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Continuous variable entanglement swapping in a thermal environment*

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With the application of a Wigner function description of a two-mode squeezed state, we study a protocol of continuous variable entanglement swapping in a noisy quantum channel. Assuming that the two initial entangled states are coupled with the same environment, we analyse the quantum state emerging from the swapping and obtain the inseparable condition for the output correlation. It is found that there is more noise resulting from the entanglement swapping than from the direct transmission, no matter what values of displacement gain are chosen.

Keywords: Continuous variable, entanglement swapping, squeezed state, Wigner function
PACC: 4250, 0367, 0365

1. Introduction

Quantum entanglement, first noted by Einstein, Podolsky and Rosen (EPR)[1] and Schrödinger,[2] is one of the important features of quantum mechanics. Entanglement describes a system composed of two or more particles,[3,4] exhibiting the novel property that the result of measurement on one particle cannot be specified independently of the parameters of the measurements on other particles. The resource of entanglement has a number of useful applications in quantum communication and quantum computation.

Sharing a non-locally entangled state of the EPR type between a sender and a receiver makes it possible to transmit an unknown quantum state from the sender to the receiver through direct transmission of only classical information. This process, first proposed by Bennett *et al.*[5] in the spin-$\frac{1}{2}$ system, is called quantum teleportation. Recently, Vaidman,[6] Braunstein and Kimble[7] have suggested the teleportation of continuous-variable states, such as the quadrature amplitudes of the electromagnetic field, in which the concept of teleportation of discrete variables was extended to that of continuous variables belonging to states of infinite-dimensional systems. With a high detection efficiency of the homodyne measurement, the teleportation of arbitrary coherent states was implemented experimentally[8] using the parametric down-conversion as an EPR source. Since then, more and more attention has been paid to the study of continuous variables in quantum information processing, such as secret key distribution,[9] teleportation,[10] dense coding,[11] error correction,[12] entanglement purification,[13,14] quantum computation[15] and cloning.[16] In addition, a necessary and sufficient condition for the inseparability of all Gaussian states was studied in Refs.[17] and [18] which turned out to be a sufficient inseparable criterion for continuous variable states of any two particles.

It is worth mentioning that the entanglement swapping[19] has a number of useful purposes, such as entangling two quantum states which have never interacted directly with each other, constructing a quantum communication network, speeding up the distribution of entanglement, and serving as a kind of entanglement purification. Another important application of entanglement swapping is the quantum repeater, suggested by Briegel *et al.*[20] in which the exponential degradation of entanglement with a noisy channel length can be overcome with a polynomial increase in time and a logarithmic increase in resources. The above-mentioned studies concerning entanglement swapping have only referred to discrete-variable systems, i.e. two-level systems. More recently, Polkinghorne and Ralph[21] have put forward the first

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scheme for continuous-variable entanglement swapping, in which polarization entangled states of single photons from two independently entangled source are teleported with the use of squeezed state entanglement. Subsequently, two other schemes for entanglement swapping in continuous-variable systems have been proposed. Tan[22] confirmed the nonlocal entanglement of the teleported signal beam with the idler beam from the same parametric amplifier. Van Loock and Braunstein[23] investigated the unconditional teleportation of continuous variable entanglement by means of squeezed light and linear optical devices. Note that up until now, such reports of continuous-variable entanglement swapping have only discussed pure quantum states as the quantum channel. In fact, as the quantum channel is exposed to the environment,[24,25] the decoherence effect will turn the pure quantum state into a statistical mixture and destroy the entanglement property. Therefore, it is important to consider the influence of environment on the entanglement swapping. Inspired by Ref.[24], we discuss continuous-variable entanglement swapping in a mixed quantum channel in this paper. The two initial entangled states are assumed to couple with the same thermal environments. Based on the inequality in Ref.[17] about the sufficient-inseparability criterion for continuous-variable systems, we investigate the quantum correlation emerging from entanglement swapping. We show that there is more noise resulting from the entanglement swapping than from the direct transmission, no matter what values of displacement gain are chosen.

