Vol 11 No 4, April 2002
1009-1963/2002/11(04)/0383-06

Chinese Physics

© 2002 Chin. Phys. Soc.
and IOP Publishing Ltd

Critical properties of the transverse ferromagnetic spin system with random single-ion anisotropy*

Deng Ling-Ling(邓玲玲)[a] and Yan Shi-Lei(晏世雷)[a,b,c]†

[a] *Department of Physics, Suzhou University, Suzhou* 215006, *China*
[b] *Provincial Laboratory of Thin Film Materials, Suzhou University, Suzhou* 215006, *China*
[c] *CCAST (World Laboratory), Beijing* 100080, *China*

(Received 5 July 2001; revised manuscript received 9 December 2001)

A transverse ferromagnetic spin-1 system with a random single-ion anisotropy is considered in the framework of an Ising model. The effective field theory and decoupling approximation are applied to the derivation of the expressions of magnetizations for a honeycomb lattice. Special emphasis is placed on the critical properties of the system. New critical properties are obtained in a certain range of single-ion anisotropy, random concentration, and transverse field. We discuss in detail the influence of the random concentration and transverse field on the critical properties. Some phenomena have not been discovered in previous reports. Detailed descriptions of the phase transition and magnetization curves are presented.

Keywords: critical properties; transverse ferromagnetic spin system; random single-ion anisotropy
PACC: 7510D, 7510H, 7510J

1. Introduction

In the past few decades there has been an increasing number of studies dealing with the critical properties of the various magnetic spin systems.[1−4] The Ising model has been extensively investigated and widely proven to be useful for various systems. Besides the ordinary exchange interactions, the model takes into account the single-ion anisotropy, transverse field and some disorder distributions.[5−7] When these factors are considered for a magnetic system simultaneously, very rich phase transition features and some interesting phenomena can be found. It has been recognized that the outstanding features not only are purely theoretical, but have also been proposed as possible factors to describe certain ferromagnetic (ferri-)materials studied experimentally.[8,9] Meanwhile, some sophisticated theoretical methods have been used to examine the critical properties.[10−13]

The study of a ferromagnetic Ising spin system is one of the most successful works. Two models have frequently been used in many previous works. The first is often called the Blume–Capel model and is suited for spin-S ($S > 1/2$). This model considers the ferromagnetic spin system with single-ion anisotropy. In the presence of single-ion anisotropy, some studies have shown that the phase diagram can exhibit a tricritical point in the system with a coordination number $z \geq 3$ at which the phase transition changes from second to first order.[14,15] Of course, the tricritical point can also be observed in the three-dimensional kinetic Ising spin system.[16] The second model is the so-called transverse Ising model (TIM). This model has been introduced to explain the phase transition and order–disorder phenomena when the tunnelling effect is present. The critical properties of the spin $\geq 1/2$ TIM have been studied by using a variety of techniques.[17−18] The transverse field gives rise to possible spin flip transitions and hence the work against the ordering. Therefore, a finite-temperature phase transition can be depressed to occur at zero temperature under a critical value of the transverse field. However, many authors have introduced various random distributions in the above two models. It was learnt that the form of random distribution plays an important role in the determination of the order of phase transition and may lead to some new results.[19−20]

*Project supported by the Natural Science Foundation of the Education Bureau of Jiangsu Province (Grant No 00SJB140003) and by the Thin Film Materials Key Laboratory Open Foundation of Jiangsu Province, China (Grant No K2022).
†Corresponding author: e-mail: slyan@suda.edu.cn; fax: 0512-5111907.

