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How to control a geometric quantum gate*

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In this paper, we detail the theoretical ideas which are used to explain the mechanism of the laser controlling the geometric quantum gates introduced in the work by Ekert *et al.* (Ekert A, Ericsson M, Hayden P, Inanori H, Jones J A, Oi D K L and Vedral V 2000 *J. Mod. Opt.* **47** 2501). We have introduced a two-level Hamiltonian system, and directed to solve this system, and then obtained the probability distribution of this two-level system. We also show the relationships between the external laser fields and the transition of the qubit in the two-qubit controlled-phase gate, and how the transition of the qubit depends on the external laser fields and the states of the controlled qubit.

Keywords: laser controlling mechanism, geometric quantum gate, two-level system
PACC: 0365, 0367

1. Introduction

Recently, an exciting development in the field of quantum mechanics has been the discovery that quantum systems can be used to perform novel information processing tasks, including computations which are more efficient than any algorithm known in classical computing.[1−5] Quantum information processing requires the ability to execute conditional dynamics[6] between two quantum bits (qubits), where the state of one qubit influences the evolution of the other qubit during a quantum computation. Simple quantum information processing has been demonstrated using nuclear magnetic resonance (NMR)[7−10] and trapped ions.[11]

Another very surprising effect recently discovered in the same field is that a quantum system retains a memory of its motion when it undergoes a cyclic evolution.[12−17]

Recent experimental work has managed to combine these two quantum phenomena in the form of geometric quantum computation,[18−20] and in Ref.[21] the theoretical ideas behind geometric quantum computation are detailed. In Ref.[21], Ekert *et al.* have shown that the Berry phase[12] may be used to implement conditional phase shifts, and thus any quantum gate.[22] In this paper, we try to study in detail the mechanism of the laser controlling the geometric quantum gates. In particular, we show the relationship between the external laser field and the transition of the qubit in a two-qubit controlled-phase gate. We begin with a brief introduction to a two-qubit controlled-phase gate, proceeding to analyse the mechanism of the laser controlling the geometric quantum gate in order to see in detail how the transition of the qubit in this gate depends on the external laser fields. Finally, we give a conclusion about the laser controlling mechanism on the quantum gate.

2. Brief introduction to the geometric quantum gate

In order to discuss how to control a geometric quantum gate conveniently, we firstly introduce the definition of a geometric quantum gate.[21] Ekert *et al.*[21] used the geometric quantum phase[12] to implement a two-qubit controlled-phase gate that they called the geometric quantum gate. They begin with a system of two non-interacting spin-half particles S_a and S_b and consider these spin-half particles in a reference frame aligned with the static field. Then the Hamiltonian reads

$$H_0 = \hbar\omega_a S_{az} \otimes \mathbf{1}_b + \hbar\omega_b \mathbf{1}_a \otimes S_{bz}, \qquad (1)$$

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where the frequencies $\omega_a/2\pi$ and $\omega_b/2\pi$ are the transition frequencies of the two spins, and the spin operators S_i have been scaled as the Pauli operators $\sigma_i/2$.

When the two particles are sufficiently close to each other, they will interact, creating an additional splitting between the energy levels of the composite system composed of two subsystems, A and B. Because only two spin-half particles are considered, the magnetic field of one spin may directly or indirectly affect the energy levels of the other; the energy of the composite system is increased by $\pi\hbar J/2$ if the two spins are parallel, and it is decreased by $\pi\hbar J/2$ if the two spins are anti-parallel. The Hamiltonian of the composite system, in which this interaction is taken into account, reads

$$H = H_0 + 2\pi\hbar J S_{az} \otimes S_{bz}, \tag{2}$$

where $\otimes$ means that these operators S_a and S_b operate on two particles A and B, respectively, and J is the coupled constant.

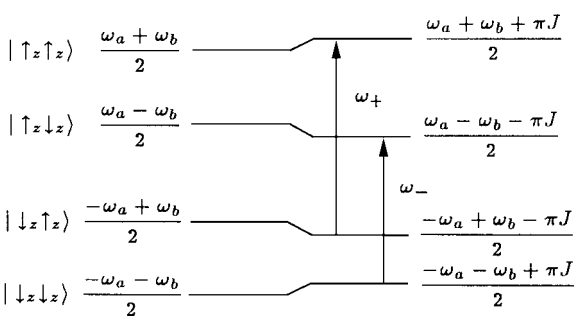


Fig.1. The energy diagram of two interacting spin-half particles. The transition frequency of the first spin depends on the state of the second.