This paper is arranged as follows. In Sec.2, we give a brief review of the time evolution of a two-mode squeezed vacuum in a noisy channel. In Sec.3, the whole process for continuous-variable entanglement swapping is described and the inseparable condition for the output correlation is obtained. In Sec.4, we compare the decoherence property of the entanglement swapping with that of direct transmission. Finally, remarks and conclusions are given in Sec.5.

2. Two-mode squeezed vacuum in a noisy quantum channel

Before studying the scheme of entanglement swapping, we briefly review the evolution of the two-mode squeezed vacuum in a thermal environment. A two-mode squeezed vacuum state, generated by a parametric down-converter,[26] can be described by the Wigner function

$$W(\alpha_a;\alpha_b) = \frac{4}{\pi^2}\exp\{-\mathrm{e}^{-2r}[(x_a+x_b)^2+(p_a-p_b)^2] - \mathrm{e}^{2r}[(x_a-x_b)^2+(p_a+p_b)^2]\}, \quad (1)$$

where r is the squeezing coefficient and the complex quadrature phase variable $\alpha_j = x_j + ip_j$ $(j = a, b)$. In reality, due to the unavoidable light transmission loss, the initial pure squeezed state will evolve to a two-mode mixed squeezed state. The time evolution of the Wigner function can be described by the Fokker–Planck equation in the interaction picture[27]

$$\frac{\partial W(\alpha_a;\alpha_b;t)}{\partial t} = \frac{\gamma}{2}\sum_{i=a,b}\left[\frac{\partial}{\partial\alpha_i}\alpha_i + \frac{\partial}{\partial\alpha_i^*}\alpha_i^* + (1+2\overline{n})\frac{\partial^2}{\partial\alpha_i\partial\alpha_i^*}\right]W(\alpha_a;\alpha_b;t), \quad (2)$$

where γ is the damping rate for the two field modes, and $\overline{n}$ is the average thermal photon number. For the zero-temperature environment, $\overline{n} = 0$. By solving the Fokker–Planck equation, the Wigner function for the mixed squeezed state[24] is given by

$$W(\alpha_a;\alpha_b;T) = \frac{4}{\pi^2(E^2-F^2)} \times \exp\left[-\frac{2E}{E^2-F^2}(x_a^2+p_a^2+x_b^2+p_b^2) + \frac{4F}{E^2-F^2}(x_ax_b-p_ap_b)\right], \quad (3)$$

where $E = T(1+2\overline{n}) + (1-T)\cosh 2r$ and $F = (1-T)\sinh 2r$ with the normalized time $T(t) = 1 - \exp(-\gamma t)$. For $T = 0$, the mixed squeezed state is the initial pure squeezed vacuum state defined in Eq.(1), as expected. The initial squeezed state will eventually assimilate with the thermal environment, i.e. the quantum nonlocality is lost completely, when F is zero for the limit of $t \to \infty$.

3. Process of entanglement swapping

In our protocol, a pair of entangled states, source A and source B, are needed to perform entanglement swapping. Both sources A and B are made to emit a two-mode squeezed vacuum state, which can be generated by a nonlinear medium.[28] The Wigner function of the two two-mode pure squeezed states is given respectively by

$$W(\alpha_{1a};\alpha_{1b}) = \frac{4}{\pi^2}\exp\{-\mathrm{e}^{-2r_1}[(x_{1a}+x_{1b})^2+(p_{1a}-p_{1b})^2] - \mathrm{e}^{2r_1}[(x_{1a}-x_{1b})^2+(p_{1a}+p_{1b})^2]\}, \quad (4)$$

$$W(\alpha_{2a};\alpha_{2b}) = \frac{4}{\pi^2}\exp\{-\mathrm{e}^{-2r_2}[(x_{2a}+x_{2b})^2+(p_{2a}-p_{2b})^2] - \mathrm{e}^{2r_2}[(x_{2a}-x_{2b})^2+(p_{2a}+p_{2b})^2]\}, \quad (5)$$