A more general strategy in studying the magnetic spin system combines the Blume–Capel model and transverse Ising model mainly because a real spin system ($S > 1/2$) can have single-ion anisotropy and transverse field simultaneously. A few years ago, Jiang *et al.*[21] developed the general expressions of the second-order phase transition in the spin-1 ferromagnetic system and obtained a number of meaningful results. Jiang *et al.*[22,23] and Wang and Li[24] gave further consideration to the second-order phase transition in the above system by virtue of bond dilution, random mixed bond and trimodel transverse field distribution, separately. Although the critical properties of a spin-1 system with random single-ion anisotropy have been discussed,[25−28] to our knowledge, in previous works few researchers have ever considered the influence of random single-ion anisotropy on the critical properties of the transverse spin-1 ferromagnetic system. In this paper, we give a discussion on the transverse field, single-ion anisotropy and random concentration dependence of the critical properties. The calculated results show that the critical properties of the system not only give a correction in a limit case,[25] but also obtain some new information. The obtained results are attributed to the competition of the above-mentioned three factors. We expect our results to be valid at least qualitatively for further theoretical and experimental research. Theoretical expressions for solving the present problem are be derived based on the effective field theory and decoupling approximation.

2. Theory

For a spin-1 transverse ferromagnetic system with random single-ion anisotropy, the total Hamiltonian is given by

$$H = -\sum_{<ij>} J_{ij} S_i^z S_j^z - \Omega \sum_i S_i^x - \sum_i D_i (S_i^z)^2, \quad (1)$$

where S_i^x and S_j^z are spin-1 Pauli matrices associated with the ith site. The analysis is performed only for the case of the nearest-neighbour interaction. The first summation runs only over all pairs of nearest-neighbour sites. The second and the third summations involve all sites of the lattice. $J_{ij} = J$ is the exchange interaction between the nearest-neighbour sites and we let $J > 0$. Ω represents the uniform transverse field. D_i is a random single-ion anisotropy, assumed to be negative, which satisfies an independent random distribution of probability

$$P(D_i) = t\delta(D_i - D) + (1 - t)\delta(D_i) \quad (2)$$

where $0 \leq t \leq 1.0$. We let t indicate the random concentration. The problem can be treated usually in three steps. The Hamiltonian (1) is first written as an effective Hamiltonian H_i which includes all parts of H associated with the lattice. The second step is to calculate the eigenvalues of H_i. The third step is to introduce the differential operator technique within the framework of the effective field theory. The standard procedure then leads to the following expectation expressions of averaged magnetizations in the lattice

$$m = < S_i^z > = \Big\langle \prod_{i=1}^{z} [(S_i^z)^2 \cosh(J\nabla) + S_i^z \sinh(J\nabla) + 1 - (S_i^z)^2] \Big\rangle F(x)|_{x=0}, \quad (3)$$

while the quadrupolar moment is given by

$$q = < (S_i^z)^2 > = \Big\langle \prod_{i=1}^{z} [(S_i^z)^2 \cosh(J\nabla) + S_i^z \sinh(J\nabla) + 1 - (S_i^z)^2] \Big\rangle G(x)|_{x=0}, \quad (4)$$

where $\nabla = \partial/\partial x$ is a differential operator, and the $< \cdots >$ denotes the canonical thermal average. Functions $F(x)$ and $G(x)$ are defined by

$$F(x) = \int P(D_i) f(x, D_i) \mathrm{d}D_i \quad (5)$$

and

$$G(x) = \int P(D_i) g(x, D_i) \mathrm{d}D_i \quad (6)$$

where $P(D_i)$ is random single-ion anisotropy distribution for Eq.(2). $f(x, D_i)$ and $g(x, D_I)$ are as follows

$$f(x, D_i) = \sum_{n=1}^{3} \left[\exp\left[2\beta C E_1(n) + \frac{2\beta D_i}{3} \right] \left[\frac{2x}{3C} E_1(n) + \frac{2}{27} \frac{D_i^3 x - D_i x^3 + 3.5 D_i \Omega^2 x}{BC} E_2(n) \right] \right] \times \left[\sum_{n=1}^{3} \exp\left[2\beta C E_1(n) + \frac{2\beta D_i}{3} \right] \right]^{-1} \quad (7)$$

