Vol 11 No 12, December 2002
1009-1963/2002/11(12)/1319-05

Chinese Physics

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Optically-thick accretion discs with advection*

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(Received 10 April 2002; revised manuscript received 13 August 2002)

The structures of optically-thick accretion discs with radial advection have been investigated by the iteration and integration algorithms. The advective cooling term changes mostly the inner part of disc solution, and even results in an optically-thick advection-dominated accretion flow (ADAF). Three distinct branches—the outer Shakura–Sunyaev disc (SSD), the inner ADAF and the middle transition layer—are found for a super-Eddington disc. The SSD–ADAF transition radius can be estimated as $18(\dot{M}/\dot{M}_E)R_G$ where R_G is the Schwarzschild radius, $\dot{M}$ is the mass accretion rate and $\dot{M}_{\rm E}$ is the Eddington accretion rate. SSD solutions calculated with the iteration and integration methods are identical, while ADAF solutions obtained by these two methods differ greatly. Detailed algorithms and their differences have been analysed. The iteration algorithm is not self-consistent, since it implies that the dimensionless advection factor ξ is invariant, but in the inner ADAF region the variation of ξ is not negligible. The integration algorithm is always effective for the whole region of an optically-thick disc if the accretion rate is no smaller than $10^{-4}\dot{M}_E$. For optically-thin discs, the validity of these two algorithms is different. We suggest that the integration method be employed to calculate the global solution of a disc model without assuming ξ to be a constant. We also discuss its application to the emergent continuum spectrum in order to explain observational facts.

Keywords: accretion disc, black hole physics, SSD, ADAF
PACC: 9710H, 9760L

1. Introduction

Accretion discs around black holes,[1] neutron stars and white dwarfs play a major role in the research of binaries, cataclysmic variables and active galactic nuclei (AGNs). It is generally accepted that the dissipation of gravitational energy through accretion on to a compact body is the central engine of quasars and the source of violent activities of many objects. To date, four types of typical disc models have been found and discussed by Chen *et al.*[2] Among them, the most famous is the standard thin disc model, called the Shakura–Sunyaev disc (SSD).[3] The accreting gas forms a geometrically thin, optically-thick disc with relatively low temperatures ($T \sim 10^4 - 10^6$K), and it produces a quasi-blackbody spectrum. The viscosity responsible for both the energy release and the angular momentum outward transfer is described with a phenomenological α-viscosity law.[3] The heating energy produced by viscous dissipation is purely balanced by local radiative cooling process. The SSD model suffers from thermal and secular instabilities in the radiation-pressure dominated region.

Starting with the initial work by Muchotrzeb and Paczynski,[4] it has gradually been realized that radial advection may become dominant compared with radiation. Thus, optically-thin advection-dominated accretion flows (ADAFs) and optically-thick ADAFs, corresponding to low and high mass accretion rates respectively, have been studied extensively to account for the hard spectrum component. The advection process physically means that the generated viscous energy is restored as entropy rather than radiated locally. It may change the disc structure significantly, modify the emergent spectrum and avoid the instabilities that SSD has in the radiation pressure region. Much work[5−10] has been performed on these models stemming from the original paper, Ref.[4]. However, accretion disc models with radial advection are governed by a set of stiff differential equations that are very difficult to solve numerically. It has been assumed the dimensionless advection factor ξ is a constant, so that the advection term is localized.[2,11−13]

*Project supported by the Special Funds for Major State Basic Research Programs of China (Grant No G2000077604).

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However, inconsistency occurred since the advection, a global energy transport term, was not calculated from the global structure. The iteration and integration algorithms have been introduced to calculate the global solutions of optically-thin discs in Ref.[4]. It is not very clear why the iteration is valid only for the Shapiro–Lightman–Eardley (SLE) disc and the integration is valid just for ADAF. And what is the case for optically-thick discs? In the present paper, by numerically solving optically-thick accretion flows with radial advection, the essence of the two algorithms and their different scopes in utilization are analysed. The fact that a super-Eddington disc has three branches may be of great significance in the application to the saturated accreting systems, such as narrow-line Seyfert 1 (NLS1) galaxies and some ultraviolet x-ray sources.

2.Calculation algorithms

We consider the optically-thick, geometrically slim, axisymmetric disc formalism with radial advection described in Ref.[2]. In this model the following assumptions are made.

(1) The vertical half-thickness $H = c_s/\Omega_K \leq R$, where R is the distance from the central object, c_s is the local sound speed and Ω_K is the Keplerian angular velocity.