where r_1 and r_2 are the degrees of squeezing, and the quadrature variable $\alpha_{ka} = x_{ka} + ip_{ka}$, $\alpha_{kb} = x_{kb} + ip_{kb}$ $(k = 1, 2)$. Note that if $r_1 \to \infty$, the quantum state described by Eq.(4) will become exactly the maximally entangled continuous-variable state (EPR state), while the state described by Eq.(5) will also become the EPR state if $r_1 \to \infty$. Supposing that the two sources are exposed to the same thermal environment, the entanglement property of the two initial squeezed states will be deteriorated. According to Sec.2, the time evolution of the two pure quantum states, sources A and B, influenced by the noisy channel, can be written respectively as

$$W(\alpha_{1a};\alpha_{1b};T_1) = \frac{4}{\pi^2(E_1^2-F_1^2)}\exp\left[-\frac{2E_1}{E_1^2-F_1^2}(x_{1a}^2+p_{1a}^2+x_{1b}^2+p_{1b}^2) + \frac{4F_1}{E_1^2-F_1^2}(x_{1a}x_{1b}-p_{1a}p_{1b})\right], \quad (6)$$

$$W(\alpha_{2a};\alpha_{2b};T_2) = \frac{4}{\pi^2(E_2^2-F_2^2)}\exp\left[-\frac{2E_2}{E_2^2-F_2^2}(x_{2a}^2+p_{2a}^2+x_{2b}^2+p_{2b}^2) + \frac{4F_2}{E_2^2-F_2^2}(x_{2a}x_{2b}-p_{2a}p_{2b})\right], \quad (7)$$

where the parameters $E_i = T_i(1+2\overline{n}) + (1-T_i)\cosh 2r_i$, $F_i = (1-T_i)\sinh 2r_i$ $(i = 1, 2)$, and the normalized interaction time $T_i(t) = 1 - \exp(-\gamma_i t)$. Here γ_i is the loss rate.

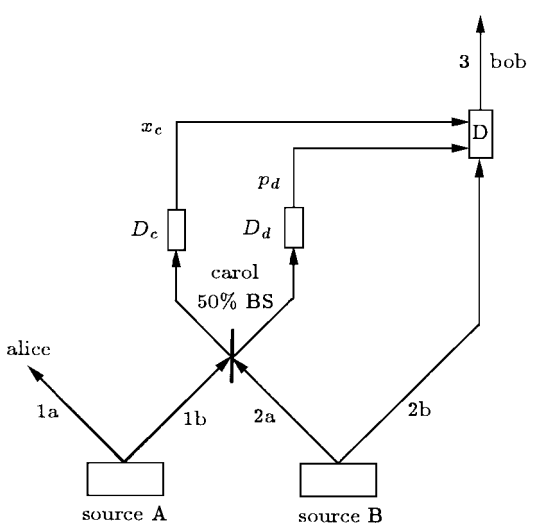


Fig.1. Scheme for continuous-variable entanglement swapping between two two-mode squeezed vacuum states influenced by the thermal environment. Sources A and B are two optical parametric down-converters, each emitting a two-mode squeezed state. Before the measurement, Alice and Carol share the mixed squeezed state of mode $1a$ and mode $1b$ while Carol and Bob share the other mixed squeezed state between modes $2a$ and $2b$.

