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Photon statistics of the micromaser with a Kerr medium*

Wu Shu-Dong(吴曙东), Zhan Zhi-Ming(詹志明), and Jin Li-Xia(金丽霞)

Department of Physics, Huazhong University of Science of Technology, Wuhan 430074, *China*

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We have established the master equation for the micromaser with a Kerr medium. We have studied the photon statistics of the micromaser with a Kerr medium field and analysed the influence of the Kerr effect on the photon statistics. The influence of the Kerr effect on the photon statistics is different in two regimes. In the thermal-atom regime, the Kerr effect produces quantum noise, and decreases the mean photon number. In the ultracold-atom regime, with the increase of the nonlinear parameter of the Kerr medium, the stability of the mean photon number and the normalized variance enhances the mean photon number, and the normalized variance exhibits collapse-revival phenomena periodically, their resonance peaks become lower, and the photon statistics of more and more regions are sub-Poissonian.

Keywords: photon statistics, micromaser, Kerr medium
PACC: 4250, 4252

1. Introduction

The micromaser is a fertile ground for observing basic interactions between light and matter. Many quantum electrodynamic effects have been found in the micromaser.[1−4] With the advent of laser cooling techniques, we have obtained very cold atoms and found interesting properties of ultracold atoms.[5−7] Studies of the micromaser have yielded much information, such as, for example, sub-Poissonian photon statistics,[8−12] trapping states,[13] atomic bunching and antibunching,[14] and nondemolition measurement of small photon number states among others.[15] Recently, the photon statistics of the micromaser has received considerable attention and continues to be an active area of research. The influence of the Kerr effect really exists in the actual systems, therefore the study of its effect on photon statistics is very necessary. In this paper, we establish the master equation for the reduced density operator of the micromaser with a Kerr medium. We deal with a modified Jaynes–Cummings model (JCM) in which we consider the micromaser cavity filled with a Kerr nonlinear medium[16−19] and we study photon statistics of the micromaser with a Kerr medium in the thermal-atom and ultracold-atom regimes.

2. Derivation of the master equation

We now consider a beam of two-level atoms moving along the z-direction, and entering into the Kerr medium cavity which contains a single-mode quantum radiation field. The atomic injection rate is low enough to ensure that there is just a single atom inside the cavity at one time. The atom is initially in the upper state $|a\rangle$ and the Kerr cavity field is in the superposition state $\sum_n C_n|n\rangle$. By forming the atom–field density matrix and tracing over the internal and external atomic degrees of freedom, we obtain the reduced density matrix of the cavity field[10]

$$\rho(t) = \sum_{i=a,b} \int \mathrm{d}z \langle i, z|\psi(t)\rangle\langle\psi(t)|i, z\rangle. \qquad (1)$$

The coarse-grained equation of motion for the radiation field is governed by[10]

$$\dot{\rho}(t) = r\delta\rho(t) + L\rho(t), \qquad (2)$$

where $\delta\rho(t)$ is the change of $\rho(t)$ due to the interaction with a single atom. In principle, the master equation for a nonlinear oscillator should differ from that of linear oscillator. However, for weak decay and for very low temperature (very small n_b), the same master

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equation may be conveniently used, so field damping is described by the Liouville operator[20]

$$L\rho(t) = -\frac{C}{2}(n_b+1)(a^+a\rho+\rho a^+a-2a\rho a^+) - \frac{C}{2}n_b(aa^+\rho+\rho aa^+-2a^+\rho a), \quad (3)$$

where C is the cavity decay rate and n_b is the number of photons in thermal equilibrium. We have neglected the dissipation during the atom–field interaction.

The equations of motion for the reduced density matrix elements can be written as

$$\dot\rho_{n,n'}(t) = a_{n,n'}\rho_{n,n'} + b_{n-1,n'-1}\rho_{n-1,n'-1} + c_{n+1,n'+1}\rho_{n+1,n'+1}, \quad (4)$$

where

$$a_{n,n'} = r(R_{a,n}R^*_{a,n'} + T_{a,n}T^*_{a,n'} - 1) - \frac{C}{2}(n_b+1)(n+n') - \frac{C}{2}n_b(n+n'+2), \quad (5)$$

$$b_{n,n'} = r(R_{b,n+1}R^*_{b,n'+1} + T_{b,n+1}T^*_{b,n'+1}) + Cn_b\sqrt{(n+1)(n'+1)}, \quad (6)$$

$$c_{n,n'} = C(n_b+1)\sqrt{nn'}. \quad (7)$$

Here $R_{a,n}(k)$, $T_{a,n}(k)$, $R_{b,n+1}(k)$ and $T_{b,n+1}(k)$ are the amplitudes of reflection (R) and transmission (T) and simultaneous transition into the lower state $|b\rangle$ or remaining in the upper state $|a\rangle$ of an incident excited atom, respectively. In Ref.[19] we have