(2) The kinematic viscosity coefficient $\nu = 2\alpha c_s H/3$, where α is the viscosity parameter.

(3) Keplerian motion in the pseudo-Newtonian potential[15] is adopted, so the angular velocity $\Omega_K = \sqrt{GM/R^3}/(1 - R_G/R)$, where $R_G = 2GM/c^2$ is the Schwarzschild radius, M is the mass of the central object and G is the gravitational constant.

For a steady disc there are three conservation equations

$$\nu\Sigma = \frac{\dot{M}}{3\pi} f g^{-1} \quad \text{(angular momentum equation)}, \quad (1)$$

where the surface density $\Sigma = 2H\rho$, $f = 1 - 9\Omega_* R_G^2/(\Omega_K R^2)$, $g = -(2/3)(\mathrm{d}\ln\Omega_K/\mathrm{d}\ln R)$, $\Omega_* = \Omega_K(3R_G)$ and $\dot{M}$ is the mass accretion rate.

$$\dot{M} = 2\pi R\Sigma v_R \quad \text{(mass conservation equation)}, \quad (2)$$

where v_R is the radial velocity of plasma. Viscous heating is cooled by local radiative diffusion and global radial advection, which reads

$$\frac{3\dot{M}}{8\pi}\Omega_K^2 f g = \frac{4\sigma T^4}{3\tau} + \frac{\dot{M}}{4\pi R^2}\frac{p}{\rho}\xi, \quad (3)$$

where

$$\xi = -(12 - 10.5\beta)(\mathrm{d}\ln T/\mathrm{d}\ln R) + (4 - 3\beta)(\mathrm{d}\ln\Sigma/\mathrm{d}\ln R)^*, \quad (4)$$

$\tau = 0.5(\kappa_{es} + \kappa_{ff})\Sigma$ (κ_{es} and κ_{ff} are the opacity for electron scattering and free–free absorption respectively) is the mean Rosseland optical depth, σ is the Stefan–Boltzmann constant, T is the midplane temperature and β is the ratio of gas to total pressure. For optically-thick accretion discs, the total pressure is

$$p = \frac{\Re\rho T}{\mu} + \frac{4\sigma T^4}{3c}, \quad (5)$$

where $\mu = 0.617$ is the mean molecular weight, $\Re$ is the gas constant and c is the velocity of light. Throughout this paper, the dimensionless variables $r = R/R_G$, $h = H/R_G$, $m = M/M_\odot$ ($M_\odot$ is the solar mass) and $\dot{m} = \dot{M}/\dot{M}_E$ are adopted, where $\dot{M}_E = 64\pi GM/(\kappa_{es}c)$ is the Eddington accretion rate.

From Eq.(1) we have $\rho = \left(\frac{2\dot{M}}{\pi\alpha}\right)^{-\frac{1}{2}}\Omega_K^{\frac{1}{2}}(f/g)^{-\frac{1}{2}}\Sigma^{\frac{3}{2}}$, $p = \left(\frac{\dot{M}}{8\pi\alpha}\right)^{\frac{1}{2}}\Omega_K^{\frac{3}{2}}(f/g)^{\frac{1}{2}}\Sigma^{\frac{1}{2}}$. Substituting these into Eqs.(5) and (3), we obtain two nonlinear equations $f_1(r,\Sigma,T) = 0$ and $f_2(r,\Sigma,T,\xi) = 0$ if parameters $(\alpha, m, \dot{m})$ are given. To solve the equation set with first-order differential, two algorithms were introduced in Ref.[14]. They both start from the outer boundary of the disc and continue inwards to the inner edge. By first assuming a numerical value of ξ, we iterate three sets of solutions (Σ, T) at r and the two neighbouring radii $r + \Delta r$ and $r - \Delta r$ from the set of nonlinear equations. With the neighbouring (Σ, T), a new ξ is then obtained from Eq.(4). We then calculate the new values for (Σ, T) using the new ξ. The iteration of ξ is repeated until it converges. Thus, we obtain the disc solution (Σ, T, ξ) of radius r. At the next radius r', the previous ξ is used as the first guess and the same iteration as above is performed. This is the iteration algorithm to search for disc solutions. An alternative algorithm is the integration method. ξ can be reduced to contain only the derivative $\mathrm{d}\ln\Sigma/\mathrm{d}\ln r$ by differentiating f_1 with respect to r. The initial starting value of Σ is taken as that of the iteration

*The general formula is $\xi = -(12 - 10.5\beta)(\mathrm{d}\ln T/\mathrm{d}\ln R) + (4 - 3\beta)(\mathrm{d}\ln\rho/\mathrm{d}\ln R)$. We adopt the form of Eq.(4) just for consistency and comparison with Ref.[2].

