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Structural stability in discrete singular systems*

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In this paper, we study the structural stability of singular discrete systems with perturbations in coefficient matrices. Some sufficient and necessary conditions of structurally stable singular discrete systems are given. Two kinds of structurally stable normal compensators are discussed.

Keywords: singular discrete system, structural stability, normal compensator
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1. Introduction

In the past few years, there has been a growing interest in the system-theoretic problems of singular systems due to their applications in representing and resolving problems concerning many naturally occurring systems. Several important and fundamental results in normal systems have been successfully extended to singular systems.[1,2] Note that the stability problem is of both practical and theoretical significance. Many researchers have made enormous efforts in this fields (see, for example, Refs.[1] and [3-5]). Their works are developed in the continuous case and indicate that the stability problem of singular systems is much more complicated than that of normal systems. As for the stability problem of discrete singular systems, there are only few results. In fact, to deal with this problem it is necessary to note that not only stability but also regularity and causality must be considered simultaneously, while the latter two do not occur in standard state-space systems. For example, Xu and Yang[6] considered the state feedback stabilization problem of discrete-time singular systems. Using a matrix inequality approach, Xu and Yang designed state feedback controllers such that the closed-loop system is regular, casual and stable. Myszkorowski[7] has shown a general methodology for designing a stable control algorithm for a linear discrete-time system with additive disturbances.

It is well known that stability is necessary for a practical system (see, for example, Ref.[8]). In this paper, we study the structural stability of singular discrete systems with perturbations (or uncertainties, deviations, errors in other terms) in coefficient matrices, as well as their compensation strategies. Some important results in Ref.[1] will be generalized to singular discrete systems.

2. Structural stability in discrete singular systems

We consider a discrete singular system

$$Ex(k+1) = Ax(k) + Bu(k),$$

$$y(k) = Cx(k), \tag{1}$$

where E, $A \in R^{n\times n}$, rank $E \leq n$, $x \in R^n$, k=0, 1, 2, $\cdots$. To guarantee the uniqueness and existence of the state solution, system (1) is always assumed to be regular, i.e., $|sE - A| \not\equiv 0$.

Definition 2.1[1] A discrete singular system (1) is termed stable if its free response ($u(k) \equiv 0$) satisfies

$$\|x(k)\| \leq \alpha\beta^k \|x(0)\|,\ k = 1, 2, \cdots, \alpha > 0, 0 < \beta < 1,$$

for any admissible initial condition $x(0)$ and $k \geq 0$.

System (1) is called stabilizable if there exists a feedback

$$u(k) = Kx(k) + v(k)$$

such that closed-loop system

$$Ex(k+1) = (A + BK)x(k) + Bv(k)$$

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$$y(k) = Cx(k)$$

is stable. It is called detectable if its dual system

$$E^T x(k+1) = A^T x(k) + C^T u(k)$$

$$y(k) = B^T x(k)$$

is stabilizable.

Lemma 2.1[1] A discrete singular system (1) is stable if its finite pole set $\sigma(E, A)$ is within the unit circle on the complex plane, which is denoted by U^+.

It is well known that, if the normal system

$$x(k+1) = Ax(k) + Bu(k)$$

$$y(k) = Cx(k) \tag{2}$$

is stable, i.e., $\sigma(I, A) = \sigma(A) \subset U^+$, a positive constant $\delta > 0$ exists such that

$$\sigma(A + \delta A) \subset U^+$$

for an arbitrary deviation δA in the coefficient matrix A, provided $\|\delta A\| \delta$. Here $\|A\|$ is any consistent matrix norm; that is, the system

$$x(k+1) = (A + \delta A)x(k) + Bu(k)$$

is still stable. This property is termed structural stability. Thus, a stable normal system is structurally stable. But this statement is unfortunately not applicable to a singular system.

Example: Consider a singular system:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} x(k+1) = \begin{bmatrix} -\frac{1}{2} & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} x(k).$$

Since the pole set is $\sigma(E, A) = \left\{ -\frac{1}{2} \right\} \subset U^+$, this system is stable. Taking the perturbation in coefficient matrix A as

$$\delta A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\varepsilon \end{bmatrix}, 1 > \varepsilon > 0,$$

we have $\sigma(E, A + \delta A) = \left\{ -\frac{1}{2}, \frac{1}{\varepsilon} \right\} \not\subset U^+$. Thus, the perturbed system $x(k+1) = (A + \delta A)x(k)$ is unstable no matter how small ε (thus δA) is.