and

$$g(x,D_i)=\sum_{n=1}^{3}\left[\exp\left[2\beta CE_1(n)+\frac{2\beta D_i}{3}\right]\left[\frac{2D_i}{9C}E_1(n)+\frac{2}{27}\frac{0.5\Omega^2x^2-D_i^2x^2+x^4-0.5\Omega^4}{BC}E_2(n)+\frac{2}{3}\right]\right]$$
$$\times\left[\sum_{n=1}^{3}\exp\left[2\beta CE_1(n)+\frac{2\beta D_i}{3}\right]\right]^{-1}. \quad (8)$$

Here

$$E_1(n)=\cos\frac{2\pi(n-1)+\theta}{3}, \quad (9)$$

$$E_2(n)=\sin\frac{2\pi(n-1)+\theta}{3}, \quad (10)$$

$$B=\frac{1}{9}\Big[3(D_i^2x-x^3)^2+\frac{3}{4}D_i^2\Omega^4+15D_i^2\Omega^2x^2+9\Omega^4x^2+9\Omega^2x^4+3D_i^6\Big]^{1/2}, \quad (11)$$

$$C=\left[\frac{1}{9}D_i^2+\frac{1}{3}\Omega^2+\frac{1}{3}x^2\right]^{1/2}, \quad (12)$$

and

$$\theta=\arccos\lfloor(C^6-B^2)^{1/2}/C^3\rfloor. \quad (13)$$

It is obvious that Eqs.(3) and (4) deal with multi-spin correlations. We cannot treat exactly all the spin–spin correlations. Therefore, a decoupling approximation is frequently adopted: $<S_i^z(S_j^z)^2\cdots S_k^z>\approx<S_i^z><(S_j^z)^2>\cdots<S_k^z>$, for $i\neq j\neq\cdots\neq k$. With such a procedure, the averaged magnetization m and quadrupolar moment q can be evaluated from the following coupled equations

$$m=[q\cosh(J\nabla)+m\sinh(J\nabla)+1-q]^zF(x)|_{x=0}, \quad (14)$$

$$q=[q\cosh(J\nabla)+m\sinh(J\nabla)+1-q]^zG(x)|_{x=0}, \quad (15)$$

where z is the lattice coordination number. Here, we are interested in studying the critical properties of the system. However, we need to expand the right-hand side of Eqs.(14) and (15) and to replace q in Eq.(14) by Eq.(15). Then we can obtain the self-consistent equation with respect to the averaged magnetization m

$$m=am+bm^3+cm^5+\cdots \quad (16)$$

In the vicinity of the second-order phase transition line, m is small enough according to Landau theory. Hence we retain only the linear term to determine the second-order phase transition equation and take only the first and the second terms to obtain an explicit expression of the magnetization m, i.e.

$$a=1 \quad (17)$$

and

$$m=[(1-a)/b]^{1/2}. \quad (18)$$

The right-hand side of Eq.(18) must be positive. If this is not the case, the phase transition is first-order. Therefore, $b<0$ must be satisfied. Thus, the point at which a=1 and b=0 is a tricritical point. While a and b denote the following expressions

$$a=z\sinh(J\nabla)[q_0\cosh(J\nabla)+1-q_0]^{z-1}F(x)|_{x=0}, \quad (19)$$

$$b=z(z-1)q_1\sinh(J\nabla)[\cosh(J\nabla)-1]\times[q_0\cosh(J\nabla)+1-q_0]^{z-2}F(x)|_{x=0}+\frac{z(z-1)(z-2)}{3!}\sinh^3(J\nabla)\times[q_0\cosh(J\nabla)+1-q_0]^{z-3}F(x)|_{x=0}, \quad (20)$$

where q_0 and q_1 are the solutions of

$$q_0=[q_0\cosh(J\nabla)+1-q_0]^zG(x)|_{x=0} \quad (21)$$

and

$$q_1=h/(1-e), \quad (22)$$

in which e and h are defined by

$$e=z[\cosh(J\nabla)-1]\times[q_0\cosh(J\nabla)+1-q_0]^{z-1}G(x)|_{x=0}, \quad (23)$$

$$h=\frac{z(z-1)}{2!}\sinh^2(J\nabla)\times[q_0\cosh(J\nabla)+1-q_0]^{z-1}G(x)|_{x=0}. \quad (24)$$

