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Analytical method for yrast line states in the interacting two-component Bose-Einstein condensate

Xie Bing-Hao(解炳昊) and Jing Hui(景 辉)

Theoretical Physics Division, Nankai Institute of Mathematics, Nankai University, Tianjin 300071, *China*

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The yrast spectrum for the harmonically trapped two-component Bose–Einstein condensate (BEC), omitting the difference between the two components, has been studied using an analytical method. The energy eigenstates and eigenvalues for L=0,1,2,3 are given. We illustrate that there are different eigenstate behaviours between the even L and odd L cases for the two-component BEC in two dimensions. Except for symmetric states, there are antisymmetric states for the permutation of the two components, which cannot reduce to those in a single condensate case when the value of L is odd.

Keywords: Bose–Einstein, condensate, yrast line states
PACC: 0375, 6740, 0530

1. Introduction

Over the last few years, much attention has been paid to experimental investigations[1] and theoretical studies[2] for systems consisting of two- and multi-component Bose–Einstein condensates (BECs), since such systems give rise to the possibility of observing a rich set of new macroscopic quantum phenomena which do not exist in a single condensate. As is well known, for a single condensate much research has recently been carried out on the low-lying excitations of atomic BECs in harmonic traps, particularly on the low-lying excitations when the condensate has a non-zero total angular momentum.[3−7] Mottelson[4] and Wilkin *et al.*[5] studied the yrast line of N-boson systems with weak interactions, i.e. the ground states at a non-vanishing angular momentum L. Bertsch and Papenbrock[6] investigated numerically the yrast line for harmonically trapped N-boson systems with weak contact interactions and made a conjecture on the yrast line expression for $L \ll N$. Recently, Wu *et al.*[8] and Jackson *et al.*[9] have studied the yrast line states using different analytical methods. But what happens for the yrast spectrum of two- or multi-component BECs? In this paper, we extend the method developed in Ref.[8] to a two-component BEC.

2. The analytical method and the results for low angular momentum

The general second quantized Hamiltonian for a two-component interacting BEC in a trap (assuming the interactions do not change the internal state of the atoms) is described by[10]

$$\hat{H} = \int \mathrm{d}^3 r \Big\{ \sum_{i=a,b} \Big[\hat{\psi}_i^+(\boldsymbol{r}) \hat{h}_i \hat{\psi}_i(\boldsymbol{r}) + \frac{1}{2} g_{ii} \hat{\psi}_i{}^+(\boldsymbol{r}) \hat{\psi}_i{}^+(\boldsymbol{r}) \hat{\psi}_i(\boldsymbol{r}) \hat{\psi}_i(\boldsymbol{r}) \Big] + g_{ab} \hat{\psi}_a{}^+(\boldsymbol{r}) \hat{\psi}_b{}^+(\boldsymbol{r}) \hat{\psi}_b(\boldsymbol{r}) \hat{\psi}_a(\boldsymbol{r}) \Big\}. \quad (1)$$

Here h_i is the single-particle Hamiltonian for atoms in internal states i, and g_{ij} is the two-body interaction strength between an atom in state i and an atom in state j. In terms of corresponding scattering lengths they are given by $g_{ij} = \dfrac{4\pi\hbar^2}{a_{ij}m}$. In this paper, we study a harmonically trapped two-component BEC under the two-dimensional rotation, i.e. $h_i = -\dfrac{\hbar^2}{2m}\nabla_i^2 + \dfrac{1}{2}m\omega^2 r_i^2$. For simplicity, we consider the case $g_{aa} = g_{bb} = g_{ab} = 2g$ and omit the difference single-particle energy between the two components (this is allowed if the difference energy is much smaller than the potential energy). So here we discuss a simplified Hamiltonian for the two-component BEC which can be considered as a reasonable approximation. Thus, our Hamiltonian can be written in the

following form

$$\hat{H}=\sum_{j=0}^{\infty}j\omega(a_j^+a_j+b_j^+b_j)+\sum_{i,j,k,l}^{\infty}\delta_{i+j,k+l}V_{ijkl}\times(a_i^+a_j^+a_ka_l+b_i^+b_j^+b_kb_l+2a_i^+b_j^+b_ka_l),\quad(2)$$

