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Scheme for teleporting an unknown atomic state to any node in a quantum communication network

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We propose a scheme for teleporting an unknown atomic state. In order to realize the teleportation to any node in a quantum communication network, an n-atom Greenberger–Horne–Zeilinger (GHZ) state is needed, which is utilized as the quantum channel. From this n-atom GHZ state, two-node entanglement of processing and receiving teleported states can be obtained through the quantum logic gate manipulation. Finally, for the unequally weighted GHZ state, probabilistic teleportation is shown.

Keywords: teleportation, n-particle GHZ state, probabilistic teleportation, quantum logic gate manipulation

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1. Introduction

Quantum teleportation, proposed by Bennett *et al.*[1] is the process in which an unknown two-state particle, or a qubit, is transmitted from a sender (Alice) to a receiver (Bob) through a quantum channel with the help of some classical information. In their scheme, such a quantum channel is represented by a maximally entangled Bell state. On the one hand, based on the quantum teleportation protocol described in Ref.[1], the group headed by Zeilinger[2] has experimentally demonstrated quantum teleportation for a photon in an unknown polarization state. Davidovich *et al.*,[3] Cirac *et al.*,[4] and Zheng *et al.*[5] have also proposed schemes for teleporting an unknown atomic state. Moussa,[6] Zheng *et al.*[7] and Xu *et al.*[8] have also presented schemes for teleporting certain cavity field states. The teleportation of continuous variables corresponding to states of an infinite-dimensional system has also been presented.[9,10] On the other hand, for teleporting the quantum state, a number of quantum channels are used, such as the Einstein–Podolsky–Rosen (EPR) pair of two-particle,[1] the GHZ state of three particles,[11] and the squeezed vacuum states.[12] In this paper, we propose a scheme to implement teleportation in which the quantum channel is a GHZ state of an n-distant-atom [13,14] and the state that we wish to teleport is the state of an atom initially prepared in an unknown linear superposition of a two-level atom. The objective of this paper is to realize the teleportation of an unknown atomic state to any node in a quantum communication network.

This paper is organized as follows. In section 2, we describe how to prepare the GHZ state of an n-distant-atom, namely a quantum channel of an n-distant-atom; moreover, the teleportation of an unknown atomic state is discussed in this section. Next, in section 3, we generalize this method for an unequally weighted GHZ state of an n-distant-atom, and then we discuss probabilistic teleportation. A concluding summary and discussion are given in section 4.

2. Teleportation of an unknown atomic state

2.1. Preparation of the GHZ state of an n-distant-atom

Let us review the preparation of the GHZ state of an n-distant-atom.[13] For this, we need to share, respectively, a pair of atoms to nodes 1, 2, $\cdots$, n,

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while each pair of the n pairs is in a long-distant EPR pair,[15] i.e., 1 has the EPR pair $(1,1')$, 2 has the EPR pair $(2,2')$, $\cdots$, n has the EPR pair (n,n'). The state of a system composed of such n EPR pairs is $\prod_{i,i'}^{n} |\Psi\rangle_{ii'}$. Then we perform a joint GHZ state measurement for the n atoms $1',2',\cdots,n'$, and another n atom collapses into the following state

$$|\Psi\rangle^{\pm}_{n\text{-GHZ}} = \frac{1}{\sqrt{2}}\left(\prod_{i=1}^{n}|e_i\rangle \pm \prod_{i=1}^{n}|g_i\rangle\right), \qquad (1)$$

where $|e_i\rangle$ and $|g_i\rangle$ denote the excited and ground states of the ith atom, respectively. The state of Eq.(1) is the GHZ state of the n-distant-atom. This state will be used as a quantum channel in our scheme.