algorithm. For a given Σ, T is calculated from equation f_1. Then the derivative $\mathrm{d}\ln\Sigma/\mathrm{d}\ln r$ is obtained from equation f_2, and Σ of the next integration point can be evaluated by the standard fourth-order Runge–Kutta integration procedure. In this paper, the outer boundary is set at 3×10^5R_G. As usual treatments, the last stable orbit $3R_G$ is taken to be the inner disc edge. Since the transonic radius $r<3.6$, the innermost region that the disc model cannot describe well is so narrow that it does not affect the overall disc structure. And the radius of the inner edge can even be treated as a free parameter.[16,17]

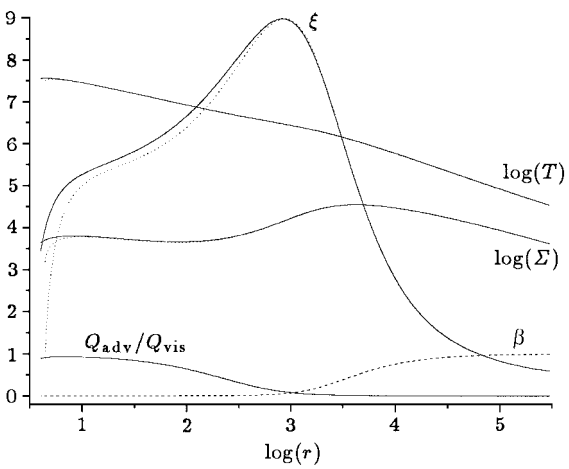


Fig.1. The different disc solutions obtained by iterations (dotted) and integrations (solid). ξ, T, Σ, β and $Q_{\rm adv}/Q_{\rm vis}$ are, respectively, the dimensionless advection factor, mid-plane temperature, surface density, ratio of gas to total pressure and the ratio of advective cooling to viscous heating, corresponding to the mass accretion rate $\dot{m}=10$. Here $\alpha=0.1$ and $m=10$, and the units of T and Σ are Kelvin and g·cm^{-2} (also for Figs.2 and 3).

For an optically-thick disc, the solutions obtained by the above two algorithms are presented in Fig.1. In the outer gas-pressure dominated domain, the results of these two algorithms are completely identical. But when the total pressure becomes radiation dominant, the two solutions begin to separate, and the temperature and surface density even differ greatly near the inner disc edge. Then which is the real solution of the disc model? It should be noted that in the iteration algorithm the method to obtain the new ξ may be questionable. The Σ and T of neighbouring points are calculated with the same ξ at the radius r, which is only accurate when the variation of ξ is sufficiently small. It is proven that the iteration algorithm equals the method that assumes ξ to be a constant in Refs.[2] and [11]–[13]. Although ξ is iterated from the vicinity, the iteration algorithm of this paper still does not treat the radial advection globally and is not self-consistent, because it implies $\mathrm{d}\xi/\mathrm{d}r=0$. It is certain that the integration algorithm can be used to derive the real global disc solution for an optically-thick accretion flow, if the advection term is not too small.

3. Radial disc structures

The radial disc structures with mass accretion rates $\dot{m}$ ranging from 10^{-5} to 10^6 are depicted in Figs.1 and 2. With the advent of global advective cooling, the structures in the inner radiation part change considerably compared with those predicted by the SSD theory. The curve of ξ in Fig.1 has a peak at $r\sim10^3$, where β is just slightly larger than zero. When $r\gg10^3$, the disc is gas-pressure supported, the advective cooling is negligible and the solution is almost the same as that of SSD. With decreasing r, the advection becomes more and more important. When $Q_{\rm adv}/Q_{\rm vis}$ approaches 1, an optically-thick ADAF forms. This new type of solution is different from the SSD solution in many respects. Firstly, because much heating energy is stored as entropy, the accreting plasma is so hot that the temperature reaches 10^7-10^8K. Secondly, due to photon trapping effects the radiative efficiency is very small. Thirdly, the optically-thick ADAF is both thermally and secularly stable, so it exhibits more physical significance than SSD in the inner disc region. Between SSD and ADAF is the middle transition region. The transition is represented by the variations of β and $Q_{\rm adv}/Q_{\rm vis}$. These three typical branches are for the disc with a super-Eddington accretion rate. Advective cooling cannot exceed radiation if the accretion rate $\dot{m}<1$, and all the region $r<3\times10^5$ is full of ADAF solutions once $\dot{m}>10^4$.