Figure 1 illustrates the energy levels of the composite system. When spin S_b is in state $|\uparrow_z\rangle$ or $|\downarrow_z\rangle$, the transition frequency of the spin S_a is

$$\omega_\pm = \omega_a \pm \pi J. \tag{3}$$

In addition to the static field, if one applies a rotating field that is slowly varied, the Berry phase acquired by the spin S_a, which depends on its transition resonance frequency, will be different for the two possible states of spin S_b. Indeed, when spin S_b is in state $|\uparrow_z\rangle$ or $|\downarrow_z\rangle$, the Berry phase acquired by the spin S_a is $\gamma_\pm = \mp\pi(1-\cos\theta_\pm)$, with a negative or positive sign in front of the expression, depending on whether spin S_a is up or down, respectively, and

$$\cos\theta_\pm = \frac{\omega_\pm - \omega}{\sqrt{(\omega_\pm-\omega)^2+\omega_1^2}}, \tag{4}$$

where $\omega/2\pi$ and $\hbar\omega_1$ are the frequency and the amplitude of the oscillating field, respectively. If one defines the differential Berry phase shift

$$\begin{aligned}\Delta\gamma &= \gamma_+ - \gamma_- \\ &= \pi\left(\frac{\omega_+-\omega}{\sqrt{(\omega_+-\omega)^2+\omega_1^2}} - \frac{\omega_--\omega}{\sqrt{(\omega_--\omega)^2+\omega_1^2}}\right),\end{aligned} \tag{5}$$

then the net transformation, up to global phases, is given by

$$\begin{pmatrix} e^{2i\Delta\gamma} & 0 & 0 & 0 \\ 0 & e^{-2i\Delta\gamma} & 0 & 0 \\ 0 & 0 & e^{-2i\Delta\gamma} & 0 \\ 0 & 0 & 0 & e^{2i\Delta\gamma} \end{pmatrix}. \tag{6}$$

Thus, one succeeds in engineering a conditional evolution since the state of the qubit S_b influences the phase acquired by the qubit S_a.[23] This geometric quantum gate (for more details, see Ref.[21]), which introduces a phase of $e^{2i\Delta\gamma}$ if the two spins are aligned, or a phase of $e^{-2i\Delta\gamma}$ if they are anti-aligned, is equivalent to the controlled-phase gate introduced in Ref.[20].

3. Laser controlling mechanism

As mentioned above, when the spin S_b is in state $|\uparrow_z\rangle$, the transition frequency of and the Berry phase acquired by the spin S_a are ω_+ and γ_+ respectively. However, when the spin S_b is in state $|\downarrow_z\rangle$, the transition frequency of and the Berry phase acquired by the spin S_a are ω_- and γ_- respectively. Therefore, the state of the qubit S_b influences the phase acquired by a second qubit S_a which introduces a Berry phase. They consist of the controlled-phase gate–geometric quantum gate described by Eq.(6). Now we ask how the geometric quantum gate works and how it is realized. The key to the geometric quantum gate is the transition under the control of the laser fields. Now we analyse the mechanism of laser control on this geometric quantum gate.

In order to implement a two-qubit controlled-phase gate, we use two types of field that act on the two-spin-half system: one is a static magnetic field which is much stronger and acts on this system in the z-axis direction; the other is a laser field which acts on the system in the x-axis direction. The former (static magnetic field) acts on both particles S_a and S_b, but the latter acts only on particle S_a or particle S_b. Since we want to discuss the transition action of the spin-half particle S_a on which the laser field acts, we can let

the spin-half particle S_b be in its eigenstate so that the two-body problem changes into a one-body problem.

Since the transition frequency of the spin-half particle S_a depends on the state of the qubit S_b, we discuss, in general, the transition ω_+ of the spin-half particle S_a when S_b is in state $|\uparrow_z\rangle$. In this case, the Hamiltonian (2) can be reduced to

$$H = \hbar(\omega_a + \pi J)S_{az} = \hbar\omega_+ S_z, \tag{7}$$

where in the second equality, we omit the subscript a of S_{az} since we only discuss one spin-half particle hereafter.

