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Elastic analysis of a mode II crack in a decagonal quasi-crystal*

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The elastic analysis of a mode II Griffith crack penetrating through a decagonal quasi-crystal along the periodic axis is made within the context of the continuum theory. By using a general solution obtained previously, the problem in the case of uniform shear stress at infinity is solved, and the analytical expressions for the entire stress field disturbed by an internal crack are derived in an explicit form. The asymptotic fields of the displacement and stress around a crack tip in both phonon and phason fields indicate that the stresses near a crack tip exhibit the square-root singularity. The formula for evaluating the energy release rate is also given. If imposing that the phason field is absent, the well-known results of a mode II crack in a conventional material are recovered from the present results.

Keywords: quasi-crystal, crack, elastic field, energy release rate, singularity
PACC: 6140M, 6220M, 4630N

1. Introduction

The physical, chemical and mechanical properties of quasi-crystals have been investigated intensively and extensively by many researchers. Elasticity is one of the interesting properties of quasi-crystals. Great progress has been made in the study of the elastic properties of quasi-crystals. For example, within the framework of the Landau theory, the elastic energies of quasi-crystals have been formulated by Bak[1,2] and Levine *et al.*[3] Hydrodynamics equations of motion have been suggested by Lubensky *et al.*[4] Since quasi-crystals are brittle solids at room temperature[5] it is convenient to adopt the linear elasticity theory of quasi-crystals[6] (based on the notion of a continuum) to analyse the mechanical property of quasi-crystals.

It is well known that dislocations have a significant effect on most behaviours of a conventional material. So far, many experiments have shown that elastic and plastic deformations of a quasi-crystal can be accounted for by the motion of dislocations.[7,8] Therefore, the dislocation-induced elastic field in a quasi-crystal is important in understanding the mechanical behaviour of a quasi-crystal. Along this line, the analytical expressions for the displacement field induced by a dislocation in several classes of quasi-crystals have been derived.[9−13] A united treatment of elastic analysis for a straight dislocation in quasi-crystals has also been made by Ding *et al.*,[14] who extended the Stroh method in conventional anisotropic materials to quasi-crystals. For a three-dimensional (3D) quasi-crystal such as the icosahedral and cubic phases, due to the complicated coupling relations of the phonon and phason fields, it seems unlikely that an exact analytic solution can be obtained. Instead, the elastic Green's function and elasticity problems have been determined approximately via perturbation theory.[15−18] The energy ratio between the phonon and the phason part of dislocations and the dislocation dissociation have been analysed by Ovid'ko[19] and Wollgarten *et al.*[20] The problem involving a dislocation propagating in a one-dimensional hexagonal quasi-crystal has been studied by Fan *et al.*[21]

Recently, some experimental evidence has revealed that some two-dimensional (2D) defects such as planar defects and cracks produced by cleavage are detected in quasi-crystals.[22,23] In addition, some macroscopic cracks or flaws inevitably exist in quasi-crystalline solids. The mechanical properties of quasi-crystals such as fracture toughness, fracture strength, etc, against temperature have been investigated by Yokoyama *et al.*[24] and Watanabe *et al.*[25] Since the introduction of the phason field in quasi-crystals, which gives rise to the significant discrepancy of the

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elastic energy density, and also Hooke's law, between the conventional linear elasticity theory and the linear elasticity theory of quasi-crystals, the usual fracture criteria, strictly speaking, are no longer suitable for quasi-crystals. Some characteristics related to the phason field of crack propagation in a quasi-crystal have been pointed out by Mikulla *et al.*[26] Hence the analysis of cracks in quasi-crystals becomes more complicated than for conventional materials owing to the introduction of the phason filed. Therefore, to quantitatively describe the influence of cracks on the mechanical behaviour of quasi-crystals, crack mechanics in quasi-crystals, including the determination of the elastic field, the energy release rate and so forth, is a prerequisite. A Griffith crack embedded in a decagonal quasi-crystal subjected to a remote uniform tension has been studied by Li *et al.*[27] In general, shear deformation is more interesting for investigating the plastic process of materials.