$$R_{a,n}(k) = (\sin\theta_n)^2\rho_n^+(k) + (\cos\theta_n)^2\rho_n^-(k), \quad (8)$$

$$T_{a,n}(k) = (\sin\theta_n)^2\tau_n^+(k) + (\cos\theta_n)^2\tau_n^-(k), \quad (9)$$

$$R_{b,n+1}(k) = (\sin\theta_n)(\cos\theta_n)[\rho_n^+(k) - \rho_n^-(k)], \quad (10)$$

$$T_{b,n+1}(k) = (\sin\theta_n)(\cos\theta_n)[\tau_n^+(k) - \tau_n^-(k)], \quad (11)$$

$$\tan\theta_n = \frac{(\Delta-2\chi n)/2 + \sqrt{[(\Delta-2\chi n)/2]^2 + g^2(n+1)}}{g\sqrt{n+1}}. \quad (12)$$

Here χ indicates the nonlinear parameter of the Kerr medium, Δ is the detuning of the system and g is the atom–cavity mode coupling constant. In the following, we only consider a special case of a mesa mode function, and we have the following forms

$$\rho_n^\pm = i\Delta_n^\pm \sin(K_n^\pm l)\tau_n^\pm, \quad (13)$$

$$\tau_n^\pm = [\cos(K_n^\pm l) - i\sum_n^\pm \sin(K_n^\pm l)]^{-1}, \quad (14)$$

with

$$K_n^\pm = \sqrt{k^2 \mp (k_n^\pm)^2}, \qquad k_n^\pm = \kappa\mu_n^\pm, \quad (15)$$

$$\Delta_n^\pm = \frac{1}{2}\left(\frac{K_n^\pm}{k} - \frac{k}{K_n^\pm}\right), \qquad \sum_n^\pm = \frac{1}{2}\left(\frac{K_n^\pm}{k} - \frac{k}{K_n^\pm}\right), \quad (16)$$

$$\mu_n^\pm = \left[\pm n^2\chi/g + \sqrt{\left(\frac{\Delta/g - 2n\chi/g}{2}\right) + (n+1)}\right]^{1/2}, \quad (17)$$

$$\kappa = \sqrt{2Mg/\hbar}, \quad r = k/\kappa, \quad L = \kappa l. \quad (18)$$

Here k is a c.m. momentum, r is the atomic injection rate, L is the interaction length of the cavity, and M is the atomic mass.

3. Photon statistics

The equation of motion for the photon-number distribution $p(n) = \rho_{n,n}$ follows from Eqs.(4)–(7) under the steady-state condition $\dot\rho_{n,n}$=0. Therefore we can find the following solution

$$p_n = p_0\prod_{m=1}^{n} X_m, \quad (19)$$

where

$$X_m = \frac{1}{n_b+1}\left[n_b + \frac{N_{\rm ex}}{m}P_{\rm emission}(m-1)\right], \quad (m \geq 1). \quad (20)$$

Here $N_{\rm ex} = \dfrac{r}{C}$ is the normalization injection rate, and the normalization condition and the atomic emission probability are given, respectively, by

$$p_0 = \left[1 + \sum_{n=1}^{\infty}\prod_{m=1}^{n} X_m\right]^{-1}, \quad (21)$$

and

$$P_{\rm emission}(n) = |R_{b,n+1}(k)|^2 + |T_{b,n+1}(k)|^2. \quad (22)$$

The mean number of photons and the normalized variance are given by $\langle n\rangle = \sum_{n=0}^{\infty} np_n$ and $\sigma = \sqrt{(\langle n^2\rangle - \langle n\rangle^2)/\langle n\rangle}$, respectively. In the following, taking into account the Kerr effect, we study photon statistics in the thermal-atom and ultracold-atom regimes.

In Fig.1, we show the plots of the mean number of photons $\langle n\rangle$ and the normalized variance σ for different values of dimensionless nonlinear Kerr parameter χ/g versus the interaction length L in the thermal-atom regime (r=100). In a zero-temperature cavity, i.e. in the absence of thermal photons (n_b=0), the Kerr

effect (χ/g=0.05) leads to rapid oscillation in the resonance peaks where the little indentation represents quantum noise. As χ/g increases, the mean number of photons $\langle n\rangle$ decreases as a whole.

In Fig.2, we show the plots of the mean number of photons $\langle n\rangle$ and the normalized variance σ for different values of dimensionless nonlinear Kerr parameter χ/g versus the interaction length L in the ultracold-atom regime (r=0.01). In a zero-temperature cavity ($n_b = 0$), with the increase of the nonlinear parameter of the Kerr medium χ/g, the stability of the mean photon number and the normalized variance enhances, $\langle n\rangle$ and σ become more regular and collapse-revival phenomena appear periodically, the resonance peaks become lower, the number of resonance peaks decreases, and photon statistics of more and more regions L are sub-Poissonian ($\sigma < 1$).