We consider a laser field acting only on the spin-half particle S_a. The Hamiltonian of the system that takes into account the interaction reads

$$H = H_0 + H' = \hbar\omega_+ S_z + \frac{e\hbar}{\mu c}\boldsymbol{S}\cdot\boldsymbol{B}, \tag{8}$$

where $\boldsymbol{B}$ is the external magnetic field which comes from the external laser fields $\boldsymbol{A}$

$$\boldsymbol{A} = \frac{1}{2}\boldsymbol{e}\left(A_0 \mathrm{e}^{-\mathrm{i}(\omega_{\mathrm{L}}t - \boldsymbol{k}\cdot\boldsymbol{r})} + A_0^* \mathrm{e}^{\mathrm{i}(\omega_{\mathrm{L}}t - \boldsymbol{k}\cdot\boldsymbol{r})}\right), \tag{9}$$

and is expressed as

$$\boldsymbol{B} = \bigtriangledown \times \boldsymbol{A} = \frac{1}{2}\boldsymbol{B}_0\left[\mathrm{e}^{-\mathrm{i}\omega_{\mathrm{L}}t} + \mathrm{e}^{\mathrm{i}\omega_{\mathrm{L}}t}\right]. \tag{10}$$

Here $\boldsymbol{B}_0 = \mathrm{i}A_0\boldsymbol{e}\times\boldsymbol{k}$, and $\boldsymbol{e}$ is the polarization direction unit vector of the external laser field $\boldsymbol{A}$. In Eq.(10), we have made a zero-order correction to function $\mathrm{e}^{\pm\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}$ on the space coordinates $\boldsymbol{r}$.

Now we assume that the direction of the laser field $\boldsymbol{A}$ points to the y-direction with the laser wave travelling along the z-direction, then the magnet field $\boldsymbol{B}$ points to the x-direction. So the Hamiltonian (8) can be rewritten as

$$H = H_0 + H'(t) = \hbar\omega_+ S_z + \frac{e\hbar}{\mu c}S_x B_x(t). \tag{11}$$

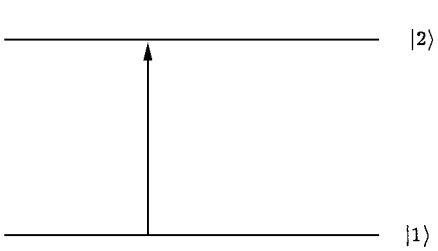


Fig.2. The two-level system.

It is obvious that the Hamiltonian H of the system is a 2×2 matrix and the system described by Eq.(11) is a two-level system. Now we return to solving this two-level system directly so that the control of the transition of the spin-half particle can be solved.

The time evolution of a general quantum system is governed by the Schödinger equation,

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\psi(\boldsymbol{r},t) = H(\boldsymbol{r},t)\psi(\boldsymbol{r},t), \tag{12}$$

where $H(\boldsymbol{r},t)$ is the Hamiltonian and can usually be written as

$$H(\boldsymbol{r},t) = H_0(\boldsymbol{r}) + H'(\boldsymbol{r},t). \tag{13}$$

Equations (8) or (11) are in this form. Using the eigenvector $\psi_n(\boldsymbol{r},t)$ of the static Hamiltonian $H(\boldsymbol{r})$, the solution $\psi(\boldsymbol{r},t)$ to Eq.(12) can be expanded as

$$\begin{aligned}\psi(\boldsymbol{r},t) &= \sum_n C_n(t)\psi_n(\boldsymbol{r},t)\\ &= \sum_n C_n(t)|u_n(\boldsymbol{r})\rangle \mathrm{e}^{-\mathrm{i}E_n t/\hbar}.\end{aligned} \tag{14}$$

Substituting Eq.(14) into Eq.(12) and using the orthogonal normalized relations

$$\langle u_m(\boldsymbol{r})|u_n(\boldsymbol{r})\rangle = \delta_{mn}\delta^3(\boldsymbol{r}-\boldsymbol{r}), \tag{15}$$

yield

$$\mathrm{i}\hbar\frac{\mathrm{d}}{\mathrm{d}t}C_m(t) = \sum_n C_n(t)\mathrm{e}^{-\mathrm{i}(E_n - E_m)t/\hbar}\langle u_n|H'(t)|u_n\rangle. \tag{16}$$

After obtaining the eigenvectors and eigenvalues of $H_0(\boldsymbol{r})$, and the coefficient $C_n(t)$ determined by the above equation, we can completely describe the time evolution of the wavefunction $\psi(\boldsymbol{r},t)$ under the governing of $H(\boldsymbol{r},t)$.