2.2. The scheme of teleportation

Now suppose that the $(n-1)$th node (called Alice) wants to send a quantum state

$$|\Psi_a\rangle = a|e\rangle + b|g\rangle \quad \left(|a|^2+|b|^2=1\right) \qquad (2)$$

to any other node of the quantum communication network described by Eq.(1). For example, Alice wants to send this state of Eq.(2) to the nth node, which is called Bob. According to the original scheme proposed by Bennett *et al.*,[1] to teleport a single-atom state, a two-atom maximally entangled state (an EPR singlet state) is eventually used. So we need to obtain a purely quantum channel from the n-GHZ state of Eq.(1). In the discussion below, we will use two methods for obtaining the quantum channel of the two-atom EPR state through quantum logic manipulation. Firstly, in order to obtain the EPR singlet state from Eq.(1), we use a linear polarization analyser with two outcomes $|a_i'\rangle$ ($a_i = e$ or g) to the $(n-1)$th and nth atoms. The state $|a_i'\rangle$ can be written as

$$|a_i'\rangle = \begin{cases} |g'\rangle = (|g\rangle + |e\rangle)/\sqrt{2}; \\ |e'\rangle = (|g\rangle - |e\rangle)/\sqrt{2}. \end{cases} \qquad (3)$$

According to Eq.(3), the states $|g\rangle$ and $|e\rangle$ can be represented as

$$|g\rangle = (|g'\rangle + |e'\rangle)/\sqrt{2};$$
$$|e\rangle = (|g'\rangle - |e'\rangle)/\sqrt{2}. \qquad (4)$$

From the states of Eq.(4), it is easy to expand the state of Eq.(1) as

$$\begin{aligned} |\Psi\rangle_{n\text{-GHZ}} =& |\Phi'\rangle^{+}_{(n-1),n} \otimes |\Psi\rangle^{\pm}_{(n-2)\text{-GHZ}} \\ & - |\Psi'\rangle^{+}_{(n-1),n} \otimes |\Psi\rangle^{\pm}_{(n-2)\text{-GHZ}}, \end{aligned} \qquad (5)$$

where $|\Psi\rangle^{\pm}_{(n-2)\text{-GHZ}}$ are two GHZ states of $(n-2)$ atoms: $1,2,\cdots,n-2$. $|\Psi\rangle^{\pm}_{(n-2)\text{-GHZ}}$ can be written as

$$|\Psi\rangle^{\pm}_{(n-2)\text{-GHZ}} = \frac{1}{\sqrt{2}}\left(\prod_{i=1}^{n-2}|e_i\rangle \pm \prod_{i=1}^{n-2}|g_i\rangle\right), \qquad (6)$$

while $|\Phi'\rangle^{+}_{(n-1),n}$ and $|\Psi'\rangle^{+}_{(n-1),n}$ are two Bell states of the $(n-1)$th and nth atoms. The two Bell states are

$$|\Phi'\rangle^{+}_{(n-1),n} = \frac{1}{\sqrt{2}}\left(|e'_{n-1}e'_n\rangle + |g'_{n-1}g'_n\rangle\right), \qquad (7)$$

and

$$|\Psi'\rangle^{+}_{(n-1),n} = \frac{1}{\sqrt{2}}\left(|e'_{n-1}g'_n\rangle + |g'_{n-1}e'_n\rangle\right). \qquad (8)$$

To measure on the $(n-2)$ atoms $(1,2,\cdots,n-2)$, we need a $(n-3)$ two-qubit controlled NOT gate C and one single-qubit rotation gate R. The rotation gate R performs the following unitary transformation[16,17] on atom 1

$$R_1\left(\frac{\pi}{4}\right)|g_1\rangle = (|e_1\rangle + |g_1\rangle)/\sqrt{2};$$
$$R_1\left(\frac{\pi}{4}\right)|e_1\rangle = (|e_1\rangle - |g_1\rangle)/\sqrt{2}, \qquad (9)$$

and the operation of the controlled NOT gate is described by [14,18,19]

$$C_{ij}|e_i\rangle|u_j^c\rangle = |e_i\rangle|u_j^c\rangle;$$
$$C_{ij}|g_i\rangle|u_j\rangle = |g_i\rangle|u_j\rangle, \qquad (10)$$

where the first subscript of C_{ij} refers to the control bit and the second to the target bit, and $u_i \in \{e,g\}$ and its complement u_i^c (defined as $1-u_i = \{g,e\}$). In terms of the above relations, the results $|\Psi\rangle_{\text{out}}$ of measuring on $(n-2)$ atoms can be represented by