Let us suppose that Alice wants to transfer mode $1b$ of source A to mode $2b$ in Bob's side with source B providing the entanglement for the teleportation in Fig.1. Of course, initially Alice and Bob do not share any entangled state. The total original state is a product of the state of source A and that of source B, which can be represented by the total Wigner function

$$W(\alpha_{1a};\alpha_{1b};T_1;\alpha_{2a};\alpha_{2b};T_2) = W(\alpha_{1a};\alpha_{1b};T_1)\otimes W(\alpha_{2a};\alpha_{2b};T_2). \quad (8)$$

In order to realize the entanglement swapping, Alice sends mode $1b$ to Carol, and keeps mode $1a$ at her side. At the same time, Bob does the same thing: sending mode $2a$ to Carol and keeping mode $2b$. After receiving mode $1b$ from Alice and mode $2a$ from Bob simultaneously, Carol performs a Bell detection by combining the two modes on a 50/50 beam splitter $\beta_{c,d} = \frac{1}{\sqrt{2}}(\alpha_{1b} \mp \alpha_{2a})$, and setting homodyne detectors at the two output ports of the beam splitter. The two conjugate quadrature variables $x_c = Re\beta_c = \frac{1}{\sqrt{2}}(x_{1b} - x_{2a})$ and $p_d = Im\beta_d = \frac{1}{\sqrt{2}}(p_{1b} + p_{2a})$ are measured by choosing the phases of local oscillators for the homodyne detectors, while all information about the observables corresponding to the imaginary part of β_c and the real part β_d is lost. After the beam splitter, the Wigner function of the total state can be

written as

$$W(\alpha_{1a};\beta_c;\beta_d;\alpha_{2b};T_1;T_2)$$
$$=W(\alpha_{1a};\frac{\beta_c+\beta_d}{\sqrt{2}};T_1)\otimes W(\frac{\beta_d-\beta_c}{\sqrt{2}};\alpha_{2b};T_2), \quad (9)$$

which displays an entanglement between the modes $1b$ and $2a$.

Integrating x_d and p_c out from Eq.(9) results in a Wigner function related to the variables $(\alpha_{1a};x_c;p_d;\alpha_{2b};T_1;T_2)$. Now, Carol sends the measurement results x_c and p_d to Bob through a classical communication. Based on the classical results, Bob displaces mode $2b$ as $x_{2b}\to x_3=x_{2b}+\lambda\sqrt{2}x_c$, $p_{2b}\to p_3=p_{2b}+\lambda\sqrt{2}p_d$. The parameter λ describes a displacement gain for the local operation. Note that it is impossible to teleport mode $1b$ to mode $2b$ without any information from Carol. After x_c and p_d are integrated out from the Wigner function $W(\alpha_{1a};x_c;p_d;\alpha_3;T_1;T_2)$, we obtain the joint time-dependent Wigner function of mode $1a$ and mode 3

$$\begin{aligned}W(\alpha_{1a};\alpha_3)=&\frac{4}{\pi^2\delta}\exp\{-\frac{2}{\delta}[E_1(x_3^2+p_3^2)\\&+[E_2+\lambda^2(E_1+E_2)-2\lambda F_2](x_{1a}^2+p_{1a}^2)\\&+2\lambda F_1(p_{1a}p_3-x_{1a}x_3)]\},\end{aligned} \quad (10)$$

where the parameter $\delta=(E_1^2-F_1^2)\lambda^2+E_1E_2(1+\lambda^2)-2\lambda E_1F_2$.

Equation (10) is a Wigner function with variables $(\alpha_{1a};\alpha_3;T_1;T_2)$, describing a correlation between mode $1a$ and the teleported mode 3. The correlation, initially an unentangled state, must be proven to be nonclassical if continuous variable entanglement swapping is achieved. Therefore, we use the inequality about the inseparability criterion in Ref.[17] to verify the correlation. The criterion is shown to be true for all Gaussian states; not only for the pure two-mode squeezed vacuum state, but also for the mixed squeezed state. In this protocol, the sufficient condition for inseparability is that the total variance of $\widehat{x}_{1a}-\widehat{x}_3$ and $\widehat{p}_{1a}+\widehat{p}_3$ operators with the quantum state described by Eq.(10) satisfies the inequality

$$\langle\Delta(\widehat{x}_{1a}-\widehat{x}_3)^2\rangle+\langle\Delta(\widehat{p}_{1a}+\widehat{p}_3)^2\rangle<1. \quad (11)$$

