Vol 11 No 2, February 2002
1009-1963/2002/11(02)/0120-06

Chinese Physics

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The fine structure splitting of the level of lithium in Rydberg states*

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(Received 16 July 2001; revised manuscript received 11 October 2001)

The Hamiltonian of the four-body problem for a lithium atom is expanded in series. The level shift and level formula of a lithium atom in Rydberg states are achieved by means of the calculation of polarization of the atomic core (including the contribution of dipole, quadrupole and octupole components). We also consider the effect of relativity theory, the orbital angular momentum L and the spin angular momentum S coupling scheme (LS coupling) and high-order correction of the effective potential to the level shift. The fine structure splitting (N=5–12, L=4–9, $J=L\pm1/2$) and level intervals in Rydberg states have been calculated by the above-mentioned formula and compared with recent experimental data.

Keywords: energy levels in Rydberg states, polarization of atomic core, level interval, fine structure
PACC: 3100

1. Introduction

The Rydberg states of a lithium atom consist of a two-electron (helium-like) core and a distant Rydberg electron. These states are more complicated than the Rydberg states of helium. In Refs.[1] and [2] precise measurements of the energy intervals of the lithium atom—$10G_{9/2}$–$10H_{11/2}$, $10G_{7/2}$–$10H_{9/2}$, $10H_{11/2}$–$10I_{13/2}$ and $10H_{9/2}$–$10I_{11/2}$—have been made by using precision radio-frequency measurements (with an accuracy of approximately 10^{-4} parts per MHz). As is well known, if the atom is in the Rydberg state, its level may be described by a Rydberg formula of atomic spectra, but it will appear as a function of the effective quantum number n^* and the quantum defect $\Delta_{\rm i}$; in general, n^* and $\Delta_{\rm i}$ may be achieved by means of experiment in spectral analysis.[1,3,4] We are interested in theoretically finding the relation[5] between the quantum numbers n^*, $\Delta_{\rm i}$ and n, L, J. In this paper, the Hamiltonian of a four-body problem for lithium atoms is expanded in series. Then, the integrated expression for the dipole, quadrupole and octupole of the atomic core is obtained, and the corresponding effective potential is further derived from the method of adiabatic approximation, pseudostate summation and Hylleraas-style wavefunctional analysis.[6] The fine structure splitting and the level shift of a lithium atom in Rydberg states are obtained by means of the calculation of the Hamiltonian matrix diagonal elements, the consideration of the leading relativistic correction, the LS coupling and high-order correction of the effective potential to level shift. We find that the calculated result of the energy intervals of lithium atoms in Rydberg states is close to the newest test data.

2. Separation of $\hat{H}$ and expansion in Rowland series

When the lithium atom is in the high-excited state of one electron, this electron does not penetrate the core. Its exchange with the core electrons is negligible, and it may be taken to be distinguishable from them, hence only the polarization of the core is considered. We assume that electron 3 is the valence electron and is much farther from the nucleus than the other two. Then the four-body Hamiltonian (in Rydberg state) including the nucleus is

$$\hat{H} = -\nabla_1^2 - \nabla_2^2 - \nabla_3^2 - \frac{1}{M}\nabla_M^2$$

*Project supported by the Natural Science Foundation of Chongqing Education Committee of China (Grant No. 990213) and by the Basic Research of Application of Chongqing Science and Technology Committee, China (Grant No. 199966).

$$-\frac{2z}{|\boldsymbol{r}_1-\boldsymbol{r}_M|}-\frac{2z}{|\boldsymbol{r}_2-\boldsymbol{r}_M|}-\frac{2z}{|\boldsymbol{r}_3-\boldsymbol{r}_M|}+\frac{2}{|\boldsymbol{r}_1-\boldsymbol{r}_2|}+\frac{2}{|\boldsymbol{r}_1-\boldsymbol{r}_3|}+\frac{2}{|\boldsymbol{r}_2-\boldsymbol{r}_3|}. \quad (1)$$