Up to now, we have derived the expressions for the transverse ferromagnetic spin-1 system with a random single-ion anisotropy. Here for simplicity, we only study the honeycomb lattice with z=3, although any lattices with an arbitrary coordination number z can be discussed in the same way. The next step is to solve Eqs.(17) and (18) numerically and to discuss the second-order phase transition and magnetization curves of the present system. We find that the present system can show some new critical phenomena, which depend on the transverse field, the single-ion anisotropy and the random concentration. Hence,

in the following section, we examine the influence of three important factors on the critical properties.

3. Results and discussion

In this section, we discuss the phase transition and magnetization curves in the transverse ferromagnetic spin-1 system with a random single-ion anisotropy in the honeycomb lattice. Clearly, it is necessary to solve Eqs.(17) and (18) numerically. Then we can obtain the solutions for the above-mentioned equations.

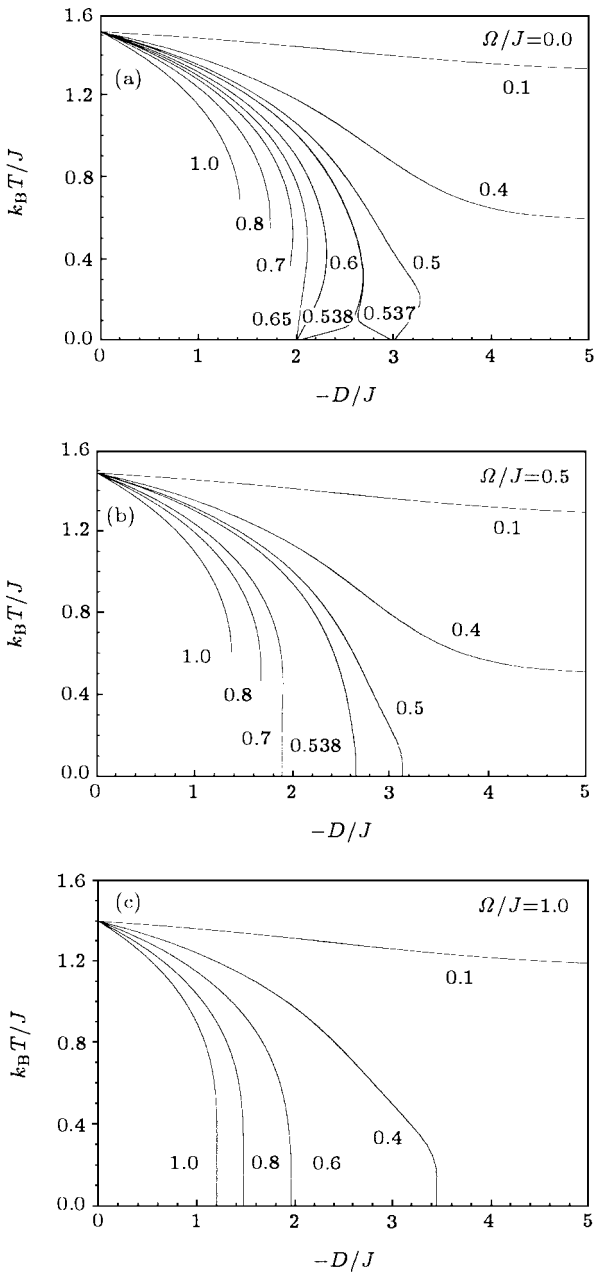


Fig.1. Single-ion anisotropy dependences of the Curie temperature for the honeycomb lattice, for selected values of Ω/J: (a) 0.0; (b) 0.5; (c) 1.0. The numbers on the curves are the values of random concentration t.