In this paper we are concerned with a through mode II (shear) crack along the periodic axis in a decagonal quasi-crystal. Based on a general solution in terms of an auxiliary function F, a mixed boundary value problem associated with a mode II crack is reduced to dual integral equations. By solving the resulting equations, the analytical expressions for the entire stress field and the singular behaviour of the stress field around a crack tip are determined. Moreover, the asymptotic displacement field and the energy release rate around a crack tip are given. All the results obtained here in the absence of phason field will reduce to the well-known results of a mode II crack in a conventional material.

2.General solution

According to the description of an n-dimensional quasi-crystal as a quasi-periodic structure which is periodic in a $(3+n)$-dimensional space ($1\leq n \leq 3$), the $(3+n)$-dimensional space can be divided into two orthogonal subspaces, one being 3D physical space and the other being n-dimensional perpendicular space. Therefore, for each quasi-crystal, in addition to the usual phonon displacements u_i describing the local shifts of atoms in the physical space, there are the phason displacements w_i describing the local order of atoms in the perpendicular spaces.[1−3] As suggested by Ding *et al.*,[6] the phonon stress components σ_{ij} in the physical space and the phason stress components H_{ij} in the perpendicular space must satisfy the generalized Hooke's law in quasi-crystals

$$\begin{aligned}\sigma_{ij} &= C_{ijkl}\varepsilon_{kl} + R_{ijkl}w_{kl},\\ H_{ij} &= K_{ijkl}w_{kl} + R_{klij}\varepsilon_{kl},\end{aligned} \tag{1}$$

where $\varepsilon_{ij} = (\partial_j u_i + \partial_i u_j)/2$, $w_{ij} = \partial_j w_i$, $\partial_j = \partial/\partial x_j$, C_{ijkl} and K_{ijkl} are the elastic constants of the phonon and phason fields, respectively, and R_{ijkl} are the phonon–phason coupling elastic constants. These elastic constants have been derived by group representation theory.[28,29]

In particular, for a 2D decagonal quasi-crystal with point groups $10/mmm$, $10mm$, 1022 and $\overline{10}m2$, where the periodic direction is assumed to be orientated in the x_3-axis direction and the quasi-periodic plane is parallel to the x_1x_2-plane, there are only five displacement components, fifteen stress components, and nine independent elastic constants.[30,31]

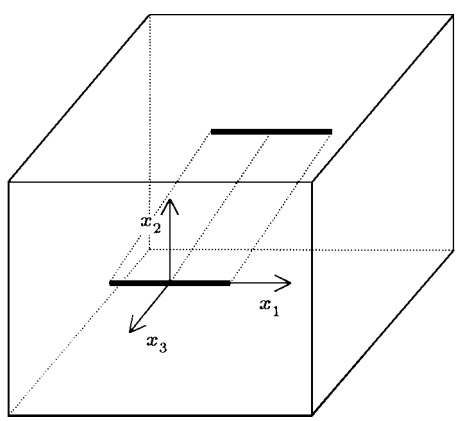


Fig.1. A Griffith crack penetrating through a decagonal quasi-crystal along the periodic direction.

In what follows we consider a decagonal quasi-crystal containing a through Griffith crack along the periodic direction, i.e. the crack front is parallel to the x_3-axis, as shown in Fig.1. In this case, the elastic field along the periodic direction in the phonon field is decoupled from the elastic field in the quasi-periodic plane, and the former can be treated easily as in the classical elasticity theory. Consequently, in the present study we are concerned with the latter. In the absence of body forces, the governing equations of plane elasticity problems for the quasi-periodic plane in a decagonal quasi-crystal are

$$\begin{aligned}&2C_{11}\partial_1^2u_1 + 2C_{66}\partial_2^2u_1 + (C_{11}+C_{12})\partial_1\partial_2u_2\\ &+ 2R[(\partial_1^2 - \partial_2^2)w_1 + 2\partial_1\partial_2w_2] = 0,\end{aligned} \tag{2a}$$

$$\begin{aligned}&(C_{11}+C_{12})\partial_1\partial_2u_1 + 2C_{66}\partial_1^2u_2 + 2C_{11}\partial_2^2u_2\\ &+ 2R[(\partial_1^2 - \partial_2^2)w_2 - 2\partial_1\partial_2w_1] = 0,\end{aligned} \tag{2b}$$

$$K_1\partial_1^2 w_1 + K_1\partial_2^2 w_1 + R[(\partial_1^2 - \partial_2^2)u_1 - 2\partial_1\partial_2 u_2] = 0, \tag{2c}$$

$$K_1\partial_1^2 w_2 + K_1\partial_2^2 w_2 + R[(\partial_1^2 - \partial_2^2)u_2 + 2\partial_1\partial_2 u_1] = 0. \tag{2d}$$

