm EPR}\rangle_{45} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{45}, \tag{5}$$

$$|\Psi_{\rm GHZ}\rangle_{678} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{678}. \tag{6}$$

The total initial state is $|\Psi_{\rm t}\rangle = |\Psi_W\rangle_{123}|\Psi_{\rm EPR}\rangle_{45}|\Psi_{\rm GHZ}\rangle_{678}$, where particles 1, 2, 3, 4 and 6 are on Alice's side while 5, 7 and 8 are on Bob's side. To teleport the state of $|\Psi_W\rangle_{123}$ to Bob, using the technique of entanglement swapping, Alice performs two Bell measurements on particles 3, 4 and 1, 6 respectively. There are sixteen possible outcomes

$$\begin{aligned}&{}_{16}\langle\Phi^+|_{34}\langle\Phi^\pm|\Psi_{\rm t}\rangle\\ =&\frac{1}{4\sqrt{3}}(|1000\rangle \pm |0100\rangle + |0011\rangle)_{2578},\end{aligned} \tag{7}$$

$$\begin{aligned}&{}_{16}\langle\Phi^-|_{34}\langle\Phi^\pm|\Psi_{\rm t}\rangle\\ =&\frac{1}{4\sqrt{3}}(|1000\rangle \pm |0100\rangle - |0011\rangle)_{2578},\end{aligned} \tag{8}$$

$$\begin{aligned}&{}_{16}\langle\Psi^+|_{34}\langle\Phi^\pm|\Psi_{\rm t}\rangle\\ =&\frac{1}{4\sqrt{3}}(|1011\rangle \pm |0111\rangle + |0000\rangle)_{2578},\end{aligned} \tag{9}$$

$$\begin{aligned}&{}_{16}\langle\Psi^-|_{34}\langle\Phi^\pm|\Psi_{\rm t}\rangle\\ =&\frac{1}{4\sqrt{3}}(|1011\rangle \pm |0111\rangle - |0000\rangle)_{2578},\end{aligned} \tag{10}$$

$$\begin{aligned}&{}_{16}\langle\Phi^+|_{34}\langle\Psi^\pm|\Psi_{\rm t}\rangle\\ =&\frac{1}{4\sqrt{3}}(|1100\rangle \pm |0000\rangle + |0111\rangle)_{2578},\end{aligned} \tag{11}$$

$$\begin{aligned}&{}_{16}\langle\Phi^-|_{34}\langle\Psi^\pm|\Psi_{\rm t}\rangle\\ =&\frac{1}{4\sqrt{3}}(|1100\rangle \pm |0000\rangle - |0111\rangle)_{2578},\end{aligned} \tag{12}$$

$$\begin{aligned}&{}_{16}\langle\Psi^+|_{34}\langle\Psi^\pm|\Psi_{\rm t}\rangle\\ =&\frac{1}{4\sqrt{3}}(|1111\rangle \pm |0011\rangle + |0100\rangle)_{2578},\end{aligned} \tag{13}$$

$$\begin{aligned}&{}_{16}\langle\Psi^-|_{34}\langle\Psi^\pm|\Psi_{\rm t}\rangle\\ =&\frac{1}{4\sqrt{3}}(|1111\rangle \pm |0011\rangle - |0100\rangle)_{2578},\end{aligned} \tag{14}$$

where $|\Phi^\pm\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$ and $|\Psi^\pm\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$ are Bell states. After these two Bell measurements, the entanglement between particles 1, 2 and 3 has been converted into the entanglement between particles 2, 5, 7 and 8. In order to achieve the three-particle entangled state, Alice carries out a Hadamard operation on particle 2 to separate particle

2 from the entangled state. For instance, if the result of the two Bell measurements is

$$\begin{aligned}&{}_{16}\langle\Psi^+|_{34}\langle\Phi^-|.\Psi_{\rm t}\rangle\\&=\frac{1}{4\sqrt{3}}(|1011\rangle-|0111\rangle+|0000\rangle)_{2578},\end{aligned}\tag{15}$$

the Hadamard operation

$$H|0\rangle=\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle),\tag{16}$$

$$H|0\rangle=\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle),\tag{17}$$

will transform Eq.(15) into

$$\begin{aligned}&\frac{1}{4\sqrt{6}}\left[(|000\rangle+|011\rangle-|111\rangle)_{578}|0\rangle_2\right.\\&\left.+(|000\rangle-|011\rangle-|111\rangle)_{578}|1\rangle_2\right].\end{aligned}\tag{18}$$

