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Implementation of nonlocal quantum swap operation on two entangled pairs*

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We propose a scheme for the implementation of nonlocal quantum swap operation on two spatially separated entangled pairs and we show that the operation can swap two qubits of these entangled pairs. We discuss the resources of the entangled qubits and classical communication bits required for the optimal implementation of the nonlocal quantum swap operation. We also put forward a scheme for probabilistic implementation of nonlocal swap operation via a nonmaximally entangled quantum channel. The probability of a successful nonlocal swap operation is obtained by introducing a collective unitary transformation.

Keywords: quantum nonlocality, swap operation, nonmaximally entangled
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1. Introduction

The most essential features of quantum information and computation are making use of quantum coherence and quantum nonlocality to realize information processing. By using quantum coherence the parallel computation can be performed, and it gives a quantum computer marvellous power in some algorithms.[1,2] Moreover, the quantum nonlocality, i.e. there are nonlocal correlations among quantum systems, plays a central role in quantum information. Many novel results have been obtained by using quantum nonlocality of entangled particles, such as the protocols of superdense coding,[3] quantum teleportation,[4,5] and telecloning.[6] However, most research on quantum nonlocality has been devoted to the issue of nonlocality of quantum states. In fact, another important issue is that of nonlocality of quantum operations. This situation arises, for example, in the context of distributed quantum computation,[7] where nonlocal operations between different quantum computers are required. Two other examples are quantum network communication[8] and the production of multiparticle entangled states.[9] For these alluring potential applications, some fundamental research on nonlocal quantum operations has been presented.[8,10,11]

Although it is well known that control-not (CNOT) gates together with a set of general one-qubit gates are sufficient to implement any multiqubit unitary transformation.[12,13] Yet some other gates that can achieve the implementation of two or more logic operations simultaneously are important for practical applications, because they would be useful in reducing operational time in circuits that require a large number of logical operations. The swap operation is just one of these efficient gates. It is used to exchange the states of a pair of qubits completely, and might be useful during the course of a computation when qubits need to be permuted. Experimental implementation of the local swap operation in NMR has been presented by Madi *et al*[14] and Linden *et al.*[15] Protocols for the implementation of the swap operation in a quantum dot quantum computer have been offered by Sousa *et al.*[16] In this paper, we investigate how the nonlocal swap operation can be implemented by using

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local operations and classical communication, and previously shared entanglement. In particular, the nonlocal swap operation on two entangled pairs is discussed in detail, and we show that this operation can swap two qubits of two entangled pairs completely. The consumption of resources of ebits (maximally entangled pairs of qubits) and cbits (bits of classical communication) is compared for two protocols of implementation of nonlocal swap operation. Finally, we propose a scheme of implementation of nonlocal swap operation via a nonmaximally entangled quantum channel, and the probability of successful nonlocal swap operation is given.

2. Nonlocal swap operation

The swap operation is defined by

$$U_{\rm swap}|\psi\rangle_A \otimes |\phi\rangle_B = |\phi\rangle_A \otimes |\psi\rangle_B. \tag{1}$$

For a two-qubit system, the swap operation may be constructed by concatenating three CNOT operations

$$\begin{aligned} &U_{\rm swap}(\psi;\phi)\\ =&U_{\rm CNOT}(\phi;\psi)U_{\rm CNOT}(\psi;\phi)U_{\rm CNOT}(\phi;\psi)\\ =&\begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 \end{pmatrix}. \end{aligned} \tag{2}$$

It is not difficult to check that this unitary transformation has the effect of interchanging the states of a pair of qubits completely. For example, suppose Alice and Bob prepare two entangled pairs locally

$$a|00\rangle_{A_1A_2}+b|11\rangle_{A_1A_2} \text{ and } c|00\rangle_{B_1B_2}+d|11\rangle_{B_1B_2}, \tag{3}$$

where Alice's particles are labelled A_1 and A_2 and Bob's are B_1 and B_2. Now performing the swap operation represented by Eq.(2) on particles A_2 and B_2, the straightforward calculation gives

$$\begin{aligned} &U_{\rm swap}(A_2;B_2)[(a|00\rangle_{A_1A_2}+b|11\rangle_{A_1A_2})\\ &\otimes(c|00\rangle_{B_1B_2}+d|11\rangle_{B_1B_2})]\\ =&(a|00\rangle_{A_1B_2}+b|11\rangle_{A_1B_2})\\ &\otimes(c|00\rangle_{B_1A_2}+d|11\rangle_{B_1A_2}). \end{aligned} \tag{4}$$

For the case of local operation we have completed the required operation. But if Alice and Bob are situated far from each other, how could they perform the operations on to each other's particles to carry out the desired interchange? In fact, if Alice and Bob share some prior entanglement and are allowed to communicate with classical bits, the nonlocal swap operation can be accomplished by two entanglement swapping[17,18] operations.