Now we consider the homogeneous system of system (1):

$$Ex(k+1) = Ax(k)$$

$$y(k) = Cx(k), \tag{3}$$

where $E, A \in R^{n \times n}$, rank $E \leq n$, $x \in R^n$, k=0, 1, 2, ...

Definition 2.2 Data point $P(E, A)$ for system (3) is defined as a vector formed by listing the elements of E and A in an arbitrary order.

Definition 2.3 Assume that system (3) is stable. If there exists a certain δ neighbourhood $U_\delta(P(E, A))$ at data point $P(E, A)$ such that the stability of system (3) is preserved in the presence of arbitrary perturbations of structural parameters in this neighbourhood, we will term system (3) structurally stable at data point $P(E, A)$.

By definition, a structurally stable system is stable, and hence is regular.

Theorem 2.1 Assume that system (3) is stable. It is structurally stable at data point $P(E)$ if and only if rank $E = n$.

Proof. Sufficiency: If rank $E = n$, then system (3) becomes a normal system. Thus, it is structurally stable at data point $P(E)$.

Necessity: Let system (3) be structurally stable at data point $P(E)$. If rank $E = n$ is false, rank $E < n$. Since system (3) is regular, there exist two nonsingular matrices Q and P such that system (3) is a restricted system equivalent (r.s.e.) to:

$$x_1(k+1) = A_1 x_1(k),$$

$$y_1(k) = C_1 x_1(k), \tag{4}$$

$$Nx_2(k+1) = x_2(k),$$

$$y_2(k) = C_2 x_2(k), \tag{5}$$

in which the coordinate transformation is

$$x(k) = P \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix},$$

$$x_1(k) \in R^{n_1},\ x_2(k) \in R^{n_2},, n_1 + n_2 = n,$$

and

$$QEP = \begin{bmatrix} I_{n_1} & \\ & N \end{bmatrix},\quad QAP = \begin{bmatrix} A_1 & \\ & I_{n_2} \end{bmatrix},$$

$$CP = [C_1\ C_2],$$

where $A_1 \in R^{n_1 \times n_1}$, $N \in R^{n_2 \times n_2}$ is nilpotent with nilpotent index h. The stability of system (3) implies that $\sigma(E, A) \subset U^+$, therefore, $\sigma(A_1) = \sigma(I_{n_1}, A_1) \subset U^+$.

Without loss of generality, we assume that N is in the form

$$N = \begin{bmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & 0 & \ddots & \\ & & & \ddots & 1 \\ & & & & 0 \end{bmatrix} \in R^{n_2 \times n_2}.$$

Let the perturbation be

$$\delta E = Q^{-1} \begin{bmatrix} 0_{n_1 \times n_1} & 0 & 0 \\ 0 & 0_{(n_2-1)\times 1} & 0 \\ 0 & \varepsilon & 0_{1\times(n_2-1)} \end{bmatrix}$$

$$P^{-1} \in R^{n\times n}, \delta A = 0,$$

where $\varepsilon > 0$ is a sufficiently small scalar. Then

$$\begin{aligned} &|z(E+\delta E) - A| \\ =&|Q^{-1}| \cdot |P^{-1}| \cdot |zI_{n_1} - A_1| \cdot (-1)^{n_2} \cdot (1 - \varepsilon z^{n_2-1}), \end{aligned}$$

indicating that the perturbed system has an unstable pole $\frac{1}{\varepsilon}$ no matter how small $\varepsilon > 0$ is. So, system (3) is not structurally stable at data point $P(E)$, which conflicts with the assumption. Thus, rank $E = n$.

Theorem 2.2 System (3) is structurally stable at data point $P(A)$ if and only if it is stable (implying regular) and $\deg|zE - A| = \mathrm{rank}E$ (or in other words, system (3) is causal).

Proof. Sufficiency: If $\deg|zE - A| = \mathrm{rank}E$, there must exist two nonsingular matrices Q and P such that

$$QEP = \begin{bmatrix} I_q & \\ & 0 \end{bmatrix}, \ QAP = \begin{bmatrix} A_1 & \\ & I \end{bmatrix}, \ q = \mathrm{rank}E.$$

Denote

$$Q\delta AP = \begin{bmatrix} \delta A_{11} & \delta A_{12} \\ \delta A_{21} & \delta A_{22} \end{bmatrix}.$$