As a result, the key of the task is to determine the elastic field in the quasi-periodic plane. To accomplish this, it is convenient to employ the formulae obtained previously by Li and Fan.[32] That is, for an elasticity problem in the quasi-periodic plane governed by Eqs.(2), if we choose a function F satisfying the following equation

$$\Delta^4 F(x, y) = 0, \tag{3}$$

where

$$\Delta = \partial_1^2 + \partial_2^2, \tag{4}$$

the phonon and phason displacements can be expressed in terms of F as

$$u_1 = R\partial_1\partial_2[c_2\Pi_1 + c_1\Pi_2]\Delta F, \tag{5a}$$

$$u_2 = R[c_2\partial_2^2\Pi_1 - c_1\partial_1^2\Pi_2]\Delta F, \tag{5b}$$

$$w_1 = -Rc_0\partial_1\partial_2\Pi_1\Pi_2 F, \tag{5c}$$

$$w_2 = -[C_{11}c_2\partial_2^2\Pi_1^2 + C_{66}c_1\partial_1^2\Pi_2^2]F, \tag{5d}$$

where

$$c_0 = (C_{11} + C_{12})R/2, \;\; c_1 = C_{11}K_1 - R^2, \;\; c_2 = C_{66}K_1 - R^2, \tag{6}$$

$$\Pi_1 = 3\partial_1^2 - \partial_2^2, \quad \Pi_2 = 3\partial_2^2 - \partial_1^2. \tag{7}$$

With the aid of the generalized Hooke's law, the phonon and phason stress components can also be expressed in terms of F, and these expressions are omitted here for saving space, which can be found in Ref.[32]. It is noted that C_{11} and C_{12} in the above expressions have replaced $2\mu + \lambda$ and λ used in Ref.[32], respectively. On the other hand, it is readily verified that substitution of Eqs.(5) into the governing equations (2) can immediately indicate that these equations are fulfilled automatically.

3. Stress field induced by a mode II crack

We consider a decagonal quasi-crystal solid with a Griffith crack, which penetrates through the medium along the periodic direction (Fig.1). In view of the analysis in the preceding section, it is sufficient to study a crack of length $2a$ occupying the interval, $-a \le x \le a$, y=0, in the quasi-periodic plane. Here we denote x and y as x_1 and x_2, respectively. Let a uniform shear stress $\sigma_{12} = q$ in the physical space act on the quasi-crystal at infinity. Since the phason stress H_{22} belongs to the perpendicular space, no external loads are exerted in the perpendicular space, namely we assume that the phason stress H_{12} at infinity is free, H_{12}=0. It is natural to restrict our attention to the upper half plane due to the symmetry of the problem. Owing to the crack faces being stress-free, the mixed boundary conditions associated with the problem can be stated as

$$\sigma_{12}(x, 0) = 0, \quad H_{12}(x, 0) = 0; \quad |x| \le a, \tag{8a}$$

$$u_1(x, 0) = 0, \quad w_1(x, 0) = 0; \quad |x| \ge a, \tag{8b}$$

$$\sigma_{22}(x, 0) = 0, \;\; H_{22}(x, 0) = 0; \quad -\infty < x < +\infty, \tag{8c}$$

and

$$\sigma_{12}(x, y) \to q, \;\; H_{12}(x, y) \to 0, \quad \sqrt{x^2 + y^2} \to \infty. \tag{9}$$