$$\begin{aligned} |\Psi\rangle_{\text{out}} =& R_1\left(\frac{\pi}{4}\right) C_{1,[(n-2)-1]}C_{1,[(n-2)-2]}\cdots C_{1,2} \\ & \times \frac{1}{\sqrt{2}}\left(\prod_{i=1}^{n-2}|e_i\rangle \pm \prod_{i=1}^{n-2}|g_i\rangle\right) \\ =& \cdots = R_1\left(\frac{\pi}{4}\right)\frac{1}{\sqrt{2}}\left(|e_1\rangle \pm |g_1\rangle\right)|g_2\rangle|g_3\rangle\cdots|g_{n-2}\rangle \\ =& \begin{cases} |e_1\rangle|g_2\rangle|g_3\rangle\cdots|g_{n-2}\rangle & \text{(taking +)}; \\ -|g_1\rangle|g_2\rangle|g_3\rangle\cdots|g_{n-2}\rangle & \text{(taking −)}. \end{cases} \end{aligned} \qquad (11)$$

Assuming that the state $|\Psi\rangle_{\text{out}}$ is detected in the above state of Eq.(11), then Alice and Bob can obtain the two atoms of nth and $(n-1)$th in the state $|\Phi'\rangle^{+}_{(n-1),n}$ or $|\Psi'\rangle^{+}_{(n-1),n}$, i.e., a quantum channel of the two-atom EPR state.

Secondly, in this method, instead of expanding the state of Eq.(1) by terms of the Bell state of $(n-1)$th and nth atoms, we manipulate $(n-2)$ atoms by using $(n-3)$ control NOT gates. This procedure is described as follows

$$\begin{aligned}&|\Psi\rangle'_{\text{out}}\\ =&C_{(n-1),[(n-2)-1]}C_{(n-1),[(n-2)-2]}\cdots C_{(n-1),2}C_{(n-1),1}\\ &\times\frac{1}{\sqrt{2}}\left(\prod_{i=1}^{n}|e_i\rangle\pm\prod_{i=1}^{n}|g_i\rangle\right).\end{aligned}\tag{12}$$

In Eq.(9), the $(n-1)$th atom is used as the control bit, $1,2,\cdots(n-2)$ atoms are all the target bit. For the nth atom, no manipulation is performed. Using Eqs.(9) and (10), Eq.(12) becomes

$$\begin{aligned}|\Psi\rangle'_{\text{out}}=&\frac{1}{\sqrt{2}}\left(|e_{n-1}e_n\rangle\pm|g_{n-1}g_n\rangle\right)|g_1\rangle|g_2\rangle\cdots|g_{n-2}\rangle\\ =&|\Phi\rangle^{\pm}_{n-1,n}\otimes|g_1\rangle|g_2\rangle\cdots|g_{n-2}\rangle.\end{aligned}\tag{13}$$

By measuring on $(n-2)$ atoms, the quantum channel of the two-atom EPR state can be also obtained.

In the next step, the teleportation of the unknown state of atom a is carried out. According to the teleportation theorem provided by Bennett *et al*,[1] the joint product state of $|\Psi_a\rangle\otimes|\Phi\rangle^{+}_{n-1,n}$ can be rewritten as

$$\begin{aligned}&|\Psi_a\rangle\otimes|\Phi\rangle^{+}_{n-1,n}\\ =&\frac{1}{2}[|\Phi^{+}_{a,n-1}\rangle\otimes(a|g\rangle_n-b|e\rangle_n)\\ &+|\Phi^{-}_{a,n-1}\rangle\otimes(a|g\rangle_n+b|e\rangle_n)\\ &+|\Psi^{+}_{a,n-1}\rangle\otimes(-a|e\rangle_n+b|g\rangle_n)\\ &+|\Psi^{-}_{a,n-1}\rangle\otimes(-a|e\rangle_n-b|g\rangle_n)],\end{aligned}\tag{14}$$