It is believed that a two-mode state with the density operator ρ has the following characteristic function[28]

$$\begin{aligned}&\chi^{(w)}(\xi_1;\xi_2)\\=&\mathbf{tr}[\rho\exp(\xi_1\hat{a}_1^+-\xi_1^*\hat{a}_1)\exp(\xi_2\hat{a}_2^+-\xi_2^*\hat{a}_2)]\\=&\mathbf{tr}\{\rho\exp[i2(\xi_1^I\hat{x}_1-\xi_1^R\hat{p}_1)]\exp[i2(\xi_2^I\hat{x}_2-\xi_2^R\hat{p}_2)]\},\end{aligned} \quad (12)$$

where $\xi_l=\xi_l^R+i\xi_l^I$ and the annihilation operator $\widehat{a}_l=\widehat{x}_l+i\widehat{p}_l$ $(l=1,2)$. The quadrature amplitudes $\widehat{x}_l$ and $\widehat{p}_l$ satisfy the commutators $[\widehat{x}_l,\widehat{p}_{l'}]=\frac{i}{2}\delta_{ll'}$ $(l,l'=1,2)$. In terms of the characteristic function, the average value of an observable for the two-mode state can be obtained

$$\langle(\widehat{a}_p^+)^m\widehat{a}_q^n\rangle=\frac{\partial^m}{\partial\xi_p^m}\frac{\partial^n}{\partial(-\xi_q^*)^n}\chi^{(w)}(\xi_1,\xi_2)|_{\xi_1=\xi_2=0}, \quad (13)$$

where $p,q=1,2$. Admittedly, the Wigner function of the density operator ρ is the Fourier transform of its symmetrically ordered characteristic function. In this case, the characteristic function $\chi^{(w)}(\xi_1,\xi_2)$ can also be derived from the inverse Fourier transform of the Wigner function

$$\begin{aligned}&\chi^{(w)}(\xi_1;\xi_2)\\=&\int\mathbf{d}^2\alpha_1\int\mathbf{d}^2\alpha_2W(\alpha_1,\alpha_2)\exp(\xi_1\alpha_1^*-\xi_1^*\alpha_1)\\&\times\exp(\xi_2\alpha_2^*-\xi_2^*\alpha_2).\end{aligned} \quad (14)$$

Clearly, there are two different expressions for the same characteristic function described by Eqs.(12) and (14). Using Eqs.(10) and (14), we have the characteristic function

$$\begin{aligned}&\chi^{(w)}(\eta_{1a};\eta_3)\\=&\exp\{-\frac{E_1}{2}[(\eta_{1a}^R)^2+(\eta_{1a}^I)^2]\\&-\frac{E_1^2\lambda^2+E_1E_2(1+\lambda^2)-2\lambda E_1F_2}{2E_1}[(\eta_3^R)^2+(\eta_3^I)^2]\\&+\lambda F_1(\eta_{1a}^R\eta_3^R-\eta_{1a}^I\eta_3^I)\},\end{aligned} \quad (15)$$

where $\eta_k=\eta_k^R+i\eta_k^I$ for $k=1a,3$.

Substituting Eq.(15) into Eq.(13), we obtain the total variance of the $\widehat{x}_{1a}-\widehat{x}_3$ and $\widehat{p}_{1a}+\widehat{p}_3$ operators as

$$\begin{aligned}&\langle\Delta(\widehat{x}_{1a}-\widehat{x}_3)^2\rangle+\langle\Delta(\widehat{p}_{1a}+\widehat{p}_3)^2\rangle\\=&\frac{1}{2}[(1+\lambda^2)(E_1+E_2)-2\lambda(F_1+F_2)].\end{aligned} \quad (16)$$

Let $\tau_1(\overline{n},r_1,r_2,\lambda,T_1,T_2)=\langle\Delta(\widehat{x}_{1a}-\widehat{x}_3)^2\rangle+\langle\Delta(\widehat{p}_{1a}+\widehat{p}_3)^2\rangle$. Provided that the inequality $\tau_1\geq1$ is violated, as shown in Ref.[17], the correlation which is originally unentangled between modes $1a$ and 3 may be inseparable. Thereby, $\tau_1<1$ is the inseparability criterion for the output state of the entanglement swapping.