where a_j^+ and a_j are, respectively, the creation and annihilation operators of the jth boson, and V_{ijkl} characterizes two-body interactions among bosons, which is given by

$$V_{ijkl}=\frac{g(k+l)!}{2^{(k+l)}\sqrt{i!j!k!l!}}.$$

Here g can be either positive or negative (corresponding to repulsive and attractive cases respectively). We can easily see that there are two conserved quantities: the total number of bosons N and the total angular momentum L, as follows

$$N=N_a+N_b=\sum_{j=0}^{\infty}(a_j^+a_j+b_j^+b_j),\qquad(3)$$

$$L=L_a+L_b=\sum_{j=0}^{\infty}j(a_j^+a_j+b_j^+b_j).\qquad(4)$$

Note that the numbers of bosons N_a and N_b are also conserved quantities, but the angular momenta L_a and L_b are not.

Two points need clarifying further here. Firstly, the upper limit ∞ in the above summation can be replaced by L; for example, $L=\sum_{j=0}^{\infty}jn_j\rightarrow L=\sum_{j=0}^{L}jn_j$, for a fixed L. Secondly, the angular momentum L only represents one component of an angular momentum for a trapped BEC in two dimensions. The value of L is always chosen to be non-negative, i.e. the trapped particles rotate only in the anti-clockwise direction. There is a good illustration of this point in Ref.[11].

Our task is to solve the eigenvalue equation $\hat{H}|\psi\rangle=E|\psi\rangle$. To this end, following the method of Wu *et al.*,[8] we first assume that the eigenkets $|\psi\rangle$ take the following form

$$|\psi\rangle=F(a_0^+,a_1^+,\cdots,a_l^+;b_0^+,b_1^+,\cdots,b_l^+)|\mathrm{vac}\rangle,\qquad(5)$$

where $|\mathrm{vac}\rangle$ denotes the vacuum state satisfying $a_j|\mathrm{vac}\rangle=0$, $b_j|\mathrm{vac}\rangle=0$ for $j=0,1,2,\cdots$ and F is the linear combination of the terms $\Pi_i\Pi_j(a_i^+)^{n_{a_i}}(b_j^+)^{n_{b_j}}$, subject to the constraints that N_a, N_b and L are conserved quantities. $|\psi\rangle=F|\mathrm{vac}\rangle$ is the common eigenket of $\hat{H}$, $\hat{N}$ and $\hat{L}$ with eigenvalues E, N and L, respectively. The eigenvalue equation can be rewritten as $EF|\mathrm{vac}\rangle=\hat{H}F|\mathrm{vac}\rangle=[\hat{H},F]|\mathrm{vac}\rangle$ due to $\hat{H}|\mathrm{vac}\rangle=0$. Then utilizing the following relations

$$[a_k^+,F]=[b_k^+,F]=0,$$

$$[a_k,F]=\partial F/\partial a_k^+\ ,\ [b_k,F]=\partial F/\partial b_k^+,$$

$$[a_ma_j,F]|\mathrm{vac}\rangle=\partial^2F/\partial a_m^+\partial a_j^+|\mathrm{vac}\rangle\ ,$$
$$[b_mb_j,F]|\mathrm{vac}\rangle=\partial^2F/\partial b_m^+\partial b_j^+|\mathrm{vac}\rangle,$$

we can show that the polynomial function F satisfies the following differential equation

$$\sum_{j,k,l=0}^{L}V_{k+l-j,j,k,l}\left(\frac{\xi_{k+l-j}\xi_j\partial^2}{\partial\xi_k\partial\xi_l}+\frac{\eta_{k+l-j}\eta_j\partial^2}{\partial\eta_k\partial\eta_l}+\frac{2\xi_{k+l-j}\eta_j\partial^2}{\partial\xi_k\partial\eta_l}\right)F=(E-\omega L)F,\qquad(6)$$