Firstly, the Curie temperatures versus negative single-ion anisotropy for the present system with Ω/J=0, Ω/J=0.5 and Ω/J=1.0 are depicted in Figs.1(a), 1(b) and 1(c), respectively, when the value of random concentration t changes. In Fig.1(a) we see that the system exhibits tricritical points in a certain random concentration t range. From our exact calculations, the tricritical points exist in the range $1.0 \geq t \geq 0.667$. This is more or less consistent with the result in Ref.[25]. In this case, the second-order phase transition curves do not exist in the low-temperature region. However, it is worthwhile to notice that the tricritical temperature does not change abruptly near t=0.85, like Fig.2 in Ref.[25]. We find that the tricritical temperature decreases monotonically with the decrease of random concentration t. When the random concentration t=0.666, the tricritical point disappears. Hence, our result gives a correction to that of Ref.[25]. Meanwhile, the present result can show a complete phase diagram in the whole random concentration region. In fact, the phase diagram shows very rich features at low temperatures. In particular, the second-order phase transition temperature goes to zero at two values of the single-ion anisotropy D/J=−2.0 and D/J=−3.0, if the random concentration t varies in the ranges $0.666 > t \geq 0.538$ and $0.537 \geq t \geq 0.493$, respectively, while the second-order transition lines extend to $D \to \infty$ for $t \leq 0.492$. Therefore, for $t \leq 0.492$ the present system is always in a ferromagnetic state at low temperatures. On the other hand, the re-entrant phenomenon is another important problem in the phase diagram. The re-entrant phenomenon can be observed in an appropriate range of random concentration. We notice that with a decreasing concentration of the random single-ion anisotropy, the re-entrant phenomenon can show some interesting varieties. As seen from Fig.1(a), the re-entrant phenomenon becomes more and more strong from t=0.723 to 0.538, while a new zigzag transition line can be observed at t=0.537. Of course, with further decreasing the random concentration, the zigzag transition line will be depressed but the re-entrant phenomenon still exists until t=0.493. Figure 1(b) shows that two fixed values of the negative single-ion anisotropy disappear at T=0K and that the tricritical points are depressed and the re-entrant phenomenon becomes very weak in the presence of the transverse field. Obviously, the phase diagram displays larger changes due to the presence of a transverse field. Hence, we can infer that the tricritical points

and the re-entrant phenomenon will be depressed or destroyed entirely if the transverse field continues to increase. Figure 1(c) exhibits clearly this case. Moreover, we also see that the second-order transition lines, which tend previously to $D \to \infty$, go to a finite value of the single-ion anisotropy at T=0K (see curve labelled t=0.4). The influence of the transverse field can be seen from a comparison between Figs.1(a), 1(b) and 1(c). Here the role of the transverse field is to depress and to destroy the tricritical points and the re-entrant phenomenon.

In Fig.2 the transverse field dependences of the Curie temperature are plotted for a value of D/J=–0.8, with various values of $t \leq 1.0$. As shown in Fig.2, the critical temperature of the system enhances with decreasing concentration of the random single-ion anisotropy, which concurs with the prediction of Fig.1. This is mainly due to the fact that the decrease of the random concentration weakens the influence of the single-ion anisotropy on the phase transition and leads to a rise of critical temperature. We also see that the tricritical points and the re-entrant phenomenon do not appear at all in Fig.2. This is because the existence of small single-ion anisotropy and large transverse field makes it impossible for these to occur. Figure 3 shows the critical temperature dependences of the transverse field at t=0.6 for different values of the negative single-ion anisotropy. From Fig.3 we clearly see that the critical temperature is depressed when the value of the negative single-ion anisotropy becomes large. This is reasonable since a larger single-ion anisotropy plays a more powerful role for decreasing the critical temperature at a fixed random concentration t.