In order to easily separate the Hamiltonian operator $\hat{H}$, it is convenient to introduce the relative coordinates ρ_1 and ρ_2, which represent the vector from the nucleus to each core electron, and $\boldsymbol{x}$, the radius vector from the centre of mass of the core to the valence electron. These are

$$\boldsymbol{\rho}_{1,2}=m(\boldsymbol{r}_{1,2}-\boldsymbol{r}_M), \quad (2)$$

$$\boldsymbol{x}=m'\left(\boldsymbol{r}_3-\frac{\boldsymbol{r}_1+\boldsymbol{r}_2+M\boldsymbol{r}_M}{M+2}\right), \quad (3)$$

where M is the mass of the nucleus, $m=M/(M+1)$ is the reduced mass of one electron and the nucleus, while $m'=(M+2)/(M+3)$ is the reduced mass of the valence electron and the core. The Hamiltonian can be written as the sum of three parts: $\hat{H}_{\rm c}$, $\hat{H}_{\rm v}$ and V, representing the core, the valence electron, and the interaction between them, respectively. These are

$$\hat{H}=\hat{H}_{\rm c}+\hat{H}_{\rm v}+V, \quad (4)$$

$$\hat{H}_{\rm c}=-\nabla^2_{\boldsymbol{\rho}_1}-\nabla^2_{\boldsymbol{\rho}_2}-k\nabla_{\boldsymbol{\rho}_1}\cdot\nabla_{\boldsymbol{\rho}_2}-\frac{2z}{|\boldsymbol{\rho}_1|}-\frac{2z}{|\boldsymbol{\rho}_2|}+\frac{2}{|\boldsymbol{\rho}_1-\boldsymbol{\rho}_2|}, \quad (5)$$

$$\hat{H}_{\rm v}=-q\left[\nabla^2_x+\frac{2(z-2)}{x}\right], \quad (6)$$

$$V=2q\left[\frac{1}{|\boldsymbol{x}-f\boldsymbol{\rho}_1+g\boldsymbol{\rho}_2|}+\frac{1}{|\boldsymbol{x}-f\boldsymbol{\rho}_2+g\boldsymbol{\rho}_1|}-\frac{z}{|\boldsymbol{x}+g(\boldsymbol{\rho}_1+\boldsymbol{\rho}_2)|}+\frac{z-2}{x}\right]. \quad (7)$$

Here $k=2/(M+2)$, $q=m'/m$, $f=2q/(2+k)$, $g=kf/2$, and $x=|\boldsymbol{x}|$. Since there is no x in Eq.(5), $\hat{H}_{\rm c}$ has no contribution to the transition for the valence electron between Rydberg states, hence we consider $\hat{H}_{\rm c}$ to be a constant. The influence of the core on the valence electron is described by the interaction potential V. $\hat{H}_{\rm v}$ corresponds to the Hamiltonian operator of hydrogen-like atoms. Due to the little difference between q and $m_{\rm e}$, the eigenvalue of $\hat{H}_{\rm v}$ may be evaluated only by $R_{\rm Li}$ of the Rydberg constant of Li replacing $R_{\rm H}$, which appears in the level formula of the hydrogen atom. Therefore, the calculation of the energy of lithium in Rydberg can be summed up as the level shift with the interaction term in Eq.(7).

We note that

$$\frac{1}{|\boldsymbol{x}-f\boldsymbol{\rho}_1+g\boldsymbol{\rho}_2|}=\frac{1}{x}\left[\frac{1}{1-\dfrac{2f\boldsymbol{x}\cdot\boldsymbol{\rho}_1}{\boldsymbol{x}^2}+\dfrac{f^2\boldsymbol{\rho}_1^2}{\boldsymbol{x}^2}+\dfrac{g^2\boldsymbol{\rho}_2^2}{\boldsymbol{x}^2}-\dfrac{2g\boldsymbol{x}\cdot\boldsymbol{\rho}_2}{\boldsymbol{x}^2}-\dfrac{2gf\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_2}{\boldsymbol{x}^2}}\right]^{1/2}. \quad (8)$$

When the lithium atom is in the Rydberg state, it seems a long distance for x compared with $|f\boldsymbol{\rho}_{1,2}|$ and $|g\boldsymbol{\rho}_{1,2}|$. Hence Eq.(7) may be expanded in the Rowland series in negative power x^{-k}, and thus the coefficient of x^{-1} in the Rowland series is zero, and the terms of $\boldsymbol{x}\cdot\boldsymbol{\rho}_i$, $(\boldsymbol{x}\cdot\boldsymbol{\rho}_i)^2$, $(\boldsymbol{x}\cdot\boldsymbol{\rho}_1)(\boldsymbol{x}\cdot\boldsymbol{\rho}_2)$ appear on the expanding expression. By the definition of Legendre polynomials, the first three terms of the expanded Eq.(7) are