Since we want to discuss the transition of the spin-half particles, it is obviously a two-level system (see Fig.2). For this system, the wavefunction $\psi(\boldsymbol{r},t)$ described by Eq.(14) can be rewritten as

$$\psi(\boldsymbol{r},t) = C_1(t)|u_1\rangle\mathrm{e}^{-\mathrm{i}E_1 t/\hbar} + C_2(t)|u_2\rangle\mathrm{e}^{-\mathrm{i}E_2 t/\hbar}, \tag{17}$$

and the Schrödinger equation (16) can be reduced to

$$\frac{\mathrm{d}C_1(t)}{\mathrm{d}t} = -\frac{\mathrm{i}}{\hbar}\left[C_1(t)H'_{11}(t) + C_2(t)H'_{12}(t)\mathrm{e}^{\mathrm{i}\omega_{12}t}\right], \tag{18}$$

$$\frac{\mathrm{d}C_2(t)}{\mathrm{d}t} = -\frac{\mathrm{i}}{\hbar}\left[C_1(t)H'_{21}(t)\mathrm{e}^{\mathrm{i}\omega_{21}t} + C_2(t)H'_{22}(t)\right], \tag{19}$$

where the transition frequency $\omega_{12} = (E_1 - E_2)/\hbar$. The eigenvalues of H_0 in Eq.(11) are

$$E_1 = -\frac{1}{2}\hbar\omega_+, \qquad E_2 = \frac{1}{2}\hbar\omega_+, \tag{20}$$

and the eigenfunctions are

$$|u_1\rangle \equiv |\downarrow_z\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

$$|u_2\rangle \equiv |\uparrow_z\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{21}$$

Therefore, the transition frequencies read

$$\omega_{21} = -\omega_{12} = \omega_+. \tag{22}$$

Recalling Eq.(11), the matrix elements $H'_{nm}(t)$ in Eqs.(8) or (19) have the form

$$H'_{nm}(t) = \langle u_n|\frac{e\hbar}{\mu c}B_x(t)S_x|u_m\rangle. \tag{23}$$

Because the state $|u_n\rangle$ is not the eigenvector of S_x, we cannot calculate Eq.(23) directly. We now expand $|u_1\rangle \equiv |\downarrow_z\rangle$ and $|u_2\rangle \equiv |\uparrow_z\rangle$ in the eigenvectors of S_x as

$$|u_2\rangle \equiv |\uparrow_z\rangle = \frac{1}{\sqrt{2}}(|\uparrow_x\rangle + |\downarrow_x\rangle), \tag{24}$$

$$|u_1\rangle \equiv |\downarrow_z\rangle = \frac{1}{\sqrt{2}}(|\uparrow_x\rangle - |\downarrow_x\rangle). \tag{25}$$

Hence we have the following equations

$$S_x|\uparrow_z\rangle = \frac{1}{\sqrt{2}}(S_x|\uparrow_x\rangle + S_x|\downarrow_x\rangle) = \frac{1}{2}|\downarrow_z\rangle, \tag{26}$$

$$S_x|\downarrow_z\rangle = \frac{1}{\sqrt{2}}(S_x|\uparrow_x\rangle - S_x|\downarrow_x\rangle) = \frac{1}{2}|\uparrow_z\rangle. \tag{27}$$

Recalling Eq.(10) and substituting Eqs.(26) and (27) into Eq.(23) yield

$$H'_{12}(t) = H'_{21}(t) = \frac{e\hbar}{4\mu c}B_0(\mathrm{e}^{-\mathrm{i}\omega_{\mathrm{L}}t} + \mathrm{e}^{\mathrm{i}\omega_{\mathrm{L}}t}), \tag{28}$$

$$H'_{11}(t) = H'_{22}(t) = 0. \tag{29}$$

Proceeding to substitute Eqs.(28) and (29) into Eqs.(18) and (19) and taking into account the transition frequencies (22), we obtain

$$\frac{\mathrm{d}C_1(t)}{\mathrm{d}t} = -\mathrm{i}\frac{eB_0}{4\mu c}C_2(t)(\mathrm{e}^{-\mathrm{i}(\omega_+ + \omega_{\mathrm{L}})t} + \mathrm{e}^{-\mathrm{i}(\omega_+ - \omega_{\mathrm{L}})t}), \tag{30}$$

$$\frac{\mathrm{d}C_2(t)}{\mathrm{d}t} = -\mathrm{i}\frac{eB_0}{4\mu c}C_1(t)(\mathrm{e}^{\mathrm{i}(\omega_+ - \omega_{\mathrm{L}})t} + \mathrm{e}^{\mathrm{i}(\omega_+ + \omega_{\mathrm{L}})t}). \tag{31}$$