To show this, suppose Alice and Bob previously share two ebits

$$\begin{aligned} |E_1\rangle_{C_1D_1} &= \frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)_{C_1D_1}\\ \text{and } |E_2\rangle_{C_2D_2} &= \frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)_{C_2D_2}, \end{aligned} \tag{5}$$

and the initial entangled pairs are

$$\begin{aligned} |A\rangle_{A_1A_2} &= (a|00\rangle+b|11\rangle)_{A_1A_2}\\ \text{and } |B\rangle_{B_1B_2} &= (c|00\rangle+d|11\rangle)_{B_1B_2}. \end{aligned} \tag{6}$$

Firstly, Alice employs ebit $|E_1\rangle_{C_1D_1}$ and performs entanglement swapping on A_2 and C_1, which will transfer A_2 to D_1 from her side to Bob's side

$$\begin{aligned} &|A\rangle_{A_1A_2}|E_1\rangle_{C_1D_1}|B\rangle_{B_1B_2} \longmapsto |A\rangle_{A_1D_1}|B\rangle_{B_1B_2}\\ =&(a|00\rangle+b|11\rangle)_{A_1D_1}(c|00\rangle+d|11\rangle)_{B_1B_2}. \end{aligned} \tag{7}$$

Then Bob performs a local swap operation on D_1 and B_2, as shown in Eq.(4), to give

$$\begin{aligned} &|A\rangle_{A_1D_1}|B\rangle_{B_1B_2} \longmapsto |A\rangle_{A_1B_2}|B\rangle_{B_1D_1}\\ =&(a|00\rangle+b|11\rangle)_{A_1B_2}(c|00\rangle+d|11\rangle)_{B_1D_1}. \end{aligned} \tag{8}$$

Finally, Bob employs another ebit $|E_2\rangle_{C_2D_2}$ to perform another entanglement swapping on B_2 and C_2 to swap B_2 to D_2 from his side to Alice's side

$$\begin{aligned} &|A\rangle_{A_1B_2}|E_2\rangle_{C_2D_2}|B\rangle_{B_1D_1} \longmapsto |A\rangle_{A_1D_2}|B\rangle_{B_1D_1}\\ =&(a|00\rangle+b|11\rangle)_{A_1D_2}(c|00\rangle+d|11\rangle)_{B_1D_1}. \end{aligned} \tag{9}$$

Note that A_2 and B_2 have been transferred to D_1 and D_2 in the entanglement swappings, respectively. Replacing the subscripts D_1 and D_2 in Eq.(9) by A_2 and B_2, the nonlocal swap operation is completed, that is

$$|A\rangle_{A_1A_2}|B\rangle_{B_1B_2} \longmapsto |A\rangle_{A_1B_2}|B\rangle_{B_1A_2} \tag{10}$$

or

$$\begin{aligned} &(a|00\rangle+b|11\rangle)_{A_1A_2}(c|00\rangle+d|11\rangle)_{B_1B_2} \longmapsto\\ &(a|00\rangle+b|11\rangle)_{A_1B_2}(c|00\rangle+d|11\rangle)_{B_1A_2}. \end{aligned} \tag{11}$$

We have thus shown how to implement a nonlocal swap operation and seen that it consumes two ebits and two cbits in each direction.

It is remarkable that if we implement the nonlocal swap operation with three CNOT operations one after the other as shown in Eq.(2), then three ebits and three cbits in each direction are required since to implement a nonlocal CNOT operation at least one ebit and one cbit in each direction are required.[10] Furthermore, it has been proven[19] that the upper bound of the resources required to carry out an arbitrary nonlocal unitary operation efficiently is two ebits and four cbits. So it is more economical to use the nonlocal swap gate in situations that need nonlocal state interchange than to use the ordinary gate array that consists of a set of nonlocal CNOT gates and local unitary operations. This may be useful in distributed quantum computers and other quantum information processing.

3. Probabilistic nonlocal swap operation

In realistic conditions, instead of maximally entangled states, Alice and Bob may share nonmaximally ancillary entangled states. This means that the quantum channels for swapping will be imperfect, the states to be swapped will be distorted, and the protocol described above for the nonlocal swap operation will not work well.