To solve the mixed boundary value problem defined above, we utilize an integral transform technique. After cumbersome computations, we conclude that the displacement components satisfying the governing equations (2) can take Fourier integrals of the form

$$u_1 = \left(\frac{K_1}{c_2} + \frac{R}{2c_0}\right)qy - \frac{R}{\pi}\int_0^\infty \xi^{-1}X[2(c_1 - c_2)\xi^2 Y' - (6c_1 - 4c_2)\xi Y'' + (7c_1 - 3c_2)Y''']\mathrm{e}^{-\xi y}\cos\xi x\mathrm{d}\xi, \tag{10a}$$

$$u_2 = -\frac{Rqx}{2c_0} - \frac{R}{\pi}\int_0^\infty \xi^{-1}X[2(c_2 - c_1)\xi^2 Y' - (6c_2 - 4c_1)\xi Y'' + (7c_2 - 3c_1)Y''']\mathrm{e}^{-\xi y}\sin\xi x\mathrm{d}\xi, \tag{10b}$$

$$w_1 = \frac{R}{c_2}qy + \frac{c_0 R}{\pi}\int_0^\infty \xi^{-1}X[4\xi^3 Y - 12\xi^2 Y' + 15\xi Y'' - 10Y''']\mathrm{e}^{-\xi y}\cos\xi x\mathrm{d}\xi, \tag{10c}$$

$$w_2 = -\frac{c_0 R}{\pi}\int_0^\infty \xi^{-1}X[4\xi^3 Y - 12\xi^2 Y' + 15\xi Y'' - (10 + e_2 - e_1)Y''']\mathrm{e}^{-\xi y}\sin\xi x\mathrm{d}\xi, \tag{10d}$$

with

$$e_1 = \frac{2c_1c_2}{c_0k_0}, \quad e_2 = \frac{c_1c_2}{c_0k_0}\left(\frac{\overline{c}_1}{c_1} + \frac{\overline{c}_2}{c_2}\right),$$

$$\overline{c}_1 = C_{11}K_2 - R^2, \quad \overline{c}_2 = C_{66}K_2 - R^2,$$

$$k_0 = R(K_1 - K_2), \tag{11}$$

where $X = (A, B, C, D)$, $Y = (1, y, y^2, y^3)^T$, and the prime indicates differentiation with respect to y.

Here A, B, C and D are unknown functions of ξ to be determined from the given boundary conditions (8) and (9), and T stands for the transposition of a matrix. Furthermore, the stress components are also expressed as Fourier integrals. For example, we have

$$\sigma_{22} = -\frac{2c_0c_2}{\pi}\int_0^\infty X[2\xi^2Y' - 4\xi Y'' + 3Y''']\mathrm{e}^{-\xi y}\sin\xi x\mathrm{d}\xi, \tag{12a}$$

$$\sigma_{12} = \sigma_{21} = \frac{2c_0c_2}{\pi}\int_0^\infty X[2\xi^2Y' - 6\xi Y'' + 7Y''']\mathrm{e}^{-\xi y}\cos\xi x\mathrm{d}\xi + q, \tag{12b}$$

$$H_{22} = \frac{c_0k_0}{\pi}\int_0^\infty X[4\xi^3Y - 12\xi^2Y' + 15\xi Y''' - (10 - e_1)Y''']\mathrm{e}^{-\xi y}\sin\xi x\mathrm{d}\xi, \tag{12c}$$

$$H_{12} = -\frac{c_0k_0}{\pi}\int_0^\infty X[4\xi^3Y - 12\xi^2Y' + 15\xi Y''' - (10 + e_2)Y''']\mathrm{e}^{-\xi y}\cos\xi x\mathrm{d}\xi. \tag{12d}$$

Comparing the boundary conditions in Eqs.(8c) with Eqs.(12a) and (12c) when $y = 0$ yields

$$2B_1 = 4C_1 - 3D_1, \quad 4A_1 = 9C_1 - (8 + e_1)D_1, \tag{13}$$

where $A|\xi|^3$, $B\xi^2$, $2C|\xi|$, $6D$ have been replaced by A_1, B_1, C_1, D_1, respectively.