Alice measures particle 2. If the result is $|0\rangle_2$, particles 5, 7 and 8 collapse to the state

$$\frac{1}{4\sqrt{6}}(|000\rangle+|011\rangle-|111\rangle)_{578}.\tag{19}$$

Otherwise, the state of particles 5, 7 and 8 is

$$\frac{1}{4\sqrt{6}}(|000\rangle-|011\rangle-|111\rangle)_{578}.\tag{20}$$

Alice informs Bob, by classical communication, of the results of her three measurements. According to the information, Bob can perform some unitary operations on particles 5, 7 and 8 to convert the state of particles 5, 7 and 8 into the desired state $(|001\rangle+|010\rangle+|100\rangle)/\sqrt{3}$. For example, Bob can perform the operation $U=(U_{\rm CNOT})_{57}\,(\sigma_z)_5\,(\sigma_x)_8$ to convert Eq.(19) into the desired state $(|001\rangle+|010\rangle+|100\rangle)/\sqrt{3}$ with a probability $\left(1/4\sqrt{2}\right)^2=1/32$, where$(\sigma_z)_5$ and $(\sigma_x)_8$ are Pauli operations acting on particles 5 and 8 respectively, $(U_{\rm CNOT})_{57}$ is the control-not operation on particles 5 and 7. The other 31 results can be converted into the desired state in the same way. The total probability is unity.

3. Teleportation of a general W state via nonmaximal quantum channels

Now we consider the problem in a somewhat practical way. That is, the three-particle entangled W state is in an unknown form

$$\begin{aligned}|\Psi_W^{'}\rangle_{123}=&(x|001\rangle+y|010\rangle+z|100\rangle)_{123},\\&|x|^2+|y|^2+|z|^2=1,\end{aligned}\tag{21}$$

and the quantum channels are in nonmaximally entangled states

$$\begin{aligned}&|\Psi\rangle_{45}=(a|00\rangle+b|11\rangle)_{45},\\&|a|>|b|,\quad |a|^2+|b|^2=1;\end{aligned}\tag{22}$$

$$\begin{aligned}&|\Psi\rangle_{678}=(c|000\rangle+d|111\rangle)_{678,}\\&|c|>|d|,\quad |c|^2+|d|^2=1.\end{aligned}\tag{23}$$

We show that in this case the teleportation can still be completed by using entanglement swappings, but with a probability of less than one.

The total initial state now is $|\Psi_{\rm t}^{'}\rangle=|\Psi_W^{'}\rangle_{123}\,|\Psi\rangle_{45}|\Psi\rangle_{678}$. After Alice performs Bell measurements on particles 3, 4 and 1,6 separately, the possible sixteen outcomes are

$$\begin{aligned}&{}_{16}\langle\Phi^+|_{34}\langle\Phi^\pm|\,\Psi_{\rm t}^{'}\rangle\\&=\frac{1}{2}(\pm bcx|0100\rangle+acy|1000\rangle+adz|0011\rangle)_{2578},\end{aligned}\tag{24}$$

$$\begin{aligned}&{}_{16}\langle\Phi^-|_{34}\langle\Phi^\pm|\,\Psi_{\rm t}^{'}\rangle\\&=\frac{1}{2}(\pm bcx|0100\rangle+acy|1000\rangle-adz|0011\rangle)_{2578},\end{aligned}\tag{25}$$

$$\begin{aligned}&{}_{16}\langle\Psi^+|_{34}\langle\Phi^\pm|\,\Psi_{\rm t}^{'}\rangle\\&=\frac{1}{2}(\pm bdx|0111\rangle+ady|1011\rangle+acz|0000\rangle)_{2578},\end{aligned}\tag{26}$$

$$\begin{aligned}&{}_{16}\langle\Psi^-|_{34}\langle\Phi^\pm|\,\Psi_{\rm t}^{'}\rangle\\&=\frac{1}{2}(\pm bdx|0111\rangle+ady|1011\rangle-acz|0000\rangle)_{2578},\end{aligned}\tag{27}$$