$$V_1=\frac{2q^2w}{x^2}\left[\frac{2+(z-1)k}{2+k}\right]\mathrm{P}_1(\hat{\boldsymbol{w}}\cdot\hat{\boldsymbol{x}}), \quad (9)$$

$$V_2=\frac{2q^3}{x^3}\left[\boldsymbol{\rho}_1^2\mathrm{P}_2(\hat{\boldsymbol{\rho}}_1\cdot\hat{\boldsymbol{x}})+\boldsymbol{\rho}_2^2\mathrm{P}_2(\hat{\boldsymbol{\rho}}_2\cdot\hat{\boldsymbol{x}})-\frac{4k-zk^2}{(2+k)^2}w^2\mathrm{P}_2(\hat{\boldsymbol{w}}\cdot\boldsymbol{x})\right], \quad (10)$$

$$V_3=\frac{2q^4}{x^4}\left\{\frac{2-k}{2+k}[\boldsymbol{\rho}_1^3\mathrm{P}_3(\hat{\boldsymbol{\rho}}_1\cdot\hat{\boldsymbol{x}})+\boldsymbol{\rho}_2^3\mathrm{P}_3(\hat{\boldsymbol{\rho}}_2\cdot\hat{\boldsymbol{x}})]-\frac{4k-zk^2-zk^3}{(2+k)^3}w^3\mathrm{P}_3(\hat{\boldsymbol{w}}\cdot\boldsymbol{x})\right\}, \quad (11)$$

where P_1, P_2 and P_3 are the linear, quadric and cubic Legendre polynomials, respectively, and $w=|\boldsymbol{w}|=|\boldsymbol{\rho}_1+\boldsymbol{\rho}_2|$, $\hat{\boldsymbol{w}}=(\boldsymbol{\rho}_1+\boldsymbol{\rho}_2)/|\boldsymbol{\rho}_1+\boldsymbol{\rho}_2|$, $\rho_i=|\boldsymbol{\rho}_i|$, $\hat{\boldsymbol{\rho}}_i=\boldsymbol{\rho}_i/|\boldsymbol{\rho}_i|$. By the definition of the multipole moment of the atom,[7] Eqs.(9), (10) and (11) correspond, respectively, to the integrating expressions of the dipole, the quadrupole and the octupole moments of the core of the lithium atom. Then the influence of the polarization of the core on the energy level is summed up as the contribution of the dipole, the quadrupole and the octupole moments.

3. Computation of the fine structure splitting

Using the adiabatic approximation, the pseudostatic summation and Hylleraas-style wavefunctional analysis,[8−12] the corresponding effective potential of V_1 , V_2 and V_3, acting on the valence electron of lithium in the Rydberg state, is derived from Eqs.(9), (10) and (11)

$$\begin{aligned} U(x) =& U_4(x) + U_6(x) + U_7(x) + U_8(x) \\ =& \frac{C_4}{x^4} + \frac{C_6}{x^6} + \frac{C_7}{x^7} + \frac{C_8}{x^8} \\ =& -\frac{0.1924854}{x^4} + \frac{0.097881}{x^6} + \frac{0.143125}{x^7} \\ & - \frac{0.428584 + 0.049005L(L+1)}{x^8}. \end{aligned} \tag{12}$$

As mentioned above, $\hat{H}_{\rm v}$ corresponds to the energy operator of a hydrogen-like atom. Thus the level of the lithium atom in the Rydberg state is

$$\varepsilon(n, L) = -\frac{q(z-2)R_{\rm Li}}{n^2} + \langle n, L|U(x)|n, L\rangle, \tag{13}$$

where $\langle n, L|$ is the eigenstate of $\hat{H}_{\rm v}$, and the level shift is

$$\Delta(n, L) = R_{\rm Li}(U_4 + U_6 + U_7 + U_8). \tag{14}$$

Here

$$U_k = C_k \left\langle n, L \left| \frac{1}{x^k} \right| n, L \right\rangle \qquad (k = 4, 6, 7, 8). \tag{15}$$