From the radial distributions of Σ, T and h in Fig.2, we should notice that the change of pressure has a significant effect on the radial disc structure. The three zones, i.e. SSD, ADAF and the transition layer, are shown distinctly in the figure. It is suitable to link the numerical solutions with the theoretical analytic expressions at extreme scenarios. For example, in the ADAF region of $\dot{m}=100$, the straight lines of Σ, T and h demonstrate that they are in power-law distributions. Their indices are -0.5, -0.64 and 1.0 respectively, which coincide with the self-similar results[18] $\Sigma\propto r^{-0.5}$, $T\propto r^{-5/8}$ and $h\propto r$. For the outer SSD solution, these variables can also be regarded as in power-law radial distributions with indices -0.74, -0.76 and 1.12, very close to the theoretical values

-0.75, -0.75 and $9/8$ obtained by the SSD model.[19] We also notice that the larger the mass accretion rate, the higher the midplane temperature and the thicker the accretion flow. This is in agreement with the intuitive judgement.

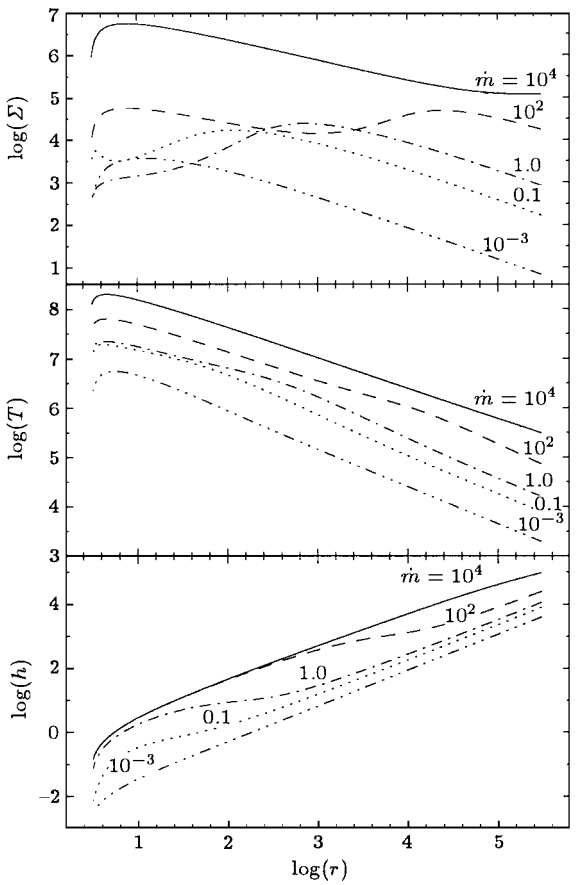


Fig.2. The radial distributions of the surface density, temperature and the vertical half-thickness of optically-thick accretion flows.

4. Discussion

The iteration algorithm employed in this work equals the iteration in Refs.[2] and [11]–[13]. Strictly speaking, it is still inconsistent and questionable in the inner ADAF region, since the term including $\mathrm{d}\xi/\mathrm{d}r$ in the advection gradient has significant influence on the global disc solution. Of course, the integration method is effective for seeking the global solution. Calculations show that for super-Eddington accretion discs, only the integration method can be employed to obtain the global solutions anywhere. But if $\dot{m} \leq 10^{-4}$, only the iteration algorithm is valid, because the advection is so small that the integration method diverges and the iteration inconsistency can be ignored. For discs with accretion rates $10^{-4} < \dot{m} < 0.1$, there are no ADAFs. So both algorithms can be used, and their results are completely identical. In the optically-thick disc, ξ has a peak where the radiation pressure just becomes dominant. This is different from the optically-thin case, where ξ is always a constant. Therefore, to assume that ξ is invariable, as Abramowicz *et al*[11] did, is not appropriate for optically-thick discs. Chen[14] has also shown that the iteration is merely valid for local radiation dominated solution SLE, and the integration is only valid for the optically-thin ADAF. The reason is related to the integration divergence and the iteration inconsistency as stated above.