From Eqs.(10) and (12) and the remaining boundary conditions (8a) and (8b), we can obtain that D_1=0 and C_1 satisfies the dual integral equations

$$\frac{4c_0c_2}{\pi}\int_0^\infty C_1\cos(\xi x)\mathrm{d}\xi = q \quad |x| \le a, \tag{14a}$$

$$\int_0^\infty \xi^{-1}C_1\cos(\xi x)\mathrm{d}\xi = 0 \quad |x| > a. \tag{14b}$$

By using the standard theory of dual integral equations (see, for example, Ref.[33]), the solution of Eqs.(14) can be found to be

$$C_1 = \frac{\pi qaJ_1(\xi a)}{4c_0c_2}, \tag{15}$$

where J_1 is the first-order Bessel function of the first kind. Hence A, B, C and D are determined from Eqs.(13) and (15), and thus the elastic field is obtainable from Eqs.(10) and (12). If we introduce three pairs of the polar coordinate systems (r, θ), (r_1, θ_1) and (r_2, θ_2) with the origin at the crack centre, at the right crack tip and the left crack tip, respectively. The Cartesian coordinates (x, y) are related to these three pairs of the polar coordinate systems by the following relationships

$$\begin{aligned} x &= r\cos\theta = a + r_1\cos\theta_1 = -a + r_2\cos\theta_2; \\ y &= r\sin\theta = r_1\sin\theta_1 = r_2\sin\theta_2. \end{aligned} \tag{16}$$

The analytical expressions for the entire stress field which is weakened by a mode II crack can be obtained explicitly, and the final results are

$$\sigma_{11}/q = \overline{r}(\overline{r}_1\overline{r}_2)^{-1/2}\sin(\theta - \overline{\theta}) - \overline{r}(\overline{r}_1\overline{r}_2)^{-3/2}\sin\theta\cos 3\overline{\theta}, \tag{17a}$$

$$\sigma_{22}/q = \overline{r}(\overline{r}_1\overline{r}_2)^{-3/2}\sin\theta\cos 3\overline{\theta}, \tag{17b}$$

$$\sigma_{12}/q = \sigma_{21}/q = \overline{r}(\overline{r}_1\overline{r}_2)^{-1/2}\cos(\theta - \overline{\theta}) - \overline{r}(\overline{r}_1\overline{r}_2)^{-3/2}\sin\theta\sin 3\overline{\theta}, \tag{17c}$$

$$H_{11}/q = 4g\overline{r}(\overline{r}_1\overline{r}_2)^{-1/2}\sin(\theta - \overline{\theta}) - 8g\overline{r}(\overline{r}_1\overline{r}_2)^{-3/2}\sin\theta\cos 3\overline{\theta} - 6g\overline{r}^3(\overline{r}_1\overline{r}_2)^{-5/2}\sin^2\theta\sin(\theta - 5\overline{\theta}), \tag{17d}$$

$$H_{21}/q = 4g\overline{r}(\overline{r}_1\overline{r}_2)^{-1/2}\cos(\theta - \overline{\theta}) - 8g\overline{r}(\overline{r}_1\overline{r}_2)^{-3/2}\sin\theta\sin 3\overline{\theta} - 6g\overline{r}^3(\overline{r}_1\overline{r}_2)^{-5/2}\sin^2\theta\cos(\theta - 5\overline{\theta}), \tag{17e}$$

$$H_{12}/q = -4g\overline{r}(\overline{r}_1\overline{r}_2)^{-3/2}\sin\theta\sin 3\overline{\theta} - 6g\overline{r}^3(\overline{r}_1\overline{r}_2)^{-5/2}\sin^2\theta\cos(\theta - 5\overline{\theta}), \tag{17f}$$

$$H_{22}/q = 4g\overline{r}(\overline{r}_1\overline{r}_2)^{-3/2}\sin\theta\cos 3\overline{\theta} + 6g\overline{r}^3(\overline{r}_1\overline{r}_2)^{-5/2}\sin^2\theta\sin(\theta - 5\overline{\theta}), \tag{17g}$$

where

$$g = \frac{k_0}{4c_2} = \frac{R(K_1 - K_2)}{4(\mu K_1 - R^2)}, \quad \overline{r} = \frac{r}{a}, \quad \overline{r}_1 = \frac{r_1}{a}, \quad \overline{r}_2 = \frac{r_2}{a}, \quad \overline{\theta} = \frac{\theta_1 + \theta_2}{2}.$$