Suppose the ancillary nonmaximally entangled pairs are

$$|E_1\rangle_{C_1D_1} = (\alpha|00\rangle + \beta|11\rangle)_{C_1D_1},$$
$$|\alpha|^2 + |\beta|^2 = 1 \qquad |\alpha| > |\beta|, \tag{12}$$
$$|E_2\rangle_{C_2D_2} = (\gamma|00\rangle + \delta|11\rangle)_{C_2D_2},$$
$$|\gamma|^2 + |\delta|^2 = 1 \qquad |\gamma| > |\delta|; \tag{13}$$

and the initial states are still $|A\rangle_{A_1A_2}$ and $|B\rangle_{B_1B_2}$ giving in Eq.(6). When Alice employs $|E_1\rangle_{C_1D_1}$ and performs a Bell measurement on A_2 and C_1 to teleport A_2 to Bob, Bob will receive the corresponding states as follows

$$_{A_2C_1}\langle\phi^{\pm}|\Psi\rangle_{A_1A_2C_1D_1} = \frac{1}{\sqrt{2}}(a\alpha|00\rangle \pm b\beta|11\rangle)_{A_1D_1}, \tag{14}$$

$$_{A_2C_1}\langle\psi^{\pm}|\Psi\rangle_{A_1A_2C_1D_1} = \frac{1}{\sqrt{2}}(a\beta|01\rangle \pm b\alpha|10\rangle)_{A_1D_1}; \tag{15}$$

where

$$|\phi^{\pm}\rangle_{A_2C_1} = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)_{A_2C_1}, \tag{16}$$

$$|\psi^{\pm}\rangle_{A_2C_1} = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)_{A_2C_1}; \tag{17}$$

are Bell states of A_2 and C_1, and

$$\begin{aligned}|\Psi\rangle_{A_1A_2C_1D_1} =& |A\rangle_{A_1A_2}|E_1\rangle_{C_1D_1}\\ =& (a\alpha|0000\rangle + a\beta|0011\rangle\\ &+ b\alpha|1100\rangle + b\beta|1111\rangle)_{A_1A_2C_1D_1}.\end{aligned}$$

After performing Bell measurement, Alice informs Bob of her result with classical communication, but Bob finds that he cannot evolve the state to the initial state which Alice wants to share with him, because the states given in Eqs.(14) and (15) include the parameters of the imperfect quantum channel, α and β. For example, Alice informs Bob that her measurement result is $|\phi^+\rangle_{A_2C_1}$, then the entangled state of A_1 and D_1 is not $(a|00\rangle + b|11\rangle)_{A_1D_1}$ but $(a\alpha|00\rangle + b\beta|11\rangle)_{A_1D_1}/\sqrt{2}$. In order to distill the former from the latter, an auxiliary qubit with the original state $|0\rangle_a$ is introduced, and a collective unitary transformation based on the basis $\{|00\rangle, |10\rangle, |01\rangle, |11\rangle\}_{D_1a}$ is constructed[20]

$$U_1 = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ \sin\theta & 0 & -\cos\theta & 0 \end{pmatrix}, \tag{18}$$

where $\cos\theta = \beta/\alpha$. Bob applies U_1 on the qubits D_1 and $|0\rangle_a$, the state $(a\alpha|00\rangle + b\beta|11\rangle)_{A_1D_1}/\sqrt{2}$ is transformed to the result

$$\frac{1}{\sqrt{2}}\Big[\beta(a|00\rangle + b|11\rangle)_{A_1D_1}|0\rangle_a + \alpha a\sqrt{1-\frac{\beta^2}{\alpha^2}}|01\rangle_{A_1D_1}|1\rangle_a\Big]. \tag{19}$$

Then a measurement on the auxiliary qubit follows. If the result is $|1\rangle_a$, teleportation fails and no information about the initial state is left. If $|0\rangle_a$ is obtained, the teleportation is successfully accessed, and Bob receives the state $(a|00\rangle + b|11\rangle)_{A_1D_1}$ with the probability $p = |\beta|^2/2$. If Alice's measurement result is $|\phi^-\rangle_{A_2C_1}$, Bob's result is

$$\frac{1}{\sqrt{2}}\Big[\beta(a|00\rangle - b|11\rangle)_{A_1D_1}|0\rangle_a + \alpha a\sqrt{1-\frac{\beta^2}{\alpha^2}}|01\rangle_{A_1D_1}|1\rangle_a\Big], \tag{20}$$

Bob obtains the state $(a|00\rangle - b|11\rangle)_{A_1D_1}$ still with the probability $p = |\beta|^2/2$.