Note that $\|\delta A_{22}\| < \|Q\delta AP\|$, we know that $\|\delta A_{22}\|$ is sufficiently small when $\|\delta A\|$ is. Therefore, $I + \delta A_{22}$ can be assumed to be nonsingular. A simple calculation shows that

$$\begin{aligned} &|zE - A - \delta A| = |Q^{-1}| \cdot |P^{-1}| \\ &\cdot \begin{vmatrix} zI_q - A_1 - \delta A_{11} & -\delta A_{12} \\ -\delta A_{21} & -I - \delta A_{22} \end{vmatrix} \\ =&|Q^{-1}| \cdot |P^{-1}| \cdot |-I - \delta A_{22}| \\ &\cdot |zI_q - A_1 - \delta A_{11} - \delta A_{12}(I + \delta A_{22})^{-1}\delta A_{21}|. \end{aligned}$$

Recalling that system (3) is stable, we have $\sigma(A_1) = \sigma(I_{n_1}, A_1) \subset U^+$. Let $\|\delta A\|$ be sufficiently small such that the perturbation

$$\delta A_{11} + \delta A_{12}(I + \delta A_{22})^{-1}\delta A_{21}$$

may be so small that

$$\sigma(A_1 + \delta A_{11} + \delta A_{12}(I + \delta A_{22})^{-1}\delta A_{21}) \subset U^+,$$

indicating that $\sigma(E, A + \delta A) \subset U^+$, i.e., system (3) is structurally stable at data point $P(A)$.

Necessity: Let system (3) be structurally stable at data point $P(A)$. Then system (3) is regular, and hence there exist two nonsingular matrices Q and P such that system (3) is a restricted system equivalent to

$$\begin{aligned} x_1(k+1) &= A_1 x_1(k), \\ y_1(k) &= C_1 x_1(k), \qquad (6) \\ Nx_2(k+1) &= x_2(k), \\ y_2(k) &= C_2 x_2(k), \qquad (7) \end{aligned}$$

where $A_1 \in R^{n_1\times n_1}$ and $N \in R^{n_2\times n_2}$ is nilpotent with nilpotent index h, $n_1 + n_2 = n$, and

$$QEP = \begin{bmatrix} I_{n_1} & \\ & N \end{bmatrix}, \ QAP = \begin{bmatrix} A_1 & \\ & I_{n_2} \end{bmatrix},$$

$$CP = [C_1 \ C_2],$$

Without loss of generality, we assume that N is in the form

$$N = \begin{bmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & 0 & \ddots & \\ & & & \ddots & 1 \\ & & & & 0 \end{bmatrix} \in R^{n_2\times n_2}.$$

Let the perturbation be

$$\delta A = Q^{-1} \begin{bmatrix} 0_{n_1\times n_1} & 0 & 0 \\ 0 & 0_{(n_2-1)\times 1} & 0 \\ 0 & -\varepsilon & 0_{1\times(n_2-1)} \end{bmatrix}$$

$$P^{-1} \in R^{n\times n}$$

where $1 > \varepsilon > 0$ is a sufficiently small scalar. Then

$$
\begin{aligned}
&|Q|\cdot|P|\cdot|zE-(A+\delta A)| \\
&= \begin{vmatrix} zI_{n_1}-A_1 & 0 & 0 & & 0 \\ & -1 & z & \ddots & \\ & 0 & -1 & \ddots & 0 \\ & & & \ddots & z \\ 0 & & & & -1 \end{vmatrix} \\
&= |zI_{n_1}-A_1|\cdot \begin{vmatrix} -1 & z & & & \\ & -1 & \ddots & & \\ & & \ddots & z & \\ & & & -1 & z \\ -\varepsilon & & & & -1 \end{vmatrix} \\
&= (-1)^{n_2}|zI_{n_1}-A_1|\cdot(1-\varepsilon z^{n_2-1}).
\end{aligned}
$$

It follows from the assumption that $\sigma(E, A+\delta A)\subset U^+$ when ε is sufficiently small. Thus $n_2=1$, and hence N=0, implying that deg$|zE-A|$=rank$E$.

Lemma 2.2[6] System (3) is regular, causal and stable if and only if there exists a nonsingular symmetric matrix $P\in R^{n\times n}$ such that the following two inequalities hold:

$$E^TPE\geq 0,\ A^TPA-E^TPE<0. \tag{8}$$

Following Lemma 2.2 and Theorem 2.2, we obtain:

Theorem 2.3 System (3) is structurally stable at data point $P(A)$ if and only if there exists a nonsingular symmetric matrix $P\in R^{n\times n}$ such that the following two inequalities hold:

$$E^TPE\geq 0,\ A^TPA-E^TPE<0.$$

3. Structural stability via state feedback

It is well known that a practical system should be structurally stable, otherwise it could not work properly or may be destroyed. In this section, we are concerned with the structural stability of a closed-loop system via a compensator.