$$\begin{aligned}&{}_{16}\langle\Phi^+|_{34}\langle\Psi^\pm|\,\Psi_{\rm t}^{'}\rangle\\&=\frac{1}{2}(\pm acx|0000\rangle+bcy|1100\rangle+bdz|0111\rangle)_{2578},\end{aligned}\tag{28}$$

$$\begin{aligned}&{}_{16}\langle\Phi^-|_{34}\langle\Psi^\pm|\,\Psi_{\rm t}^{'}\rangle\\&=\frac{1}{2}(\pm acx|0000\rangle+bcy|1100\rangle-bdz|0111\rangle)_{2578},\end{aligned}\tag{29}$$

$$\begin{aligned}&{}_{16}\langle\Psi^+|_{34}\langle\Psi^\pm|\,\Psi_{\rm t}^{'}\rangle\\&=\frac{1}{2}(\pm adx|0011\rangle+bdy|1111\rangle+bcz|0100\rangle)_{2578},\end{aligned}\tag{30}$$

$$\begin{aligned}&{}_{16}\langle\Psi^-|_{34}\langle\Psi^\pm|\,\Psi_{\rm t}^{'}\rangle\\&=\frac{1}{2}(\pm adx|0011\rangle+bdy|1111\rangle-bcz|0100\rangle)_{2578}.\end{aligned}\tag{31}$$

Alice carries out a Hadamard operation on particle 2 to separate it from the entanglement of particles 2, 5, 7 and 8, then she informs Bob of the result of her measurement via classical communication. For instance, if the state after the two Bell measurements is

$$\begin{aligned}&{}_{16}\langle\Psi^-|_{34}\langle\Phi^+|.\Psi_{\rm t}'\rangle\\ =&\frac{1}{2}\left(bdx\,|0111\rangle+ady\,|1011\rangle-acz\,|0000\rangle\right)_{2578},\end{aligned}\quad(32)$$

the Hadamard operation on particle 2 (Eqs.(16) and (17)) will transform it to

$$\begin{aligned}&\frac{1}{2\sqrt{2}}\left[\left(bdx\,|111\rangle+ady\,|011\rangle-acz\,|000\rangle\right)_{578}|0\rangle_2\right.\\&\left.+\left(bdx\,|011\rangle-ady\,|011\rangle-acz\,|000\rangle\right)_{578}|1\rangle_2\right].\end{aligned}\quad(33)$$

Alice measures particle 2. If the result is $|0\rangle_2$, particles 5, 7 and 8 collapse into the state

$$\frac{1}{2\sqrt{2}}\left(bdx\,|111\rangle+ady\,|011\rangle-acz\,|000\rangle\right)_{578}.\quad(34)$$

Otherwise, the state will be

$$\frac{1}{2\sqrt{2}}\left(bdx\,|011\rangle-ady\,|011\rangle-acz\,|000\rangle\right)_{578}.\quad(35)$$

After that, Alice informs Bob of her measurement results via a classical channel. Next, Bob will try to reconstruct the original state $|\Psi_W'\rangle$ with his particles 5, 7 and 8. In the scheme we described above for teleportation of the standard W state $|\Psi_W\rangle$, this is achieved by performing some appropriate unitary transformations on Bob's particles. In the present case, the states of particles 5, 7 and 8 in Eqs.(24)–(31) cannot be rotated back to the desired state without having any knowledge of the state parameters x, y and z. Unfortunately, x, y and z are supposed to be unknown. So Bob cannot reconstruct the original state just by performing some unitary transformations on his particles 5, 7 and 8.

It is well known that any operation in quantum mechanics can be represented by a unitary evolution together with a measurement (a unitary-reduction operation).[20,21] Based on this idea, probabilistic teleportation was proposed.[22] Though a unitary transformation suggested in Ref.[21] can be used to probabilistically teleport a two-particle entangled state[10] and a three-particle entangled GHZ state,[15] it fails for a three-particle entangled W state. Here we provide the following strategy.