With the rotating wave approximation, omitting the terms containing the factor $(\omega_+ + \omega_{\mathrm{L}})$ in the above equations, and assuming that the state of the system (17) is the ground state $|u_1\rangle \equiv |\downarrow_z\rangle$ when the system is at the initial moment (t=0), i.e. $C_1(0) = 1$ and $C_2(0) = 0$, we finally obtain

$$\begin{aligned} C_1(t) =& R_{12}^{-1}\mathrm{e}^{-\mathrm{i}(\omega_+ - \omega_{\mathrm{L}})t/2} \\ &\times \left[\mathrm{i}(\omega_+ - \omega_{\mathrm{L}})\sin\left(\frac{R_{12}}{2}t\right)\right. \\ &\left. + R_{12}\cos\left(\frac{R_{12}}{2}t\right)\right], \end{aligned} \tag{32}$$

$$C_2(t) = -\mathrm{i}2\alpha R_{12}^{-1}\mathrm{e}^{\mathrm{i}(\omega_+ - \omega_{\mathrm{L}})t/2}\sin\left(\frac{R_{12}}{2}t\right), \tag{33}$$

where $\alpha = eB_0/4\mu c$, and $R_{12} = [(\omega_+ - \omega_{\mathrm{L}})^2 + 4\alpha^2]^{1/2}$ which is called the Rabi frequency. So the time evolution of wavefunction (14) of the system can be expressed as

$$\begin{aligned} \psi(t) =& R_{12}^{-1}\mathrm{e}^{-\mathrm{i}(\omega_+ - \omega_{\mathrm{L}})t/2}\left[\mathrm{i}(\omega_+ - \omega_{\mathrm{L}})\sin\left(\frac{R_{12}}{2}t\right)\right. \\ &\left. + R_{12}\cos\left(\frac{R_{12}}{2}t\right)\right]|\downarrow_z\rangle\mathrm{e}^{\mathrm{i}\omega_+ t/2} \\ &- \mathrm{i}2\alpha R_{12}^{-1}\mathrm{e}^{\mathrm{i}(\omega_+ - \omega_{\mathrm{L}})t/2} \\ &\times \sin\left(\frac{R_{12}}{2}t\right)|\uparrow_z\rangle\mathrm{e}^{-\mathrm{i}\omega_+ t/2}. \end{aligned} \tag{34}$$

If the laser fields resonate with the spin-half particle, i.e. $\omega_+ = \omega_{\mathrm{L}}$ and $R_{12} = 2\alpha$, then Eqs.(32) and (33) can be reduced to

$$C_1(t) = \cos\left(\frac{R_{12}}{2}t\right) = \cos(\alpha t), \tag{35}$$

$$C_2(t) = -\mathrm{i}\sin\left(\frac{R_{12}}{2}t\right) = -\mathrm{i}\sin(\alpha t). \tag{36}$$

The wavefunction of the system turns into

$$\begin{aligned} \psi(t) =& \cos\left(\frac{R_{12}}{2}t\right)|\downarrow_z\rangle\mathrm{e}^{\mathrm{i}\frac{1}{2}\omega_+ t} \\ &- \mathrm{i}\sin\left(\frac{R_{12}}{2}t\right)|\uparrow_z\rangle\mathrm{e}^{-\mathrm{i}\frac{1}{2}\omega_+ t} \\ =& \begin{pmatrix} -\mathrm{i}\sin(\alpha t)\mathrm{e}^{-\mathrm{i}\frac{1}{2}\omega_+ t} \\ \cos(\alpha t)\mathrm{e}^{\mathrm{i}\frac{1}{2}\omega_+ t} \end{pmatrix}. \end{aligned} \tag{37}$$

It is entirely possible that the laser fields resonate with the spin-half particle (electron) S_a. For a far stronger static magnet $\boldsymbol{B}$ acting on the spin-half particle S_a, the transition frequency ω_+ of the spin-half particle S_a can reach $\omega_+ \sim eB/2\mu c \sim 10^{14}$Hz. Moreover, the frequency ω_{L} of the laser fields, e.g. the infrared frequency of the natrium laser fields, is of this order.