For system

$$\begin{aligned} Ex(k+1)&=Ax(k)+Bu(k),\\ y(k)&=Cx(k), \end{aligned} \tag{9}$$

here $E, A\in R^{n\times n}$, rank$E<n$, $x\in R^n$, k=0, 1, 2, $\cdots$, we consider its state feedback

$$u(k)=Gx(k)+v(k) \tag{10}$$

under which the closed-loop system is

$$Ex(k+1)=(A+BG)x(k)+Bv(k). \tag{11}$$

Definition 3.1 If the closed-loop system (11) is structurally stable at data point $P(E, A, B)$, the state feedback (10) is said to be structurally stable at data point $P(E, A, B)$.

Theorem 2.1 indicates that the state feedback (10) cannot be structurally stable at data point $P(E)$. However, it can be structurally stable at data point $P(A, B)$. In fact, from Theorem 2.2, Theorem 8-3.3[1] and Theorem 2.3 we have:

Theorem 3.1 For system (9), the following conditions are equivalent:

(1) System (9) has a structurally stable state feedback (10) at data point $P(A, B)$;

(2) The closed-loop system (11) is stable and deg$sE-(A+BG)|$=rank$E$;

(3) System (9) is stabilizable and causality controllable[1];

(4) There exists a nonsingular symmetric matrix $P\in R^{n\times n}$ such that the following two inequalities hold: $E^TPE\geq 0, (A+BG)^TP(A+BG)-E^TPE<0$.

Next, we will deal with the structural stability of a closed-loop system via a compensator. Now, we consider system (9) and the normal dynamic compensator:

$$\begin{aligned} x_c(k+1)&=A_cx_c(k)+B_cy(k),\\ u(k)&=F_cx_c(k)+Fy(k), \end{aligned} \tag{12}$$

where $x_c\in R^{n_c}$, A_c, B_c, F_c and F are constant matrices of appropriate dimensions. System (9) and compensator (12) form the closed-loop system

$$\begin{aligned} &\begin{bmatrix} E & 0 \\ 0 & I_{n_c} \end{bmatrix}\cdot\begin{bmatrix} x(k+1) \\ x_c(k+1) \end{bmatrix} \\ &=\begin{bmatrix} A+BFC & BF_c \\ B_cC & A_c \end{bmatrix}\cdot\begin{bmatrix} x(k) \\ x_c(k) \end{bmatrix}. \end{aligned} \tag{13}$$

Definition 3.2 The closed-loop system (13) is assumed to be stable. The compensator (12) is said to be structurally stable at data point $P(E, A, B, C)$ if the closed-loop system (13) is structurally stable at data point $P(E, A, B, C)$.

It is obvious that we have the following results:

Theorem 3.2 (1) System (9) has no structurally stable normal compensator defined as Eq.(12) at data point $P(E)$;

(2) Compensator (12) is a structurally stable normal dynamic compensator for system (9) at data point $P(A,B,C)$ if and only if the closed-loop system (13) is stable and the following equality holds:

$$\deg\left|z\begin{bmatrix}E & 0\\ 0 & I_{n_c}\end{bmatrix}-\begin{bmatrix}A+BFC & BF_c\\ B_cC & A_c\end{bmatrix}\right| = n_c+\mathrm{rank}E.$$

Theorem 3.3 System (9) has a structurally stable normal dynamic compensator (13) at data point $P(A,B,C)$ if and only if system (9) is stabilizable, detectable, causality controllable and causality observable.[1]

Proof. Necessity: Suppose system (9) has a structurally stable normal dynamic compensator (12). Applying Theorem 8–5.3,[1] we know that system (9) is stabilizable and detectable. On the other hand, Theorem 3.2 indicates:

$$\begin{aligned} &n_c+\mathrm{rank}E\\ &=\deg\left|z\begin{bmatrix}E & 0\\ 0 & I_{n_c}\end{bmatrix}-\begin{bmatrix}A+BFC & BF_c\\ B_cC & A_c\end{bmatrix}\right|\\ &\le n_c+\deg|zE-(A+BFC)|\le n_c+\mathrm{rank}E. \end{aligned}$$

Thus, $\deg|zE-(A+BFC)|=\mathrm{rank}E$. Using Theorem 8–3.3[1] yields the conclusion that system (9) is causality controllable and causality observable.