In practice, of special interest are the distributions of the stress and displacement around the crack tips, so we turn our attention to the vicinity of the crack tips. In calculations, some higher-order infinitesimal terms in Eqs.(17) may be neglected. Hence we can, on account of Eqs.(17), obtain the singularity

behaviour of the stress around the right crack tip ($r_1/a \ll 1$) as follows

$$\sigma_{11} = -\frac{K_{\mathrm{II}}^{\parallel}}{\sqrt{2\pi r_1}} \sin\frac{\theta_1}{2}\left(2 + \cos\frac{\theta_1}{2}\cos\frac{3\theta_1}{2}\right), \quad (18a)$$

$$\sigma_{22} = \frac{K_{\mathrm{II}}^{\parallel}}{\sqrt{2\pi r_1}} \sin\frac{\theta_1}{2}\cos\frac{\theta_1}{2}\cos\frac{3\theta_1}{2}, \quad (18b)$$

$$\sigma_{12} = \sigma_{21} = \frac{K_{\mathrm{II}}^{\parallel}}{\sqrt{2\pi r_1}} \cos\frac{\theta_1}{2}\left(1 - \sin\frac{\theta_1}{2}\sin\frac{3\theta_1}{2}\right), \quad (18c)$$

$$H_{11} = -\frac{2gK_{\mathrm{II}}^{\parallel}}{\sqrt{2\pi r_1}}\left[2\sin\frac{\theta_1}{2} + \sin\theta_1 \times \left(2\cos\frac{3\theta_1}{2} - \frac{3}{4}\sin\theta_1\sin\frac{5\theta_1}{2}\right)\right], \quad (18d)$$

$$H_{21} = \frac{2gK_{\mathrm{II}}^{\parallel}}{\sqrt{2\pi r_1}}\left[2\cos\frac{\theta_1}{2} - \sin\theta_1 \times \left(2\sin\frac{3\theta_1}{2} + \frac{3}{4}\sin\theta_1\cos\frac{5\theta_1}{2}\right)\right], \quad (18e)$$

$$H_{12} = -\frac{2gK_{\mathrm{II}}^{\parallel}}{\sqrt{2\pi r_1}}\sin\theta_1\left(\sin\frac{3\theta_1}{2} + \frac{3}{4}\sin\theta_1\cos\frac{5\theta_1}{2}\right), \quad (18f)$$

$$H_{22} = \frac{2gK_{\mathrm{II}}^{\parallel}}{\sqrt{2\pi r_1}}\cos\theta_1\left(\cos\frac{3\theta_1}{2} - \frac{3}{4}\sin\theta_1\sin\frac{5\theta_1}{2}\right), \quad (18g)$$

where $K_{\mathrm{II}}^{\parallel} = q\sqrt{\pi a}$, $K_{\mathrm{II}}^{\parallel}$ is the stress intensity factor in the phonon field for a mode II crack, defined by

$$K_{\mathrm{II}}^{\parallel} = \lim_{x\to a}\sqrt{2\pi(x-a)}\sigma_{12}(x,0). \quad (19)$$

The above results show that the distribution of the phonon stress field is in agreement with the well-known results of a mode II crack in a conventional material[34]. The phason stress field, which arises from the coupling relationship of the phonon and phason fields, is a particular case for quasi-crystals and H_{21}, H_{22} also exhibit the square-root singularity around the crack tips.

4. The asymptotic displacement field and the energy release rate

An entirely similar procedure as above can produce the displacement field of the entire plane and the asymptotic behaviour near the crack tip. For example, the asymptotic field of the displacement around the right crack tip ($r_1/a \ll 1$) can be obtained

$$u_1 = \frac{c_1 R K_{\mathrm{II}}^{\parallel}}{c_0 c_2}\sqrt{\frac{r_1}{2\pi}}\sin\frac{\theta_1}{2}\left[1 + \left(1 - \frac{c_2}{c_1}\right)\cos^2\frac{\theta_1}{2}\right], \quad (20a)$$

$$u_2 = -\frac{RK_{\mathrm{II}}^{\parallel}}{c_0}\sqrt{\frac{r_1}{2\pi}}\cos\frac{\theta_1}{2}\left[1 + \left(1 - \frac{c_1}{c_2}\right)\sin^2\frac{\theta_1}{2}\right], \quad (20b)$$

$$w_1 = w_2 = O(r_1). \quad (20c)$$

In particular, along the crack line, we obtain

$$u_1(x,0) = \begin{cases} \dfrac{Rc_1 q}{2c_0 c_2}\sqrt{a^2 - x^2} & |x| \le a \\ 0 & |x| > a \end{cases},$$