Firstly, Bob needs to establish a correspondence such that x, y and z can correspond to $|001\rangle_{578}$, $|010\rangle_{578}$, and $|100\rangle_{578}$, respectively. This can be achieved by performing a unitary transformation U_1 on particles 5, 7 and 8. For instance, a unitary transformation $(U_1)_{578}=(U_{\rm SWAP})_{58}\,(U_{\rm CNOT})_{57}\,(\sigma_x)_8$ will transform the state $(H)_2({}_{16}\langle\Psi^-|_{34}\langle\Phi^+|\,\Psi_{\rm t}')_{2578}$ in Eq.(33) into

$$\begin{aligned}&(U_1)_{578}\,(H)_2\left({}_{16}\langle\Psi^-|_{34}\langle\Phi^+|.\Psi_{\rm t}'\rangle\right)_{2578}\\=&\frac{1}{2\sqrt{2}}\left[\left(bdx\,|001\rangle+ady\,|010\rangle-acz\,|100\rangle\right)_{578}|0\rangle_2\right.\\&\left.+\left(bdx\,|001\rangle-ady\,|010\rangle-acz\,|100\rangle\right)_{578}|1\rangle_2\right],\end{aligned}\quad(36)$$

where the SWAP operation $U_{\rm SWAP}$ is defined as

$$U_{\rm SWAP}\,|u_i\rangle_A\otimes|u_j\rangle_B=|u_j\rangle_A\otimes|u_i\rangle_B.\quad(37)$$

The unitary transformations corresponding to the states of particles 5, 7 and 8 are given in Table 1.

Table 1. Unitary transformations U_1 corresponding to the states of particles 5, 7 and 8.

States of particles 5,7 and 8	Unitary transformations U_1
${}_{16}\langle\Phi^\pm|_{34}\langle\Phi^\pm|\,\Psi_{\rm t}'\rangle$	$(U_{\rm SWAP})_{58}\,(U_{\rm CNOT})_{57}\,(\sigma_x)_7$
${}_{16}\langle\Psi^\pm|_{34}\langle\Phi^\pm|\,\Psi_{\rm t}'\rangle$	$(U_{\rm SWAP})_{58}\,(U_{\rm CNOT})_{57}\,(\sigma_x)_8$
${}_{16}\langle\Phi^\pm|_{34}\langle\Psi^\pm|\,\Psi_{\rm t}'\rangle$	$(U_{\rm SWAP})_{58}\,(U_{\rm CNOT})_{57}\,(\sigma_x)_5\,(\sigma_x)_7$
${}_{16}\langle\Psi^\pm|_{34}\langle\Psi^\pm|\,\Psi_{\rm t}'\rangle$	$(U_{\rm SWAP})_{58}\,(U_{\rm CNOT})_{57}\,(\sigma_x)_5\,(\sigma_x)_8$

Secondly, Bob introduces an auxiliary particle A with its initial state $|0\rangle_a$ and performs a collective unitary transformation on particles 5, 7 and A. In order for Bob to reconstruct the original state, under the basis $\{|000\rangle_{57a}$, $|010\rangle_{57a}$, $|100\rangle_{57a}$, $|110\rangle_{57a}$, $|001\rangle_{57a}$, $|011\rangle_{57a}$, $101\rangle_{57a}$, $|111\rangle_{57a}\}$, a unitary transformation (an 8×8 matrix) may take the form

$$U_2=\begin{pmatrix}A_1 & A_2\\ A_2 & -A_1\end{pmatrix},\quad(38)$$

where A_i $(i = 1, 2)$ is a 4×4 matrix, and may be expressed, respectively, as

$$A_1 = \mathrm{diag}(a_1, a_2, a_3, a_4), \tag{39}$$

$$A_2 = \mathrm{diag}\left(\sqrt{1-a_1^2}, \sqrt{1-a_2^2}, \sqrt{1-a_3^2}, \sqrt{1-a_4^2}\right), \tag{40}$$

where a_i $(i = 1, 2, 3, 4,$ and $|a_i| \leq 1)$ depends on the state of particles 5 and 7. For Eq.(36), if we take

$$A_1 = \mathrm{diag}\left(1, \pm\frac{b}{a}, -\frac{bd}{ac}, \frac{d}{c}\right), \tag{41}$$