$R_{\rm Li}$ is the Rydberg constant of the ^{7}Li atom, and $R_{\rm Li}$= 3.289584678×10^{9}MHz. The normalized radial wavefunction of a hydrogen-like atom, $R_{nL}(x)$,[13] is

$$\begin{aligned} R_{nL}(x) =& \left[\frac{(n+L)!(n-L-1)!}{2n}\right]^{1/2} \\ & \times \left(\frac{2z}{n}\right)^{L+3/2} x^{-Zxn} \sum_{S}^{n-L-1} (-1)^S \\ & \times \frac{(2z/n)^S}{S!(2L+1-S)!(n-L-1-S)!}. \end{aligned} \tag{16}$$

Substituting this into Eq.(15), we can derive

$$\begin{aligned} U_k =& \frac{C_k(n+L)!(n-L-1)!}{2n}\left(\frac{2}{n}\right)^k \\ & \times \sum_{S,P}^{n-L-1} \frac{(-1)^{S+P}(2L+2+S+P-k)!}{S!(2L+1+S)!(n-L-1-S)!P!(2L+1+P)!(n-L-1-P)!}. \end{aligned} \tag{17}$$

Table 1 shows the level shift, which is brought about by U_k, of which the sum has been given by Eq.(14). Since we have taken consideration of U_8, it is not consistent to omit the effect of including C_4/x^4 to the second-order Δ_2 (it is proportional to x^{-8}).

According to the method of obtaining the solution Δ_2 of helium in Ref.[5], the corresponding Δ_2 of lithium, which has a 16-digit significant figure, is obtained, i.e.

$$\begin{aligned} \Delta_2 =& -4C_4^2\left[\frac{(2L-3)!!}{(2L+3)!!}\right]^2 \left\{ \frac{(2L-3)!!}{(2L+3)!!}\ \frac{1}{n^7}\left(\frac{9n^4}{f_1^3(L)} - \frac{6n^4}{f_1^2(L)}\right)[45 + 7(89f_1(L) + 520f_2(L) + 80f_3(L))] \right. \\ & \left. + \frac{(2L-3)!!}{(2L+3)!!}\ \frac{1}{n^7}[3 + 40(f_1(L) + 6f_2(L))] + \frac{1}{n^8}\left[\frac{27n^4}{f_1^2(L)} - \frac{30n^2}{f_1(L)}\right]\right\}, \end{aligned} \tag{18}$$

where

$$\begin{aligned} f_1(L) &= L(L+1), \\ f_2(L) &= (L-1)L(L+1)(L+2), \\ f_3(L) &= (L-2)(L-1)L(L+1)(L+2). \end{aligned}$$

Table 1. The corresponding level shift of the effective potential U_k of ^{7}Li (MHz).

n	L	U_4	U_6	U_7+U_8	$\Delta(n,L)=\Sigma U_k$
5	4	–3216.2390	8.7226	–0.3983	–3207.9147
6	4	–2068.0549	7.3030	–0.5003	–2061.2522
7	4	–1380.8582	5.4914	–0.4452	–1375.8120
8	4	–959.21101	4.07656	–0.36253	–955.49698
9	4	–690.12513	3.05774	–0.28809	–687.35548
10	4	–511.67472	2.33172	–0.22840	–509.45713
11	4	–389.19234	1.80936	–0.18220	–387.56516
12	4	–302.59356	1.42758	–0.14671	–301.31271
6	5	–658.01747	0.66391	6.233×10^{-3}	–657.34733
7	5	–456.66186	0.60179	1.3934×10^{-3}	–456.05867
8	5	–324.31281	0.48628	-1.0036×10^{-3}	–323.82753
9	5	–236.62803	0.38259	-1.9171×10^{-3}	–236.24735
10	5	–177.11821	0.30070	-2.1341×10^{-3}	–176.81964
11	5	–135.67732	0.23818	-2.0558×10^{-3}	–135.44119
12	5	–105.97605	0.19071	-1.8678×10^{-3}	–105.78720
7	6	–175.63918	0.08101	1.2303×10^{-3}	–175.55693
8	6	–128.69558	0.07726	9.6046×10^{-4}	–128.61735
9	6	–95.69864	0.06576	6.9022×10^{-4}	–95.63219
10	6	–72.53408	0.05409	4.9214×10^{-4}	–72.47950
11	6	–56.03569	0.04412	3.5556×10^{-4}	–55.99121
8	7	–56.626053	0.013634	1.9149×10^{-4}	–56.61222
9	7	–43.207225	0.013451	1.7303×10^{-4}	–43.19360
10	7	–33.290261	0.011899	1.4163×10^{-4}	–33.27783
11	7	–26.007712	0.010134	1.1303×10^{-4}	–25.99746
9	8	–21.025976	0.002901	3.468×10^{-5}	–21.02304
10	8	–16.554171	0.002929	3.351×10^{-5}	–16.55121
11	8	–13.119039	0.002669	2.917×10^{-5}	–13.11634
10	9	–8.712722	0.000738	7.44×10^{-6}	–8.71197
11	9	–7.032889	0.000757	7.40×10^{-6}	–7.03212