$$w_1(x,0) = 0 \quad -\infty < x < +\infty. \quad (21)$$

From the viewpoint of Griffith energy balance, as in the classical linear elastic fracture mechanics, the energy release rate, or J-integral, is taken as a significant fracture criterion. In quasi-crystals, we assume that under applied loads the crack tip advances along the crack plane from $x = a$ to $x = a + \delta$. The energy release rate during this process is defined by

$$G_{\mathrm{II}} = \lim_{\delta\to 0}\frac{1}{2\delta}\int_0^{\delta}[\sigma_{12}(r)\cdot\Delta u_1(\delta - r) + H_{12}(r)\cdot\Delta w_1(\delta - r)]\mathrm{d}r, \quad (22)$$

where r denotes the distance from the crack tip, and Δu_1 and Δw_1 are the difference between displacements across the crack faces corresponding to the phonon and phason fields, respectively. This integral can be calculated using the asymptotic expressions for the elastic field near the crack tip. After a little manipulation, the energy release rate G_{II} at the crack tip $x = a$ is found to be

$$G_{\mathrm{II}} = \frac{Rc_1}{4c_0 c_2}\pi a q^2 = \frac{(1 - C_{66}C_{11}^{-1}k_{\mathrm{coup}}^2)}{4C_{66}(1 - C_{66}C_{11}^{-1})(1 - k_{\mathrm{coup}}^2)}(K_{\mathrm{II}}^{\parallel})^2, \quad (23)$$

where the phonon–phason coupling coefficient is defined as $k_{\mathrm{coup}} = \sqrt{R^2/C_{66}K_1}$.

On the other hand, with the premise that material failure occurs when G_{II} given by Eq.(23) attains J_{c} (the critical J integral), the applied load strength is predicted to be

$$\sigma_f = 2\sqrt{\frac{J_{\mathrm{c}}C_{66}(1 - C_{66}C_{11}^{-1})(1 - k_{\mathrm{coup}}^2)}{\pi a(1 - C_{66}C_{11}^{-1}k_{\mathrm{coup}}^2)}}. \quad (24)$$

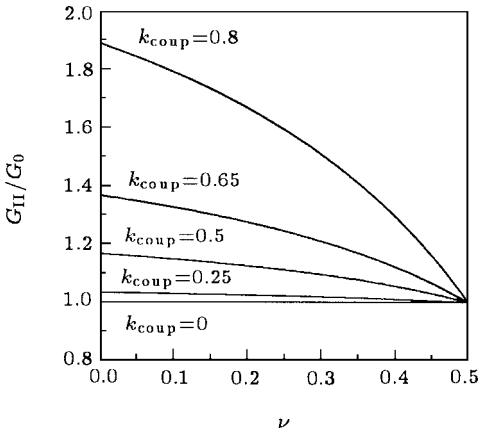


Fig.2. Variation of the dimensionless energy release rate G_{II}/G_0 versus Poisson's ratio ν for different phonon–phason coupling coefficients k_{coup}=0, 0.25, 0.5, 0.65 and 0.8.

A comparison of the energy release rate and its counterpart in a conventional material indicates that they are different, and the former depends on the phonon–phason coupling coefficient k_{coup}. If the coupling elastic constant R is so small that it can be ignored, and C_{11} and C_{66} are taken to be $2\mu + \lambda$ and μ, μ and λ being Lame constants, then the term related to phonon–phason coupling will vanish and the well-known results in conventional materials are recovered presently. Here we take $C_{11} = 2\mu + \lambda$, $C_{66} = \mu$, $\lambda = 2\nu\mu/(1-\nu)$, ν being Poisson's ratio, the variation of the dimensionless energy release rate G_{II}/G_0, G_0 being the corresponding value for $R = 0$, versus ν for different values of k_{coup}, presented graphically in Fig.2.

5. Conclusion

A decagonal quasi-crystal containing a mode II crack, which penetrates through the medium along the periodic axis is considered. With the help of a general solution, the analytical expressions for the entire stress field are derived. From the asymptotic field near the crack tip, it is seen that the stresses in phonon and phason fields around the crack tips exhibit square root singularity and that the distribution of phonon stress is in agreement with well-known results. Additionally, the displacements in phonon and phason fields along the crack line, and the energy release rate are given in explicit form.

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