$$A_2 = \mathrm{diag}\left(0, \sqrt{1-\left(\frac{b}{a}\right)^2}, \sqrt{1-\left(\frac{bd}{ac}\right)^2}, \sqrt{1-\left(\frac{d}{c}\right)^2}\right), \tag{42}$$

the unitary transformation U_2 will transform the states

$$\frac{1}{2\sqrt{2}}\left(bdx\,|001\rangle \pm ady\,|010\rangle - acz\,|100\rangle\right)_{578} \otimes |0\rangle_a \tag{43}$$

into

$$\begin{aligned}&\frac{1}{2\sqrt{2}} bd\left(x\,|001\rangle + y\,|010\rangle + z\,|100\rangle\right)_{578}|0\rangle_a \\ &+ \frac{1}{2\sqrt{2}}\Big(\pm\sqrt{a^2-b^2}\;dy|010\rangle \\ &- \sqrt{(ac)^2-(bd)^2}\;z|100\rangle\Big)_{578}|1\rangle_a.\end{aligned} \tag{44}$$

In fact, U_1 and U_2 can be incorporated into a single transformation $U = U_2(U_1 \otimes I_a)$. Table 2 gives the values of a_1, a_2, a_3 and a_4 corresponding to each state.

Table 2. a_1, a_2, a_3 and a_4 of unitary transformations U_2 corresponding to the states of particles 5, 7 and 8.

States of particles 5, 7 and 8	a_1	a_2	a_3	a_4
$U_1H_2^{(\pm)}\left({}_{16}\langle\Phi^+|_{34}\langle\Phi^\pm|\,\Psi_t'\rangle\right)$	$\pm d/c$	$(\pm)\,bd/(ac)$	b/a	1
$U_1H_2^{(\pm)}\left({}_{16}\langle\Phi^-|_{34}\langle\Phi^\pm|\,\Psi_t'\rangle\right)$	$\pm d/c$	$(\pm)\,bd/(ac)$	$-b/a$	1
$U_1H_2^{(\pm)}\left({}_{16}\langle\Psi^+|_{34}\langle\Phi^\pm|\,\Psi_t'\rangle\right)$	± 1	$(\pm)\,b/a$	$bd/(ac)$	d/c
$U_1H_2^{(\pm)}\left({}_{16}\langle\Psi^-|_{34}\langle\Phi^\pm|\,\Psi_t'\rangle\right)$	± 1	$(\pm)\,b/a$	$-bd/(ac)$	d/c
$U_1H_2^{(\pm)}\left({}_{16}\langle\Phi^+|_{34}\langle\Psi^\pm|\,\Psi_t'\rangle\right)$	$\pm bd/(ac)$	$(\pm)\,d/c$	1	b/a
$U_1H_2^{(\pm)}\left({}_{16}\langle\Phi^-|_{34}\langle\Psi^\pm|\,\Psi_t'\rangle\right)$	$\pm bd/(ac)$	$(\pm)\,d/c$	-1	b/a
$U_1H_2^{(\pm)}\left({}_{16}\langle\Psi^+|_{34}\langle\Psi^\pm|\,\Psi_t'\rangle\right)$	$\pm b/a$	$(\pm)\,1$	d/c	$bd/(ac)$
$U_1H_2^{(\pm)}\left({}_{16}\langle\Psi^-|_{34}\langle\Psi^\pm|\,\Psi_t'\rangle\right)$	$\pm b/a$	$(\pm)\,1$	$-d/c$	$bd/(ac)$

* $H_2^{(\pm)}$ denotes the Hadamard operation on particle 2, + and − in $(\pm)$ correspond to the results $|0\rangle_2$ and $|1\rangle_2$ of the measurement on particle 2.

Finally, Bob measures the state of particle A. For Eq.(44), if the result is $|1\rangle_a$, teleportation fails; if the result is $|0\rangle_a$, the state of particles 5, 7 and 8 collapses into

$$\frac{1}{2\sqrt{2}} bd\left(x\,|001\rangle + y\,|010\rangle + z\,|100\rangle\right)_{578}, \tag{45}$$

so teleportation is successfully realized, with the probability being $(1/8)\,|bd|^2$. It can be easily verified that 32 types of state have the same results, therefore the total probability of successful teleportation is $(1/8)\,|bd|^2 \times 32 = 4|bd|^2$. For the maximally entangled quantum channels, $|a| = |b| = |c| = |d| = 1/\sqrt{2}$, the total probability reaches unity.