Based on the different values of displacement gain λ, two cases are considered as follows.

(i) When $\lambda = 1$, Eq.(16) is given by

$$\begin{aligned}\tau_1 =& E_1 - F_1 + E_2 - F_2 \\ =& (1 + 2\overline{n})(T_1 + T_2) \\ &+ (1 - T_1)\mathrm{e}^{-2r_1} + (1 - T_2)\mathrm{e}^{-2r_2}. \end{aligned} \quad (17)$$

In the absence of the thermal environment, the quantum property of the two initial pure squeezed states remains unchanged before being coupled with each other on the beam splitter. This means that the inseparable condition $\tau_1 = \mathrm{e}^{-2r_1} + \mathrm{e}^{-2r_2} < 1$, which is in accordance with the criterion in Ref.[22]. From Eq.(17), we learn that if and only if both $E_1 - F_1 < 1$ and $E_2 - F_2 < 1$ are true, the inequality $\tau_1 < 1$ can be satisfied. In other words, if the quantum correlation emerging from the entanglement swapping is entangled, the two mixed squeezed states are undoubtedly inseparable before teleportation. Here, $E_j - F_j < 1$ $(j = 1, 2)$ is the inseparability criterion for the two original two-mode squeezed vacua in the thermal environment.

(ii) When an optimal displacement is $\lambda = \dfrac{F_1 + F_2}{E_1 + E_2}$, the total variance of the $\widehat{x}_{1a} - \widehat{x}_3$ and $\widehat{p}_{1a} + \widehat{p}_3$ operators is minimized and can be written as

$$\tau_1 = \frac{(E_1 + E_2)^2 - (F_1 + F_2)^2}{2(E_1 + E_2)}. \quad (18)$$

The optimal displacement corresponds to the minimal value of the total variance. Apparently the optimal gain λ is a time-dependent variable. The aim of choosing the optimal value is that we want to compare the minimal total variance with the noise factor in a direct transmission. For $t = 0$, we obtain the optimal gain $\lambda = \dfrac{\exp(2r_1 + 2r_2) - 1}{\exp(2r_1 + 2r_2) + 1}$ and the total variance $\tau_1 = \dfrac{\exp(2r_1) + \exp(2r_2)}{\exp(2r_1 + 2r_2) + 1}$.

4. Comparing the decoherence property

Actually, the losses and decoherence in the environment will turn a pure entangled state into a mixed one. In Fig.2, two different ways for distributing entanglement between the noise channel M and N are plotted. For simplicity, we assume that the environment has the same influence on each mode of the three squeezed sources and that the energy decay rate of all the modes is γ'. In addition, the quantum states emitted from the three parametric amplifiers $1'$, 1 and 2 are assumed to have the same level of squeezing s. The above two assumptions are reasonable as the temperature of the thermal environment is normally the same. From an experimental viewpoint, there are mature techniques for a parametric down-converter. The first way to build an entanglement in nodes M and N is the direct transmission of the initial two-mode squeezed state, where the state generator is located in the midpoint of the noisy channel. According to Ref.[24], $\tau = E - F$ is regarded as the noise factor of the mixed two-mode squeezed vacuum described by Eq.(3). The second way to establish the correlation is to perform a Bell detection at the middle point of the channel. We consider two parametric amplifier sources, each simultaneously emitting a pure two-mode squeezed state. Then, the entanglement swapping is used to connect the ends of the channel, sites M and N. In this case, the noise factor is defined in Eq.(16). Next, we compare the noise factors between the output state of entanglement swapping and the two-mode pure squeezed vacuum for direct transmission. Obviously, the squeezed photons in the first way travel twice as much distance as in the second way.