It is clear that the probability of the system in the excited state $|\uparrow_z\rangle$ is $|C_2(t)|^2 = \sin^2(\alpha t)$, and the probability of the system in the ground state $|\downarrow_z\rangle$ is $|C_1(t)|^2 = \cos^2(\alpha t)$. From this we know that if the width of the laser impulse is

$$\tau = \frac{\pi}{4\alpha} = \frac{2\mu c}{eB_0}\pi, \tag{38}$$

then after one impulse, called the π impulse, the probabilities change into

$$|C_2(t)|^2 = 1, \qquad |C_1(t)|^2 = 0. \tag{39}$$

That is to say, the spin-half particle is in the excited state. Therefore, there is a transition from the ground state to the excited state, as illustrated in Fig.1. From Eq.(4) we know that there is a Berry phase shift $\gamma_+ = \mp\pi(1 - \cos\theta_+)$.

Thus, we can conclude that, for the two spin-half particles described by Eq.(2), if the S_b particle is in the state $|\uparrow_z\rangle$ and there is a π impulse whose frequency is ω_+ acting on the S_a particle along the x-axis direction, then the S_a particle will produce a Berry phase shift γ_+. Similarly, if the S_b particle is in the state $|\downarrow_z\rangle$ and there is a π impulse whose frequency is ω_- acting on the S_a particle along the x-axis direction, then the S_a particle will produce a Berry phase shift γ_-. However, if the laser fields do not resonate with the energy level $\hbar\omega_+$ of the spin-half particle S_a, i.e. $\omega_+ \neq \omega_{\rm L}$, then the probability, using Eq.(33), in the excited state is

$$|C_2(t)|^2 = \frac{4\alpha^2}{R_{12}^2}\sin^2\frac{R_{12}}{2}t, \tag{40}$$

with

$$\frac{4\alpha^2}{R_{12}^2} = \frac{(eB_0/2\mu c)^2}{[(\omega_+ - \omega_{\rm L})^2 + (eB_0/2\mu c)^2]}. \tag{41}$$

We assume that $R_{12} \gg \alpha$, then $|C_2(t)|^2$ and $\overline{|C_2(t)|^2}$ are almost zero. So the probability for producing transition ω_+ (see Fig.1) is almost zero, and no Berry phase shift γ_+ is produced if the qubit S_b is in the state $|\uparrow_z\rangle$. This is the same reason that the probability produced transition ω_- (also see Fig.1) is also almost zero, and no Berry phase shift γ_- is produced either if the qubit S_b is in the state $|\downarrow_z\rangle$ and the laser fields cannot resonate with the energy level $\hbar\omega_-$ of the spin-half particle S_a.

Now we know from the above discussion that if there are two laser fields, with strengths B_0 and frequencies ω_+ and ω_- respectively, acting simultaneously on the spin-half particle S_a in the x-axis direction in the system described by Eq.(2), then the particle S_a will produce an ω_+ transition and a Berry phase shift γ_+ when the particle S_b is in the state $|\uparrow_z\rangle$, and it will produce an ω_- transition and a Berry phase shift γ_- when S_b is in the state $|\downarrow_z\rangle$. Thus, we have succeeded in explaining how the state of qubit S_b influences the phase acquired by the qubit S_a.[23] The gate (6) is equivalent to the controlled-phase gate introduced in Ref.[20].

4. Concluding remarks

We have introduced a two-level Hamiltonian system which allows us to explore the mechanism of the laser controlling the geometric quantum gate discussed by Ekert *et al.*[21] We show the relationship between the external laser fields and the transition of the qubit in the two-qubit control-phase gates, explaining how the transition of the qubit depends on the external laser fields and the states of the controlling qubit. It is worth noting that the polarized direction of the external laser field must be in the y-axis (or x-axis) direction so that the induced magnetic field $\boldsymbol{B}$ points in the x-axis (or y-axis) direction; otherwise the laser fields cannot resonate with the energy level of the spin-half particle S_a no matter what the frequency of the laser field.

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