4. Conclusions

We have investigated the problem of teleportation of a three-particle entangled W state and found that, by taking an EPR pair and a GHZ triplet as quantum channels, the standard entanglement swapping operations can be used to implement the teleportation completely. Furthermore, we have considered the three entangled particles in a general W state and taken nonmaximally entangled states as quantum channels. We have found that, by introducing a collective unitary transformation expressed as an 8×8 matrix, the teleportation can be implemented with a probability $4|bd|^2$ where $|b|^2 \leq 1/2$, $|d|^2 \leq 1/2$. According to a result that Dür *et al* obtained in Ref.[17] re-

cently, any (nontrivial) three-particle entangled state can be converted, by means of SLOCC, into one of two standard forms—either the GHZ state or the W state. Then, combining our scheme with the previous schemes[14,15] for teleportation of the GHZ state, teleportation of an arbitrary three-particle entangled state can be completed by using either maximally or nonmaximally entangled quantum channels.

References

[1] Bennett C H, Brassard G, Crepeau C, Jozsa R, Peres A and WoottersW K 1993 *Phys. Rev. Lett.* **70** 1895
Bouwmeester D, Pan J W, Matter K, Eibl M, Weinfuter H and Zeilinger A 1997 *Nature* **390** 575

[2] Bennett C H and Wiesner S J 1992 *Phys. Rev. Lett.* **69** 2881

[3] Bennett C H and Brassard G 1984 in *Proc. IEEE Int. Conference on Computers, Systems and Signal Processing, IEEE* (New York)

[4] Cleve R and Buhrman H 1997 *Preprint* quant-ph/9704026
Buhrman H, Cleve R and van Dam W 1997 *Preprint* quant-ph/9705033

[5] DiVincenzo D 1995 *Science* **270** 255
Ekert A and Jozsa R 1996 *Rev. Mod. Phys.* **68** 733

[6] Sackett C A *et al* 2000 *Nature* **404** 256
Rauschenbeutel A *et al* 2000 *Science* **288** 2024

[7] Grover L K 1997 *Preprint* quant-ph/9704012
Cirac J I, Ekert A, Huelga S F and Macchiavello C 1999 *Phys. Rev.* A **59** 064249

[8] Zukowski M, Zeilinger A, Horne M A and Ekert A 1993 *Phys. Rev. Lett.* **71** 4287
Pan J W, Bouwmeester D, Weinfurter H and Zeilinger A 1998 *Phys. Rev. Lett.* **80** 3891

[9] Gorbachev V N and Trubilko A I 1999 *Preprint* quant-ph/9906110

[10] Shi B S, Jiang Y K and Guo G C 2000 *Phys. Lett.* A **268** 161

[11] Ikram M, Zhu S Y and Zubairy M S 2000 *Phys. Rev.* A **62** 022307

[12] Lee J and Kim M S 2000 *Phys. Rev. Lett.* **84** 4236

[13] Ye L, Yao C M and Guo G C 2001 *Chin. Phys.* **10** 1001

[14] Yang C P and Guo G C 1999 *Chin. Phys. Lett.* **16** 628

[15] Lu H 2001 *Chin. Phys. Lett.* **18** 1004

[16] Bennett C H, Bernstein H J, Popescu S and Schumacher B 1995 *Preprint* quant-ph/9511030

[17] Bennett C H, Popescu S, Rohrlich D, Smolin J A and Thapliyal A V 1999 *Preprint* quant-ph/9908073

[18] Dür W, Vidal G and Cirac J I 2000 *Phys. Rev.* A **62** 062314

[19] Greenberger M, Horne M A, Shimony A and Zeilinger A 1990 *Am. J. Phys.* **58** 1131
Bouwmeester D *et al* 1999 *Phys. Rev. Lett.* **82** 1345

[20] Kraus K 1983 *States, Effects and Operations* (Berlin: Springer-Verlag)

[21] Duan L M and Guo G C 1998 *Phys. Rev. Lett.* **80** 4999

[22] Li W L, Li C F and Guo G C 2000 *Phys. Rev.* A **61** 034301