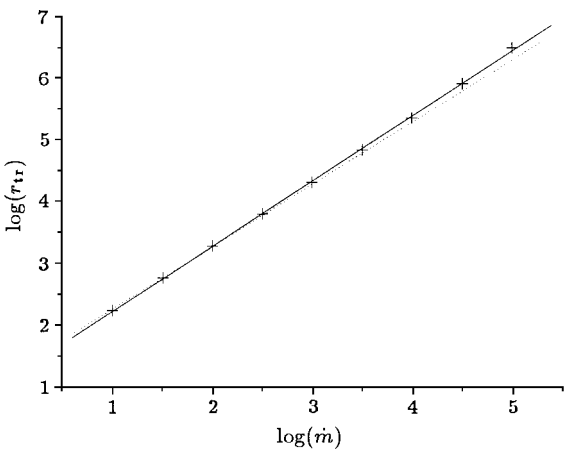


Fig.3. The relation between the SSD–ADAF transition radius $r_{\rm tr}$ and the mass accretion rate $\dot{m}$. The fitted straight line (solid) is $\log(r_{\rm tr}) = 1.166 + 1.047\log(\dot{m})$, and the theoretical line (dotted) is $\log(r_{\rm tr}) = 1.255 + \log(\dot{m})$.

The radial disc structure is related to the variation of the S-shaped curve on the $\dot{m}$–Σ plane at different radii. The inner SSD and outer ADAF coexist in an optically-thick disc. The SSD–ADAF transition radius can be determined by $Q_{\rm adv}/Q_{\rm vis} = 0.5$, and it is shown in Fig.3 as a function of the accretion rate. The fitting relation from numerical results is $\log(r_{\rm tr}) = 1.166 + 1.047\log(\dot{m})$. It can also be obtained theoretically: since $r_{\rm tr}$ is located at the radiation-pressure dominated region ($\beta \approx 0$), we have

$$3 \cdot 2^{0.25} c^2 \dot{m}^{0.5} \Sigma^{0.5} = a R_G \kappa_{es}^{0.5} \alpha^{0.5} r_{\rm tr}^{2.25} T^4 \quad (p = p_{\rm rad}), \tag{6}$$

$$12\sqrt{2}\alpha\kappa_{es} r_{\rm tr}^{0.5} \dot{m}\Sigma = \frac{32\sqrt{2}}{3c^3}\alpha\sigma R_G \kappa_{es} r_{\rm tr}^{3.5} T^4 = 512\xi\dot{m}^2 \quad (Q_{\rm vis} = 2Q_{\rm rad} = 2Q_{\rm adv}). \tag{7}$$

In addition, $\xi = 6.75 + 2.5\mathrm{d}\ln\Sigma/\mathrm{d}\ln r$, where the second term is approximately zero (see Figs.1 and 2).

Substituting $\xi = 6.75$ into Eq.(7), with Eq.(6) we obtain three equations for the three variables $r_{\rm tr}$, T and Σ. It is straightforward to derive $r_{\rm tr} = 18\dot{m}$, which is consistent with the fitting relation. $r_{\rm tr}$ is slightly larger than the photon trapping radius,[18] $r_{\rm trap} = 14.4\dot{m}$. It is interesting that the transition radius is independent of the viscosity and the mass of the central compact object, which provides an approach to estimate the mass accretion rate from observations.

Since the accretion flow is described by a set of nonlinear differential equations, the outer boundary condition (OBC) possibly plays an important role. Yuan[20] found that the OBC should be regarded as a new parameter in an optically-thin flow. Calculations in this work show that for optically-thick discs the OBC has little effect on SSD but more on ADAF; the smaller the outer boundary radius, the larger the influence of the OBC. Hence, the position of the OBC determines whether it is an important parameter for super-Eddington accretion flows. In this paper we set the OBC in the SSD region, and take the solution obtained by iterations as the boundary value. The OBC does not affect the inner global disc solution, so this treatment is suitable.

We have calculated the structures of optically-thick discs in a wide range of accretion rates using two algorithms. When the accretion rate exceeds $\dot{M}_E$, an optically-thick ADAF appears and only the integration method can give the global solution. The global disc structures obtained here are basic for emergent spectra from disc surfaces.[21] But spectra calculations are beyond the scope of this paper. It is believed that the disc model considered here has many applications to astrophysical objects. The coexistence of the outer cold SSD solution and the inner hot ADAF provides explanations for the reflected x-ray component and the cut-off hard spectrum, while the transition layer emits the observational iron Kα lines. The present disc solutions can be applied to AGNs and stellar black-hole candidates, which are powered by saturated accretion systems. Recent observations reveal a new type of AGN, the NLS1 galaxy, which is thought to have a less massive black hole with a high accretion rate exceeding the Eddington critical value. It is preferable to fit the big blue bump and explain its soft hump with the x-ray spectra calculated on global disc structures. Finally, we would like to point out that many ultraviolet x-ray sources, characterized by very strong soft x-ray emissions in the Galaxy,[17] can also be accounted for with the current disc model.

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