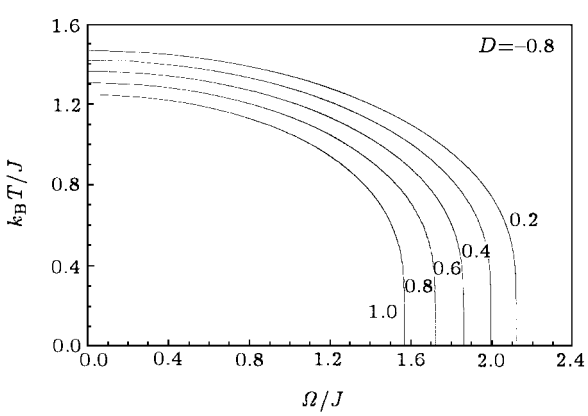


Fig.2. Transverse field dependences of the Curie temperature for the value of single-ion anisotropy, D/J=–0.8. The numbers on the curves are the values of random concentration t.

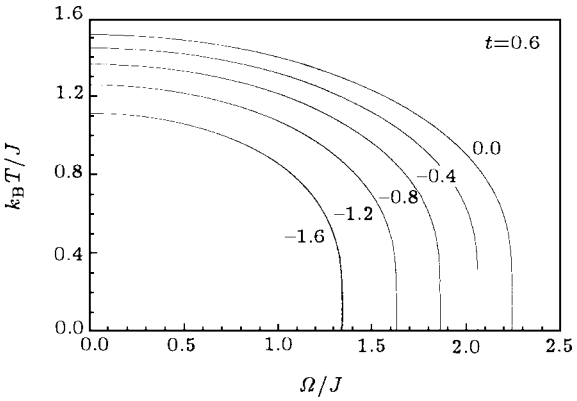


Fig.3. Transverse field dependences of the Curie temperature for the value of random concentration, t=0.6. The numbers on the curves are the values of single-ion anisotropy.

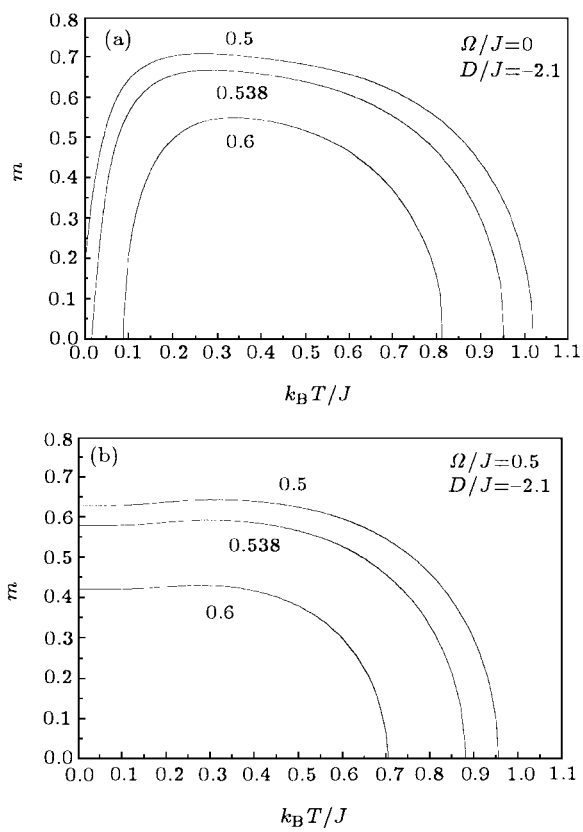


Fig.4. Temperature dependences of total magnetization per site for the value of negative single-ion anisotropy parameter, D/J=–2.1, and the values of transverse field, (a) Ω/J=0 and (b) Ω/J=0.5. The numbers on the curves are the values of random concentration t.