where $\xi_j{=}a_j^+$, $\eta_j{=}b_j^+$, $j{=}0,1,\cdots,L$ and the first term of Eq.(2) has been replaced by ωL because L is a conserved quantity. $F(\xi_0,\xi_1,\cdots,\xi_l;\eta_0,\eta_1,\cdots,\eta_l)$ is the linear combination of the terms $\Pi_i\Pi_j(\xi_i^+)^{n_{\xi_i}}(\eta_j^+)^{n_{\eta_j}}$. The problem of obtaining the energy eigenstates and energy eigenvalues has now been turned into a problem of finding the polynomial solutions to a second-order linear partial differential equation (6). Letting $x_j=\xi_j/\sqrt{j!}$, $y_j=\eta_j/\sqrt{j!}$, Eq.(6) becomes

$$\sum_{M=0}^{L}\sum_{m,k=0}^{M}\frac{M!}{2^Mm!(M-m)!}\left(\frac{x_kx_{M-k}\partial^2}{\partial x_m\partial x_{M-m}}+\frac{y_ky_{M-k}\partial^2}{\partial y_m\partial y_{M-m}}+\frac{2x_ky_{M-k}\partial^2}{\partial x_m\partial y_{M-m}}\right)F=\lambda F,\qquad(7)$$

where $\lambda=(E-\omega L)/g$, and its unit is $\omega\hbar/g$.

We use $F[N,L,\lambda]$ to denote the non-zero polynomial solution to Eq.(7), i.e. $F[N,L,\lambda]|\mathrm{vac}\rangle=|N,L,\lambda\rangle$ with $|N,L,\lambda\rangle$ describing the common (possibly non-normalized) eigenstate of the three operators $\hat{N}$, $\hat{L}$ and $\hat{H}$ with eigenvalues N, L and $E=\omega L+g\lambda$, respectively. The ket vectors $|N,L,\lambda\rangle$ are simply called energy eigenstates hereafter. Letting $P(L)$ denote the multiplicity of the partition of the non-negative inter L into two sets of positive integers, one can easily see that the polynomial function F is the linear combination of, at most, $P(L)$ terms of the form $\Pi_j\Pi_kx_j^{n_{x_j}}y_k^{n_{y_k}}$ with the constraints that N_a, N_b and L are conserved quantities. In other words, we need only to solve the eigenvalue problem of the $P(L)\times P(L)$ matrix to obtain all the energy eigenstates for a fixed L. So it is

a fairly systematical matter for us to obtain the explicit result for yrast line states with an arbitrary N. This method allows us to quickly obtain explicitly the analytical expression of low-L energy eigenstates even when the particle number N is very large. We now illustrate this point by considering $L = 0, 1, 2, 3$ with arbitrary $N \geq L$.

L=0 case: There is only one polynomial solution

$$F[N, 0, \lambda] = (a_0^+)^{N_a}(b_0^+)^{N_b}, \tag{8}$$

with

$$\lambda = N(N-1). \tag{9}$$

Here the eigenstate $F[N, 0, \lambda]|\text{vac}\rangle$ has not been normalized. We can normalize the eigenstates here and below by the follwoing method. We express F as the linear combination of the terms of the form $\Pi_{k=0}^{L}\Pi_{l=0}^{L}\xi_k\eta_l$, $\xi_k = a_k^{+\,n_{ak}}/\sqrt{n_{ak}!}$, $\eta_l = b_l^{+\,n_{bl}}/\sqrt{n_{bl}!}$ with coefficients $\alpha, \beta, \cdots$, and then we multiply F by $1/\sqrt{|\alpha|^2 + |\beta|^2 + \cdots}$ resulting in a normalized eigenstate. For the simplicity of the expression, here and below we will not normalize the eigenstates.