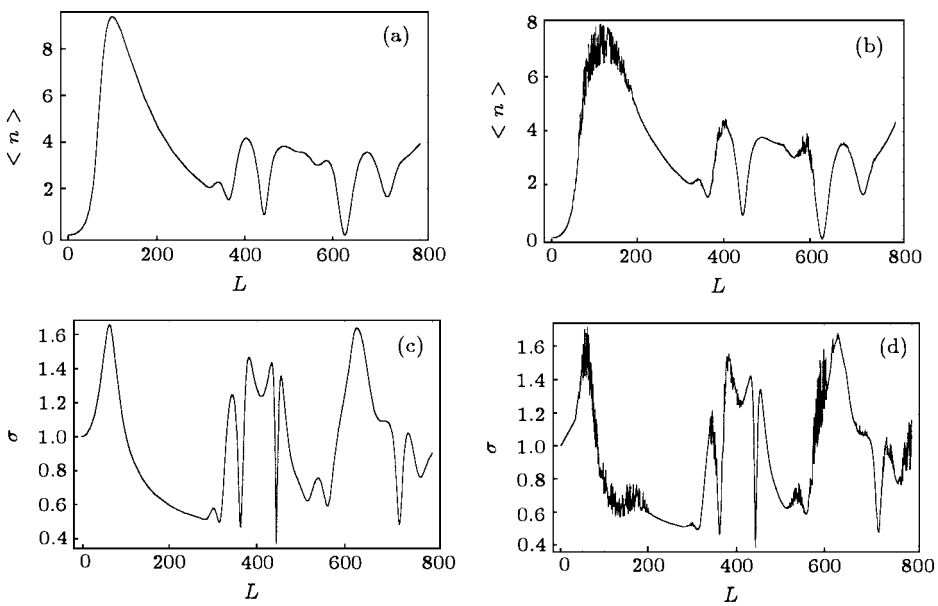


Fig.1. The mean number of photons $\langle n\rangle$ and the normalized variance σ versus the interaction length L in the ultracold-atom regime: r=100, N=10, $n_b = 0$, $\Delta = 0$. $(a, c)\chi/g = 0; (b, d)\chi/g = 0.05$.

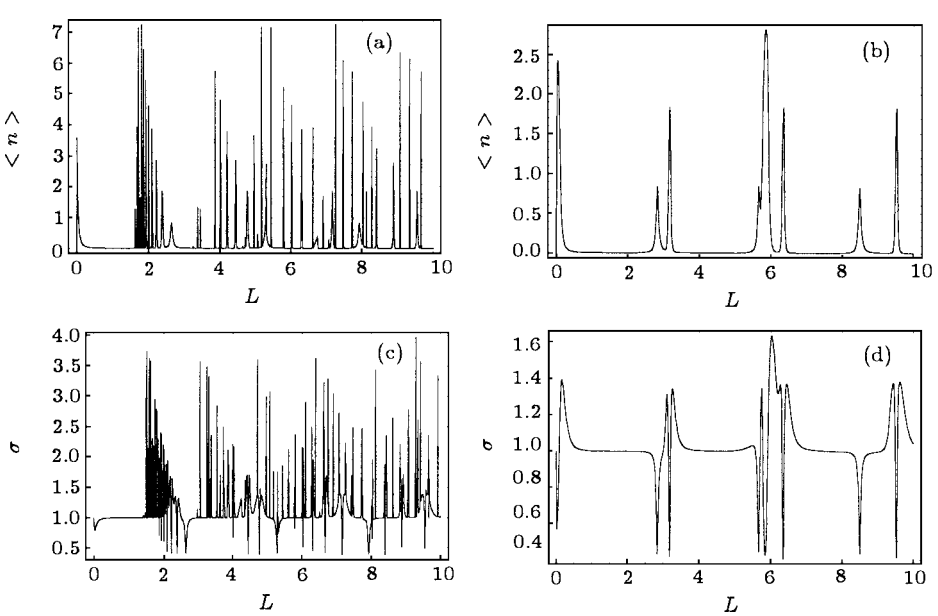


Fig.2. The mean number of photons $\langle n\rangle$ and the normalized variance σ versus the interaction length L in the ultracold-atom regime: r=0.01, N=10, $n_b = 0$, $\Delta = 0$. $(a, c)\chi/g = 0; (b, d)\chi/g = 0.2$.

4. Conclusion

In summary, we have established the master equation for the micromaser with a Kerr medium. We have studied the photon statistics of the micromaser with a Kerr medium field and analysed the influence of the Kerr effect on the photon statistics. In the thermal-atom regime, the Kerr effect produces quantum noise. In the ultracold-atom regime, with the increase of χ/g, the stability of the mean photon number and the normalized variance enhances, collapse-revival phenomena appear periodically, the resonance peaks become lower, and the photon statistics of more and more regions L are sub-Poissonian ($\sigma < 1$).

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