where

$$|\Phi^{\pm}_{a,n-1}\rangle=\frac{1}{\sqrt{2}}\left(|g_ag_{n-1}\rangle\pm|e_ae_{n-1}\rangle\right)\tag{15}$$

and

$$|\Psi^{\pm}_{a,n-1}\rangle=\frac{1}{\sqrt{2}}\left(|g_ae_{n-1}\rangle\pm|e_ag_{n-1}\rangle\right)\tag{16}$$

are the Bell basis vectors of atom a and the $(n-1)$th atom. To obtain the teleportation information, Alice uses a Bell state analyser to measure atoms a and atom $(n-1)$. With the outcome of a joint measurement on atoms a and $(n-1)$ transmitted to the receiver Bob, an appropriate rotation to atom n to reconstruct the initial state of atom a can be applied. For the two cases mentioned above, the probabilities of successful teleportation are all 50%.

3. The probabilistic teleportation of the unequally weighted GHZ state

In this section, we discuss probabilistic teleportation.[20] The single quantum channel of the n-distant-atom is not in the state of Eq.(1), but is in the following state

$$|\Psi\rangle_{n\text{-GHZ}}=\alpha|g_1g_2\cdots g_n\rangle+\beta|e_1e_2\cdots e_n\rangle,\tag{17}$$

where $|\alpha|^2+|\beta|^2=1$. According to the above procedure, we perform measurements on $(n-3)$ atoms, which do not participate in communication, i.e., atoms $1,2,\cdots,(n-3)$. The GHZ state of three atoms of communication can be obtained

$$|\Psi\rangle_{3\text{-GHZ}}=\alpha|g_{n-2}g_{n-1}g_n\rangle+(-1)^k\beta|e_{n-2}e_{n-1}e_n\rangle,\tag{18}$$

where $k=\sum_{i=1}^{n-3}a_i$, a_i is $\{g,e\}$ and k is an odd or even number. In order to realize the teleportation, a Bell state measurement on atom a and the $(n-2)$th atom is made by Alice, which will project atoms $(n-1)$ and n to the following state

$$\begin{aligned}|\Psi_a\rangle'=&\langle\Phi^{\pm}_{a,(n-2)}|\Psi\rangle_{3\text{-GHZ}}\otimes|\Psi_a\rangle\\ =&\frac{1}{\sqrt{2}}\left[b\alpha|g_{n-1}g_n\rangle\pm(-1)^k a\beta|e_{n-1}e_n\rangle\right],\end{aligned}\tag{19}$$

$$\begin{aligned}|\Psi_a\rangle''=&\langle\Psi^{\pm}_{a,(n-2)}|\Psi\rangle_{3\text{-GHZ}}\otimes|\Psi_a\rangle\\ =&\frac{1}{\sqrt{2}}\left[(-1)^k\beta b|e_{n-1}e_n\rangle\pm a\alpha|g_{n-1}g_n\rangle\right].\end{aligned}\tag{20}$$

According to the discussion in Ref.[21], if Eqs.(19) and (20) are obtained, a unitary transformation on atom $(n-1)$ is made in another basis $\{|x\rangle,|y\rangle\}$. The new basis is related to the old basis $\{|g\rangle,|e\rangle\}$ in the following manner

$$|g\rangle=\alpha|x\rangle+\beta|y\rangle,\tag{21}$$

$$|e\rangle=\beta|x\rangle-\alpha|y\rangle.\tag{22}$$

Thus Eqs.(19) and (20) are respectively transformed as follows

$$\begin{aligned}|\Psi_a\rangle'=&\frac{\alpha^2b|g_n\rangle\pm(-1)^k\beta^2a|e_n\rangle}{\sqrt{2}}|x\rangle\\ &+\frac{\alpha\beta}{\sqrt{2}}\left[b|g_n\rangle\mp(-1)^k a|e_n\rangle\right]|y\rangle,\end{aligned}\tag{23}$$

$$\begin{aligned}|\Psi_a\rangle''=&\frac{(-1)^k\beta^2b|e_n\rangle\pm\alpha^2a|g_n\rangle}{\sqrt{2}}|x\rangle\\ &+\frac{\alpha\beta}{\sqrt{2}}\left[(-1)^k b|e_n\rangle\pm a|g_n\rangle\right]|y\rangle.\end{aligned}\tag{24}$$

If a result of measurement on atom $(n-1)$ is $|x\rangle$ in Eqs.(23) and (24), the teleportation fails. However, if it is $|y\rangle$, Bob can reconstruct the initial state of atom a through an appropriate rotation to atom n, and the teleportation is successful with the probability of $2|\alpha\beta|^2$. Provided that $\alpha = \beta = 1/\sqrt{2}$, it is clear that this case returns to the former.