L=1 case: The constraint equations (3) and (4) allow us to set

$$F = \alpha(a_0^+)^{N_a-1}a_1^+(b_0^+)^{N_b} + \beta(b_0^+)^{N_b-1}b_1^+(a_0^+)^{N_a}. \tag{10}$$

Substituting this into Eq.(7), we obtain two solutions

$$\begin{aligned} F[N, 1, \lambda_1] =& N_a(a_0^+)^{N_a-1}a_1^+(b_0^+)^{N_b} \\ &+ N_b(b_0^+)^{N_b-1}b_1^+(a_0^+)^{N_a}, \end{aligned} \tag{11}$$

with

$$\lambda_1 = N(N-1), \tag{12}$$

and

$$\begin{aligned} F[N, 1, \lambda_2] =& (a_0^+)^{N_a-1}a_1^+(b_0^+)^{N_b} \\ &- (b_0^+)^{N_b-1}b_1^+(a_0^+)^{N_a}, \end{aligned} \tag{13}$$

with

$$\lambda_2 = N(N-2). \tag{14}$$

The solution $F[N, 1, \lambda_2]$ is noticeable. The value λ_1 is equal to the value λ for the single condensate case[8] with same total particle number N, but the value of λ_2 is smaller. In fact, if b is identified with a (considering the two components to be the same), the solution $F[N, 1, \lambda_1]$ will reduce to the solitary solution $F[N, 1, \lambda]$ in the single condensate case with the same energy eigenvalue. But the solution $F[N, 1, \lambda_2]$ will vanish if we identify b with a, so this solution has no counterpart in the single condensate case.

L=2 case: Using the constraint equations (3) and (4) we can let

$$\begin{aligned} F =& \alpha(a_0^+)^{N_a}(b_0^+)^{N_b-1}b_2^+ + \beta(b_0^+)^{N_b}(a_0^+)^{N_a-1}a_2^+ \\ &+ \gamma(a_0^+)^{N_a-1}a_1^+(b_0^+)^{N_b-1}b_1^+ \\ &+ \rho(a_0^+)^{N_a-2}(a_1^+)^2(b_0^+)^{N-b} \\ &+ \upsilon(b_0^+)^{N_b-2}(b_1^+)^2(a_0^+)^{N-a}. \end{aligned} \tag{15}$$

Substituting this into Eq.(7), we obtain a set of eigenfunctions with five parameters. Solving these equations we obtain the following results

$$\begin{aligned} F(N, 2, \lambda_1) =& \sqrt{2}(a_0^+)^{N_a}(b_0^+)^{N_b-1}b_2^+ \\ &+ \sqrt{2}(b_0^+)^{N_b}(a_0^+)^{N_a-1}a_2^+ \\ &+ N(a_0^+)^{N_a-1}a_1^+(b_0^+)^{N_b-1}b_1^+ \\ &+ \left(\frac{N}{2}-1\right)(a_0^+)^{N_a-2}(a_1^+)^2(b_0^+)^{N_b} \\ &+ \left(\frac{N}{2}-1\right)(b_0^+)^{N_b-2}(b_1^+)^2(a_0^+)^{N_a}, \end{aligned} \tag{16}$$

with

$$\lambda_1 = N(N-1), \tag{17}$$

and

$$\begin{aligned} & F(N, 2, \lambda_2) \\ =& \sqrt{2}(1-N)(a_0^+)^{N_a}(b_0^+)^{N_b-1}b_2^+ + \sqrt{2}(1-N) \\ & \times (b_0^+)^{N_b}(a_0^+)^{N_a-1}a_2^+ + N(a_0^+)^{N_a-1}a_1^+(b_0^+)^{N_b-1}b_1^+ \\ & + \left(\frac{N}{2}-1\right)(a_0^+)^{N_a-2}(a_1^+)^2(b_0^+)^{N_b} \\ & + \left(\frac{N}{2}-1\right)(b_0^+)^{N_b-2}(b_1^+)^2(a_0^+)^{N_a}, \end{aligned} \tag{18}$$

with

$$\lambda_2 = N(N-2). \tag{19}$$

The two solutions are in fact symmetric for the permutation of the two components. We can see that they are identified with the two solutions in the single condensate case[8] with same total particle number if we consider the two components to be the same.