Figures 4(a) and 4(b) illustrate the behaviours of the site magnetization as functions of temperature for the present system with D/J=–2.1, Ω/J=0 and Ω/J=0.5, respectively, when t changes. The magnetization curves exhibit the re-entrant phenomenon, i.e. with the increase of T the magnetization curves

first increase and then decrease to zero. The initial increase may come from the release of spin due to the thermal agitation. Comparing Figs.4(a) and 4(b), we discover easily that the re-entrant phenomenon is destroyed by the existence of the transverse field. Although the re-entrant phenomenon can still be observed in Fig.4(b), it has become weak. The reasons are very similar to those found for Fig.1.

In summary, we have studied the critical properties of the transverse ferromagnetic spin system with a random single-ion anisotropy. Some new behaviours have been discovered. We have discussed the influence of the random concentration and the transverse field on phase diagrams and magnetization curves and have shown a number of properties of the phase diagrams. Some results obtained in this system have not been reported in previous papers, but are very important for us to understand the ferromagnetic phase transition.

References

[1] Yang C Z and Li Z Y 1988 *Phys. Status Solidi* b **146** 223
[2] Gao Z, Wang Z L and Xu J H 1997 *Acta Phys. Sin.* **46** 2029 (in Chinese)
[3] Yan S L and Yang C Z 1997 *Z. Phys.* B **103** 93
[4] Laabo B, Saber M, Benab A and Kerou M 1999 *Chin. Phys.* **8** 199
[5] Zhang G M and Yang C Z 1993 *Phys. Rev.* B **48** 9452
[6] Yan S L and Yang C Z 1999 *Commun. Theor. Phys.* **31** 531
[7] Bobak A and Jascur M 1995 *Phys. Rev.* B **51** 11 533
[8] Honda S, Nawate M and Yoshima M 1987 *IEEE Trans. Magn.* **23** 2987
[9] Endl G *et al* 1991 *Collop. Digest of 13th Int. Colloq. Magn. Films and Surf.* p 155
[10] Kaneyoshi T 1986 *Phys. Rev.* B **33** 526
[11] Benyoussef A and Boccara N 1985 *J. Phys. C: Solid State Phys.* **18** 1899
[12] Li Z Y and Jiang Q 1989 *Phys. Lett.* A **138** 247
[13] Zhang G M and Yang C Z 1995 *Acta Phys. Sin.* **44** 958 (in Chinese)
[14] Siqueira A F and Fittipaldi I P 1986 *Physica* A **138** 592
[15] Janowsky S A 1988 *Phys. Lett.* A **134** 131
[16] Shao Y Z, Lan T and Lin G M 2001 *Acta Phys. Sin.* **50** 942 (in Chinese)
[17] Cuccoli A, Togentli V and Verrucchi P 1991 *Phys. Rev.* B **44** 903
[18] Yan S L and Yang C Z 1996 *Solid State Commun.* **100** 851
[19] Boccara N, Elkenz A and Saber M 1989 *J. Phys.* **1** 5721
[20] Kaneyoshi T and Michnichi J 1990 *Phys. Rev.* B **42** 4388
[21] Jiang X F, Li J L, Zhong J L and Yang C Z 1993 *Phys. Rev.* B **47** 827
[22] Jiang X F 1994 *J. Magn. Magn. Mater.* **134** 167
[23] Jiang X F, Zhong J L and Li J L 1994 *J. Phys.* **6** 523
[24] Wang Y Q and Li Z Y 1995 *Acta Phys. Sin.* **44** 811 (in Chinese)
[25] Kaneyoshi T 1986 *J. Phys. C: Solid State Phys.* **19** L557
[26] Buzano C, Maritan A and Pelizzola A 1994 *J. Phys.* **6** 327
[27] Branco N S and Boechat B 1997 *Phys. Rev.* B **56** 11673
[28] Lara D P and Plascale J A 1998 *Physica* A **260** 443