While if Alice's result is $|\psi^{\pm}\rangle_{A_2C_1}$, Bob should employ another unitary transformation U_2

$$U_2 = \begin{pmatrix} 0 & \cos\theta & \sin\theta & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & \sin\theta & -\cos\theta & 0 \end{pmatrix} \tag{21}$$

to transform $(a\beta|01\rangle \pm b\alpha|10\rangle)_{A_1D_1}/\sqrt{2}$ to

$$\frac{1}{\sqrt{2}}\Big[\beta(a|00\rangle \pm b|11\rangle)_{A_1D_1}|0\rangle_a \mp \alpha b\sqrt{1-\frac{\beta^2}{\alpha^2}}|11\rangle_{A_1D_1}|1\rangle_a\Big]. \tag{22}$$

Because the probability of successful teleportation of each transformation is $|\beta|^2/2$, the total probability of successful teleportation from Alice to Bob is $P_1 = 4\times|\beta|^2/2 = 2|\beta|^2$. In the same way, Bob employs a nonmaximally entangled pair $|E_2\rangle_{C_2D_2} = (\gamma|00\rangle + \delta|11\rangle)_{C_2D_2}$ to teleport qubit B_2 to Alice after he swaps B_2 and D_1 locally. The total probability of teleportation from Bob to Alice is $P_2 = 4\times|\delta|^2/2 = 2|\delta|^2$. Hence the probability of successful nonlocal swap operation on two qubits when the quantum channel is nonmaximally entangled is $P = P_1P_2 = 4|\beta|^2|\delta|^2$.

We thus see that, by using an auxiliary qubit and performing local unitary operations on each side, Alice and Bob are able to complete a nonlocal swap operation via nonmaximally entangled quantum channels with certain probability $P = 4|\beta|^2|\delta|^2$. This will be useful in some practical cases of distributed quantum information processing where the quantum channels are nonmaximally entangled.

4. Conclusions

In conclusion, we have investigated the nonlocal swap operation starting from the local situation. We have proposed a scheme based on entanglement swapping and local swap operations to implement the nonlocal swap operation. We have found that the resources required for accomplishing the scheme are different from the case when the scheme employs three concatenating nonlocal CNOT operations to complete the same task. That is, three ebits and three cbits in each direction are required by the latter and the former requires only two ebits and two cbits in each direction. This will be useful in situations where a large number of nonlocal permutations of states are needed, such as distributed quantum computation and quantum network communication.

We have also investigated the nonlocal swap operation via a nonmaximally entangled quantum channel. We have found that the nonlocal swap operation can be implemented by introducing an auxiliary qubit and performing a local unitary transformation on the auxiliary qubit and the swapping qubit to rectify the distorted state with certain probability. The probability is $4|\beta|^2|\delta|^2$, where β and δ are the smaller coefficients of the two ancillary entangled pairs given in Eqs.(12) and (13), respectively.

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References

[1] Deutsch D 1985 *Proc. R. Soc.* A **400** 97
[2] Shor P 1994 *Proc. 35th Annual Symp. on the Foundations of Computer Science* p 124
[3] Bennett C H and Wiesner S J 1992 *Phys. Rev. Lett.* **69** 2881
[4] Bennett C H, Brassard G, Crepeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
[5] Bouwmeester D, Pan J W, Matter K, Eibl M, Weinfuter H and Zeilinger A 1997 *Nature* **390** 575
[6] Murao M, Jonathan D, Plenio M B and Vedral V 1999 *Phys. Rev.* A **59** 156
[7] Grover L K 1997 *Preprint* quant-ph/9704012
Cirac J I, Ekert A, Huelga S F and Macchiavello C 1999 *Phys. Rev.* A **59** 064249
[8] Chefles A, Gilson C R and Barnett S M 2001 *Phys. Rev.* A **63** 2314
[9] Bose S, Vedral V and Knight P L 1998 *Phys. Rev.* A **57** 822
[10] Eisert J, Jacobs K, Papadopoulos P and Plenio M B 2000 *Phys. Rev.* A **62** 052317
[11] Collins D, Linden N and Popescu S 2001 *Phys. Rev.* A **64** 03230322
[12] Barenco A, Bennett C H *et al* 1995 *Phys. Rev.* A **52** 3457
[13] Dorai K, Arvind and Kumar A 2000 *Phys. Rev.* A **61** 042306
[14] Madi Z L, Bruschweiler R and Ernst R R 1998 *J. Chem. Phys.* **109** 10603
[15] Linden N, Barjat N, Kupce E and Freeman R 1999 *Chem. Phys. Lett.* **307** 198
[16] Sousa R, Hu X D and Sarma S 2001 *Preprint* quant-ph/0103410
[17] Zukowski M, Zeilinger A, Horne M A and Ekert A 1993 *Phys. Rev. Lett.* **71** 4287
[18] Pan J W, Bouwmeester D, Weinfurter H and Zeilinger A 1998 *Phys. Rev. Lett.* **80** 3891
[19] Huelga S F, Vaccaro J A, Chefles A and Plenio M B 2001 *Phys. Rev.* A **63** 042303
[20] Li W L, Li C F and Guo G C 2000 *Phys. Rev.* A **61** 034301