L=3 case: For simplicity, we select $N_a = N_b = \frac{N}{2}$. Because of the symmetry between a and b, it is reasonable to set

$$\begin{aligned} & F_1(N, 3, \lambda) \\ =& \alpha(a_0^+)^{\frac{N}{2}-3}(a_1^+)^3 b_0^{+\,\frac{N}{2}} + \alpha(b_0^+)^{\frac{N}{2}-3}(b_1^+)^3 a_0^{+\,\frac{N}{2}} \\ & + \beta(a_0^+)^{\frac{N}{2}-2}(a_1^+)^2 b_1^+ b_0^{+\,\frac{N}{2}-1} \\ & + \beta(b_0^+)^{\frac{N}{2}-2}(b_1^+)^2 a_1^+ a_0^{+\,\frac{N}{2}-1} \\ & + \gamma(a_0^+)^{\frac{N}{2}-2}a_1^+ a_2^+ b_0^{+\,\frac{N}{2}} + \gamma(b_0^+)^{\frac{N}{2}-2}b_1^+ b_2^+ a_0^{+\,\frac{N}{2}} \\ & + \rho(a_0^+)^{\frac{N}{2}-1}a_1^+ b_2^+ b_0^{+\,\frac{N}{2}-1} + \rho(b_0^+)^{\frac{N}{2}-1}b_1^+ a_2^+ a_0^{+\,\frac{N}{2}-1} \\ & + \upsilon(a_0^+)^{\frac{N}{2}-1}a_3^+(b_0^+)^{\frac{N}{2}} + \upsilon(b_0^+)^{\frac{N}{2}-1}b_3^+(a_0^+)^{\frac{N}{2}}. \end{aligned} \tag{20}$$

After solving the five-parameter eigenvalue functions, we obtain three solutions

1)

$$\alpha=\frac{1}{6}N^2-\frac{21}{40}N+\frac{59}{120},\quad \beta=\frac{1}{4}N^2-\frac{19}{40}N+\frac{7}{40},$$

$$\gamma=\sqrt{2}\left(\frac{2}{5}N-\frac{127}{100}\right),\ \rho=\sqrt{2}\left(\frac{3}{5}N+\frac{27}{100}\right),\ \upsilon=\frac{\sqrt{6}}{3}, \tag{21}$$

with

$$\lambda_1=N(N-1); \tag{22}$$

2)

$$\alpha=-N+2,\quad \beta=-3(N-2),$$

$$\gamma=2\sqrt{2}(N-3),\quad \rho=2\sqrt{2}(N-3),\quad \upsilon=4\sqrt{6}, \tag{23}$$

with

$$\lambda_2=N(N-2); \tag{24}$$

3)

$$\alpha=1,\quad \beta=3,$$

$$\gamma=3\sqrt{2},\quad \rho=3\sqrt{2},\quad \upsilon=2\sqrt{6}, \tag{25}$$

with

$$\lambda_3=N\left(N-\frac{5}{2}\right). \tag{26}$$

Illuminated by the solution $F[N,1,\lambda_2]$, we can try to set

$$\begin{aligned}
&F_2(N,3,\lambda)\\
=&\alpha(a_0^+)^{\frac{N}{2}-3}(a_1^+)^3b_0^{+\frac{N}{2}}-\alpha(b_0^+)^{\frac{N}{2}-3}(b_1^+)^3a_0^{+\frac{N}{2}}\\
&+\beta(a_0^+)^{\frac{N}{2}-2}(a_1^+)^2b_1^+b_0^{+\frac{N}{2}-1}\\
&-\beta(b_0^+)^{\frac{N}{2}-2}(b_1^+)^2a_1^+a_0^{+\frac{N}{2}-1}\\
&+\gamma(a_0^+)^{\frac{N}{2}-2}a_1^+a_2^+b_0^{+\frac{N}{2}}-\gamma(b_0^+)^{\frac{N}{2}-2}b_1^+b_2^+a_0^{+\frac{N}{2}}\\
&+\rho(a_0^+)^{\frac{N}{2}-1}a_1^+b_2^+b_0^{+\frac{N}{2}-1}-\rho(b_0^+)^{\frac{N}{2}-1}b_1^+a_2^+a_0^{+\frac{N}{2}-1}\\
&+\upsilon(a_0^+)^{\frac{N}{2}-1}a_3^+(b_0^+)^{\frac{N}{2}}-\upsilon(b_0^+)^{\frac{N}{2}-1}b_3^+(a_0^+)^{\frac{N}{2}}.
\end{aligned} \tag{27}$$