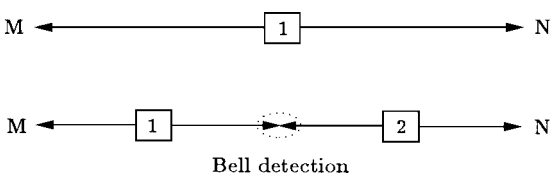


Fig.2. The two specific ways of distributing entanglement between two sites M and N of a noisy channel. Source $1'$ is made to emit a two-mode squeezed state and is positioned at the midpoint of the channel. Likewise, sources 1 and 2 are also made to produce a two-mode squeezed vacuum and are placed at the quarter position of the whole channel near nodes M and N, respectively.

(i) If $\lambda = 1$, we have

$$\begin{aligned}&\tau_1 \text{ (for time } \frac{t}{2}) - \tau \text{ (for time } t) \\ =& 2(E_1 - F_1) - (E - F) \\ =& \mathrm{e}^{-2s} + (2\bar{n} + 1 - \mathrm{e}^{-2s})(1 - \sqrt{1 - T})^2, \end{aligned} \quad (19)$$

where $T(t) = 1 - \exp(-\gamma' t)$. The right-hand side is semi-positive so that the noise resulting from the teleportation is more than that from direct transmission. This is in agreement with the protocol in Ref.[24].

(ii) If $\lambda = \dfrac{F_1 + F_2}{E_1 + E_2} = \dfrac{F_1}{E_1}$, the minimum value of the noise factor τ_1 is described by Eq.(18), and $\tau_1 - \tau$ is given by

$$\begin{aligned}
&\tau_1(\text{for time } \frac{t}{2}) - \tau(\text{for time } t) \\
=&\frac{E_1^2 - F_1^2}{E_1} - (E - F) \\
=&\frac{[(1-\sqrt{1-T})(1+2\overline{n}) + \sqrt{1-T}\mathrm{e}^{2s}][(1-\sqrt{1-T})(1+2\overline{n}) + \sqrt{1-T}\mathrm{e}^{-2s}]}{(1-\sqrt{1-T})(1+2\overline{n}) + \sqrt{1-T}\cosh 2s} \\
&- [T(1+2\overline{n}) + (1-T)\mathrm{e}^{-2s}]. \qquad (20)
\end{aligned}$$

By means of numerical computation, we find that with the increase of time, the right-hand side may be negative. In the meantime, $\tau_1 > 1$ is satisfied. That is to say, when $\tau_1 - \tau < 0$, it is impossible to preserve the entangled property of the three mixed squeezed states. This implies that the noise factor given by the output state of the swapping is larger than that by the direct transmission, even if the optimal displacement is obtained.

Thus, our results have shown that there is more noise resulting from the entanglement swapping than that from the direct transmission, regardless of the values of the displacement gain.

5. Remarks and conclusions

In this paper, we have investigated continuous-variable entanglement swapping in a thermal environment. The inseparable condition for the output of the swapping has been derived. We have also compared the noise resulting from the teleportation with that from the direct transmission. It is shown that it is impossible that the noise given by the output state of entanglement swapping is smaller than that by the direct transmission, no matter what values of displacement gain are chosen. However, the entanglement swapping enables us to entangle two quantum states which have never interacted directly with each other. It is known that entanglement plays an essential role in quantum communication, which deals with the transmission and exchange of quantum information between distant locations of a network. Hence the distribution of entangled states between distant nodes will be of great importance in the future large-scale realization of quantum communication protocols. Our protocol for the entanglement swapping in a noisy channel and the continuous-variable entanglement purification being combined, it is of significance to generalize the idea of quantum repeater from discrete variable to continuous-variable systems.

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