Sufficiency: Suppose system (9) is causality controllable and causality observable. From Theorem 8–2.1 and Theorem 3–5.1 in Ref.[1], we know that there exists a feedback control

$$u(k)=Fy(k)+v(k) \tag{14}$$

such that the closed-loop system

$$\begin{aligned} Ex(k+1)&=(A+BFC)x(k)+Bv(k)\\ y(k)&=Cx(k) \end{aligned} \tag{15}$$

is causal, i.e.,

$$\deg|zE-(A+BFC)|=\mathrm{rank}E \tag{16}$$

Then there exist two nonsingular matrices Q and P such that system (15) is a restricted system equivalent to

$$\begin{aligned} x_1(k+1)&=A_1x_1(k)+B_1v(k),\\ 0&=x_2(k)+B_2v(k),\\ y(k)&=C_1x_1(k)+C_2x_2(k), \end{aligned} \tag{17}$$

where $x_1\in R^q$, $x_2\in R^{n-q}$, q=rank$E$ and

$$QEP=\begin{bmatrix}I_q & \\ & 0\end{bmatrix},$$

$$Q(A+BFC)P=\begin{bmatrix}A_1 & \\ & I_{n-q}\end{bmatrix},$$

$$CP=[C_1\ C_2],\ QB=\begin{bmatrix}B_1\\ B_2\end{bmatrix}.$$

From Theorem 8–3.2[1], the forward subsystem of system (9)

$$\begin{aligned} x_1(k+1)&=A_1x_1(k)+B_1v(k)\\ y_1(k)&=C_1x_1(k) \end{aligned} \tag{18}$$

is stabilizable and detectable. Thus, two matrices F_c and B_c may be chosen so that

$$\sigma(A_1+B_1F_c)\subset U^+,\quad \sigma(A_1-B_cC_1)\subset U^+. \tag{19}$$

Now we will construct the normal compensator for subsystem (18)

$$\begin{aligned} x_c(k+1)&=A_cx_c(k)+B_cy(k),\\ u(k)&=F_cx_c(k)+Fy(k), \end{aligned} \tag{20}$$

where $A_c=A_1-B_cC_1+(B_1+B_cC_2B_2)F_c$.

Applying it to system (9), the closed-loop system is

$$\begin{aligned} &\begin{bmatrix}E & 0\\ 0 & I_q\end{bmatrix}\cdot\begin{bmatrix}x(k+1)\\ x_c(k+1)\end{bmatrix}\\ &=\begin{bmatrix}A+BFC & BF_c\\ B_cC & A_c\end{bmatrix}\cdot\begin{bmatrix}x(k)\\ x_c(k)\end{bmatrix}. \end{aligned} \tag{21}$$

It is obvious that

$$\begin{aligned} &\sigma\left(\begin{bmatrix}E & 0\\ 0 & I_q\end{bmatrix},\begin{bmatrix}A+BFC & BF_c\\ B_cC & A_c\end{bmatrix}\right)\\ &=\sigma(A_1+B_1F_c)\cup\sigma(A_1-B_cC_1)\subset U^+, \end{aligned}$$

implying that the closed-loop system (21) is stable. Moreover, we have

$$\begin{aligned} &\deg\left|z\begin{bmatrix}E & 0\\ 0 & I_q\end{bmatrix}-\begin{bmatrix}A+BFC & BF_c\\ B_cC & A_c\end{bmatrix}\right|\\ &=\deg|zE-(A+BFC)|+n_c=\mathrm{rank}+n_c. \end{aligned}$$

Theorem 3.2 (2) tells us that system (9) has a structurally stable normal dynamic compensator at data point $P(A,B,C)$.

As shown in Theorem 2.1, a normal compensator of the form of Eq.(12) cannot be structurally stable for system (9) at date point $P(E,A,B,C)$. So, we have to find another kind of compensator. In fact, we may consider the compensator of the following form

$$x_c(k+1) = A_c x_c(k) + B_c y(k),$$

$$u(k) = F_c x_c(k) + F y(k) + H y(k+1), \tag{22}$$

where $x_c \in R^{n_c}$, A_c, B_c, H, F_c and F are constant matrices of appropriate dimensions.

Definition 3.3 System (9) is said to be normalizable if a feedback control

$$u(k) = K_1 x(k) - K_2 x(k+1) + v(k) \tag{23}$$

may be chosen such that its closed-loop system

$$(E + BK_2)x(k+1) = (A + BK_1)x(k) + Bv(k)$$

$$y(k) = Cx(k) \tag{24}$$

is normal, i.e.,

$$|E + BK_2| \not\equiv 0. \tag{25}$$

System (9) is said to be dual normalizable if its dual system is normalizable.