Then we also have a set of equations with five parameters. Solving this set of equations we obtain following two solutions:

1)

$$\alpha=-N-17,\quad \beta=-N-7,$$

$$\gamma=\sqrt{2}\left(N+\frac{3}{2}\right),\quad \rho=-\sqrt{2}\left(N-\frac{3}{4}\right),\quad \upsilon=2\sqrt{6}, \tag{28}$$

with

$$\lambda_4=N\left(N-\frac{5}{2}\right)+\frac{7}{2}; \tag{29}$$

2)

$$\alpha=N-3,\quad \beta=-N-\frac{9}{2},$$

$$\gamma=-2\sqrt{2}(6N+7),\quad \rho=-2\sqrt{2}(N+6),\quad \upsilon=4\sqrt{6}, \tag{30}$$

with

$$\lambda_5=N(N-3)+\frac{13}{4}. \tag{31}$$

The terms, which are not more than N^{-1}, have been omitted in the calculation. We can see that for the L=3 case there are five solutions, three symmetric and two antisymmetric, for the permutation of a and b (the two components). Similar to the L=1 case, the three symmetric states coincide with, and the energy eigenvalues are equal to, the three solutions in the single condensate.[8] But the two antisymmetric solutions vanish if we identify a with b, and so have no counterpart in the single condensate case.

For the $L>3$ case, the calculation is more complicated, although the calculation principle is still simple. From the solutions with L=0,1,2,3, we know that when $L=0,2$, i.e. the value of L is even, the eigenstates for the two-component BEC are symmetric for the permutation of a and b. The eigenstates coincide with, and the energy eigenvalues are equal to, the results in the single condensate case with the same total particle number. But when L=1, 3, i.e. the value of L is odd, except for symmetric states, there are other antisymmetric states for the permutation of a and b with new energy eigenvalues. The symmetric states can reduce to the eigenstates in the single condensate case with same total particle number if we identify a with b. But the antisymmetric states have no counterparts in the single condensate case. We can also see that, for L=1, 3 ($N>6$), the values of λ in the antisymmetric case are smaller than those in the symmetric case.

Now we illustrate that, when the value of L is odd, there are either symmetric or antisymmetric eigenstates; however, when the value of L is even, there are only symmetric eigenstates for the permutation of the two components. The reason for this is simple. One can see that when the value of L is even, the number of terms in the expression of F is odd, while it will be even when the value of F is odd. When the value of L is even, one term in the expression of F is self-symmetric for the permutation of a and b, and the other terms can be split into two sets which are symmetric for the permutation. So we can only obtain symmetric states for the permutation. But, when the value of L is odd, the terms in the expression of F can be split into two sets directly which are symmetric for the permutation, so we can have either symmetric or

antisymmetric states.

3. Conclusions

In summary, an analytical method has been used to solve the yrast spectrum of a two-component BEC. Using this method, the energy eigenvalue problem for fairly universal systems with either weak or strong two-body interactions is turned into a problem of solving a second-order linear partial differential equation. Although we have only studied a simplified model, it is helpful to understand the main behaviour of the yrast spectrum for a two-component BEC. We have given all the solutions of the yrast spectrum for L=0, 1, 2, 3 cases, and discussed the difference between the eigenstates and energy eigenvalue corresponding to even L and odd L. The result is interesting. When the value of L is odd, there is a different eigenstate behaviour between the two-component and single BECs in two dimensions with same total particle number, even if we omit the difference of single-particle energy and the two-body interaction strength in the two-component case (i.e. the two components seem to be the same). The difference lies in the fact that the two-component case has antisymmetric eigenstates for the permutation of the two components with new energy eigenvalues. But when the value of L is even, the eigenvalues of the two cases are both the same and the eigenstates coincide. Although this is very possible, we are not sure whether, when the value of L is odd and N is large, there will be smaller λ for antisymmetric states than those for symmetric states. This is important because smaller λ will lead to higher and lower energy eigenvalues in the attractive ($g < 0$) case and repulsive case ($g > 0$), respectively. The problem of how to prove this point is an interesting subject to be studied further.

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