We note that Eq.(13) does not consider the relativistic correction and the LS coupling. In order to derive the leading relativistic term in the energy, the lithium atom is considered as a two-body system: the core and the valence electron. We use the Breit operator expression[14]

$$\hat{H}_1 = -\frac{\alpha^2}{4}(m')^4 R_\infty \left(1+\frac{k^3}{(2+k)^3}\right)\nabla_x^4, \qquad (19)$$

$$\hat{H}_2 = \frac{\alpha^2(m')^3 R_\infty}{2+k}\frac{1}{x}\left(\nabla_x^2+\frac{\partial^2}{\partial x^2}\right), \qquad (19)'$$

where α is the fine structure constant.

Substituting $m'=(m+2)/(m+3)=(2+k)/(2+2k)$ and $R_\infty = R(M+3)/(M+2)=R(2+2k)/(2+k)$ into the above equations, and expanding $\hat{H}_1$ and $\hat{H}_2$ in the power series of k, we obtain the following expression for the relativistic correction

$$\begin{aligned}\Delta_{\rm rel} =& \langle n,L|\hat{H}_1+\hat{H}_2|n,L\rangle \\ =& \frac{\alpha^2 R}{n^3}\left(\frac{3}{4n}-\frac{2}{2L+1}\right)+\alpha^2 Rk \\ & \times\left(-\frac{1}{8n^4}+\frac{3}{2n^3(L+1/2)}-\frac{3}{2n^3(L+1/2)}\right) \\ =& \frac{\alpha^2 R}{n^3}\left[\frac{3}{4n}\left(1-\frac{k}{6}\right)-\frac{2}{2L+1}\right]. \end{aligned} \qquad (20)$$

Since the coefficient of k^2 is zero, we obtain a corresponding correction accurate to the second order of k^2. Comparing Eq.(20) with the relativistic correction formula of a hydrogen-like atom,[15] there is only an additional term $\alpha^2 Rk/8n^3$ whose influence is inconsiderable in any case. As the amounts of the spin and orbit angular momenta of the core are both considered to be zero, so the LS coupling of the lithium atom in Rydberg states is summed up as the LS coupling of the valence electrons, spin and orbit angular momenta.

According to the Thomas formula, the level of $\varepsilon(n,L)$ is split into two levels: $\varepsilon(n,L+1/2)$ and $\varepsilon(n,L-1/2)$. Furthermore

$$\varepsilon(n,L+1/2)=\varepsilon'(n,L)+\Delta(n,j=L+1/2), \quad (21)$$

$$\varepsilon(n,L-1/2)=\varepsilon'(n,L)+\Delta(n,j=L-1/2). \quad (21)'$$

Here

$$\Delta(n,j=L+1/2)=\frac{\alpha^2 R}{2n^3(L+1)(2L+1)}, \quad (22)$$

$$\Delta(n,j=L-1/2)=-\frac{\alpha^2 R}{2n^3 L(L+1)}, \quad (22)'$$

$$\varepsilon'(n,L)=\varepsilon(n,L)+\Delta_2+\Delta_{\rm rel}. \quad (23)$$

The corresponding level shift of the fine structure is given by

$$\Delta\varepsilon(n,L+1/2)$$
$$=\Delta(n,L)+\Delta_2+\Delta_{\rm rel}+\Delta(n,j=L+1/2), \quad (24)$$

$$\Delta\varepsilon(n,L-1/2)$$
$$=\Delta(n,L)+\Delta_2+\Delta_{\rm rel}+\Delta(n,j=L-1/2). \quad (24)'$$

Table 2 shows the total amount of the level shift of the fine structure splitting, including the polarization of the atomic core, the relativistic effect and the LS coupling, i.e. a list of the calculated values of $\Delta\varepsilon(n,L\pm1/2)$. Table 3 shows the comparison between the theoretical computation of the corresponding level intervals of lithium and the experimental result from Ref.[1].