Theorem 3.4 System (9) has a structurally stable normal compensator (22) at data point $P(E,A,B,C)$ if and only if system (9) is stabilizable, detectable, normalizable and dual normalizable.

Proof. Necessity: System (9) and compensator (22) form the closed-loop system

$$\begin{bmatrix} E - BHC & 0 \\ 0 & I_{n_c} \end{bmatrix} \cdot \begin{bmatrix} x(k+1) \\ x_c(k+1) \end{bmatrix}$$

$$= \begin{bmatrix} A + BFC & BF_c \\ B_c C & A_c \end{bmatrix} \cdot \begin{bmatrix} x(k) \\ x_c(k) \end{bmatrix}. \tag{26}$$

Under the above assumption, system (26) is structurally stable at data point $P(E,A,B,C)$. From Theorem 2.1, rank(diag$(E - BHC, I_{n_c})$)=$n + n_c$ or rank$(E - BHC)$=n. Thus, system (9) is normalizable and dual normalizable. Since system (26) is stable, system (9) is stabilizable and detectable.

Sufficiency: Let system (9) be normalizable and dual normalizable. From Lemma 3–5.1[1], a feedback control

$$u(k) = Hy(k+1) + v(k) \tag{27}$$

may be chosen so that the closed-loop system

$$(E - BHC)x(k+1) = Ax(k) + Bv(k)$$

$$y(k) = Cx(k) \tag{28}$$

is normal, i.e., $|E - BHC| \neq 0$, where $v(k)$ is a new input.

System (9) can be rewritten as

$$x(k+1) = \overline{A}x(k) + \overline{B}v(k)$$

$$y(k) = Cx(k) \tag{29}$$

where $A = (E - BHC)^{-1}A$, $\overline{B} = (E - BHC)^{-1}B$. Recall that system (9) is stabilizable and detectable, Lemma 3–3.2[1] indicates system (9) is both stabilizable and detectable too. Therefore, there exist two matrices K and G such that $\sigma(\overline{A} + \overline{B}K) \subset U^+$ and $\sigma(\overline{A} - GC) \subset U^+$. Choose a compensator

$$x_c(k+1) = (\overline{A} - GC + \overline{B}K)x_c(k) + Gy(k),$$

$$u(k) = Kx_c(k) + Hy(k+1), \tag{30}$$

which, when applied to system (9), forms the closed-loop system

$$\begin{bmatrix} E - BHC & 0 \\ 0 & I \end{bmatrix} \cdot \begin{bmatrix} x(k+1) \\ x_c(k+1) \end{bmatrix}$$

$$= \begin{bmatrix} A & BK \\ GC & \overline{A} - GC + \overline{B}K \end{bmatrix} \cdot \begin{bmatrix} x(k) \\ x_c(k) \end{bmatrix}. \tag{31}$$

That is

$$\begin{bmatrix} x(k+1) \\ x_c(k+1) \end{bmatrix}$$

$$= \begin{bmatrix} \overline{A} & \overline{B}K \\ GC & \overline{A} - GC + \overline{B}K \end{bmatrix} \cdot \begin{bmatrix} x(k) \\ x_c(k) \end{bmatrix}. \tag{32}$$

The characteristic polynomial

$$\begin{vmatrix} zI - \overline{A} & -\overline{B}K \\ -GC & zI - (\overline{A} - GC + \overline{B}K) \end{vmatrix}$$

$$= \begin{vmatrix} zI - \overline{A} & -\overline{B}K \\ -zI + \overline{A} - GC & zI - \overline{A} + GC \end{vmatrix}$$

$$= \begin{vmatrix} zI - \overline{A} - \overline{B}K & -\overline{B}K \\ 0 & zI - \overline{A} + GC \end{vmatrix}.$$

Recalling that $\sigma(-\overline{A}+\overline{B}K) \subset U^+$ and $\sigma(\overline{A}-GC) \subset U^+$, we have

$$\sigma\left(\begin{bmatrix} \overline{A} & \overline{B}K \\ GC & \overline{A}-GC+\overline{B}K \end{bmatrix}\right) \subset U^+.$$

This indicates that the normal system (31) is stable, and hence is structurally stable. That is, system (9) has a structurally stable normal compensator at data point $P(E, A, B, C)$.

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