4. Conclusion

In summary, a scheme is presented for the teleportation of an unknown atomic state between any two nodes of a quantum communication network. In spite of using the n-atom GHZ state in our teleportation, we think it is necessary for our teleportation to send an unknown atomic state to any node. After achieving our teleportation, the n-atom GHZ state is destroyed. However, we have indeed realized our teleportation from one node to any other node. When the network is in a partially n-atom GHZ state, the probabilistic teleportation with probability $2|\alpha\beta|^2$ is shown. Unlike Ref.[21], the unequally weighted GHZ state of three atoms is utilized in our probabilistic teleportation. This scheme may be implemented by entangled photons,[22,23] entangled ions in a linear trap[24], and entangled unclear spins in nuclear magnetic resonance.[25] So we expect that our scheme may be used in quantum communication.

References

[1] Bennett C H, Brassard G, Crepeau C, Jozsa R, Peres A and Wooters W K 1993 *Phys. Rev. Lett.* **70** 1895

[2] Bouwmeester D, Pan J W, Mattle K, Eibl M, Weinfurter H and Zeilinger A 1997 *Nature* **390** 575

[3] Davidovich L, Zagury N, Brune M, Raimound J M and Haroche S 1994 *Phys. Rev.* A **50** R895

[4] Cirac J I and Parkins A S 1994 *Phys. Rev.* A **50** R4441

[5] Zheng S B and Guo G C 1997 *Phys. Lett.* A **232** 171

[6] Moussa M H Y 1996 *Phys. Rev.* A **54** 4661

[7] Zheng S B and Guo G C 1997 *Phys. Lett.* A **236** 180

[8] Xu X M and Luo W D 1999 *Acta Phys. Sin.* **48** 2154 (in Chinese)

[9] Vaidman L 1994 *Phys. Rev.* A **49** 1473

[10] Braunstein S L and Kimble H J 1998 *Phys. Rev. Lett.* **80** 869

[11] Marinatto L and Weber T 2000 *Preprint* quant-ph/0004054

[12] Milburn G J and Braunstein S L 1999 *Phys. Rev.* A **60** 937

[13] Song K H, Jiang Y K, Shi B S and Guo G C 1999 *Phys. Lett.* A **264** 261

[14] Song K H, Jiang Y K, Shi B S and Guo G C 2000 *J. Opt. B: Quant. Semiclass. Opt.* **2** 221

[15] Yang C P and Guo G C 1999 *Phys. Rev.* A **59** 4217

[16] Barenco A, Bennett C H *et al* 1995 *Phys. Rev.* A **52** 2457

[17] Buzek V, Braunstein S L *et al* 1997 *Phys. Rev.* A **56** 3446

[18] Barenco A, Dentsch D, Ekert A and Jozsa R 1995 *Phys. Rev. Lett.* **70** 4083

[19] Duan L M and Guo G C 1997 *Phys. Rev. Lett.* **79** 1953

[20] Li W L, Li C F and Guo G C 2000 *Phys. Rev.* A **61** 034301

[21] Shi B S, Jiang Y K and Guo G C 2000 *Phys. Lett.* A **268** 161

[22] Zeilinger A, Horne M A, Weinfurfer H and Zukowski M 1997 *Phys. Rev. Lett.* **79** 3031

[23] Cerf N J, Adami C and Kwait P G 1998 *Phys. Rev.* A **57** 1477

[24] Cirac J I and Zoller P 1995 *Phys. Rev. Lett.* **74** 4091

[25] Gershenfeld N A and Chuang I L 1997 *Science* **275** 350