Table 2. The level shift and the fine structure splitting of ^{7}Li (MHz).

(n,L,j)			Δ_2	$\Delta_{\rm rel}$	$\Delta(n,L,j)$	$\Delta(n,L)$	$\Delta\varepsilon$
9	4	9/2	–0.12859	–33.37476	5.339882	–687.35548	–715.518948
9	4	7/2	–0.12859	–33.37476	–6.674852	–687.35548	–727.533682
10	4	9/2	–0.09668	–25.78995	3.892774	–509.45713	–531.450986
10	4	7/2	–0.09668	–25.78995	–4.865967	–509.45713	–540.209727
11	4	9/2	–0.07421	–20.27370	2.924698	–387.56516	–404.988372
11	4	9/2	–0.07421	–20.27370	–3.655873	–387.56516	–411.568943
9	5	11/2	–0.01171	–23.66589	3.640828	–236.24735	–256.284122
9	5	9/2	–0.01171	–23.66589	–4.368994	–236.24735	–264.293944
10	5	11/2	–0.00899	–18.71219	2.654164	–176.81964	–192.886656
10	5	9/2	–0.00899	–18.71219	–3.184997	–176.81964	–198.725817
11	5	11/2	–0.00701	–14.95607	1.994112	–135.44119	–148.410158
11	5	9/2	–0.00701	–14.95607	–2.392935	–135.44119	–152.797205
9	6	13/2	–0.00155	–16.94437	2.640601	–95.63219	–109.937509
9	6	11/2	–0.00155	–16.94437	–3.080701	–95.63219	–115.658811
10	6	13/2	–0.00123	–13.81219	1.924998	–72.47950	–84.367922
10	6	11/2	–0.00123	–13.81219	–2.245831	–72.47950	–88.538751
11	6	13/2	–0.00098	–11.27463	1.446279	–55.99121	–65.820541
11	6	11/2	–0.00098	–11.27463	–1.687326	–55.99121	–68.954146

Table 3. The comparison between the theoretical computation of the corresponding level intervals of lithium and the experimental result.

$\Delta\varepsilon(n,j)$		Interval	Computation/MHz	Exp. result/MHz	Error/‰
$10G_{9/2}$:	–531.450986	$10G_{9/2}$–$10H_{11/2}$	338.5643	338.4778(33)	0.25
$10H_{11/2}$:	–192.886656				
$10G_{7/2}$:	–540.209727	$10G_{7/2}$–$10H_{9/2}$	341.4839	341.4027(35)	0.23
$10H_{9/2}$:	–198.725817				
$10H_{11/2}$:	–192.886656	$10H_{11/2}$–$10I_{9/2}$	108.5187	108.4819(51)	0.33
$10I_{13/2}$:	–84.367922				
$10H_{9/2}$:	–198.725817	$10H_{9/2}$–$10I_{11/2}$	110.1870	110.1566(54)	0.27
$10I_{11/2}$:	–88.538751				

4. Conclusion

1. Once the stationary state of a lithium atom in Rydberg state is given, i.e. the quantum numbers n, L and j are given, then one may evaluate U_k, $\Delta(n, L)$, $\Delta_2, \Delta_{\rm rel}$ and $\Delta(n, j = L \pm 1/2)$ by means of the expressions (14), (17), (18), (20), (22) and (22)' (only using the algebraic method) and may further evaluate the level shift. If necessary, one can evaluate the corresponding level $\varepsilon(n, L \pm 1/2)$ in the Rydberg state by means of the analytic expressions (13), (15), (23), (21) and (21)'.

2. In Table 3, the relevant experimental results of fine structure splitting in n=manifold of ^{7}Li in the Rydberg state are given: the level intervals of $10G_{9/2}$–$10H_{11/2}$, $10G_{7/2}$–$10H_{9/2}$, $10H_{11/2}$–$10I_{13/2}$ and $10H_{9/2}$–$10I_{11/2}$ correspond to 338.4778(33), 341.4027(35), 108.4819(51) and 110.1566(54)MHz, respectively. Our calculated results are 338.5646, 341.4842, 108.5173 and 110.1856MHz respectively; the error in calculation is only (0.2∼0.3) $^0/_{00}$. It is thus obvious that the results of the theoretical calculation are close to the newest data.

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