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# Magnetic ordering of the bond-diluted two-dimensional mixed spin Ising system*

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Numerical and analytical results are presented for the magnetic ordering in a bond-diluted spin-1/2 and spin-1 mixed transverse Ising system with a single-ion anisotropy on a honeycomb lattice. Special emphasis is placed on the magnetic ordering under the bond dilution and percolation threshold. We discuss in detail the influence of transverse fields of different sublattices on the normal magnetic ordering and on the magnetic ordering induced by single-ion anisotropy. We find that the magnetic ordering of a system exhibits an explicit difference when receiving the transverse field. This phenomenon has not been revealed in previous reports.

**Keywords:** magnetic ordering, mixed spin system, transverse fields of different sublattices, single-ion anisotropy

**PACC:** 7510D, 7510H, 7550G

## 1. Introduction

In the past few decades, there has been an increasing number of studies dealing with the critical properties of the various magnetic spin systems.[1−4] Recently, the mixed spin system has been a subject of extensive investigation because it helps simulate many interesting, as well as important, phenomena such as the induced magnetic ordering and multi-compensation behaviours.[5,6] One of the most successful investigations is for the mixed Ising system consisting of spin-1/2 and spin-1, located on two different sublattices. There are mainly two models in the previous works. One is the mixed spin Blume–Capel model, originally proposed to treat single spin ($S$=1) magnetic systems.[7,8] Some outstanding properties can be accounted for in terms of various theoretical methods, when the phase transition temperature is plotted as a function of single-ion anisotropy.[9−11] Another is the so-called transverse Ising model, i.e. a simple quantum spin model. This model was originally introduced by de Gennes[12] as a model for hydrogen-bonded ferroelectrics. Since then, this model has proven useful for several other systems. In fact, a transverse field term must be added in the Hamiltonian to represent the quantum tunnelling effect. For the mixed spin system, this model may introduce different transverse fields due to the presence of two different sublattices. Critical properties of the system have also been examined by a variety of techniques.[13−15] On the other hand, many authors have considered various random distributions in the above two models, such as bimodel and trimodel random fields, either site or bond dilution, and so on. It has been understood that random distributions in the system play an important role in the determination of phase transitions and magnetic properties, and that this could lead to some new results.[16−19] In particular, we have noticed that the transverse field and the single-ion anisotropy can induce new magnetic ordering.[5]

A more general strategy in studying a mixed spin system combines the Blume–Capel model and the transverse Ising model just because a real spin system can have the transverse field and the single-ion anisotropy interaction simultaneously. In this case, it is possible for the mixed spin system to give rise to a more complicated situation. Recently, some studies have made progress in the theoretical direction. Some authors have studied the phase diagrams of the mixed transverse Ising spin system on a simple cubic lattice with a random single-ion anisotropy.[20] We have discussed the effect of the transverse field on the bond-diluted mixed transverse spin system with single-ion

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anisotropy and we have found that the transverse field destroyed the re-entrant phenomenon of the system, and depressed the magnetic ordering induced by the single-ion anisotropy.[21] Meanwhile, we have shown that the transverse field can also induce the magnetic ordering at the bond percolation threshold or below the threshold on a square lattice.[22] Of course, we need to consider further the different transverse fields associated with different sublattices, whose behaviour is expected to be suitable for a more general situation. This is reasonable since transverse fields of different sublattices represent different quantum effects. Thus, magnetic ordering of the system may become very rich. In this case, the influence of transverse fields of different sublattices on the normal and induced magnetic ordering of the system will show distinctly different characteristics. To our knowledge, the above subject has not been investigated yet. Therefore, we need an overall consideration of the competition between the transverse fields of different sublattices, the bond dilution and the single-ion anisotropy in the present system.

Our aim in this paper is to study the magnetic ordering of the bond-diluted mixed spin-1/2 and spin-1 system with a single-ion anisotropy as well as transverse fields of different sublattices, and to focus attention on the influence of transverse fields of different sublattices on the magnetic ordering. This paper is divided into four sections. The next section is devoted to the theoretical derivation and gives an overview of the procedure by means of the effective field theory (EFT) with correlations introduced by Honmura and Kaneyoshi.[23] The detailed numerical results and discussion are presented in section 3. A short conclusion is given in section 4.

## 2. Theory

We consider a two-sublattice bond-diluted mixed spin-1/2 and spin-1 Ising system, in which the transverse fields of different sublattices and single-ion anisotropy are involved. The Hamiltonian of the system can be described by the following equation

$$H = -\sum_{\langle ij\rangle} J_{ij}\sigma_i^z S_j^z - \Omega_{1/2}\sum_i \sigma_i^x - \Omega_1\sum_j S_j^x - D\sum_j (S_j^z)^2. \tag{1}$$

The underlying lattice is composed of two interpenetrating sublattices $A$ and $B$. Here $\sigma_i^x$ and $\sigma_i^z$ are spin-1/2 Pauli matrices associated with the $i$th site of sublattice $A$, while $S_j^x$ and $S_j^z$ are spin-1 Pauli matrices associated with the $j$th site of sublattice $B$. The quantities $\Omega_{1/2}$ and $\Omega_1$ represent the transverse fields on the sublattices $A$ and $B$, respectively, i.e. they both indicate different quantum effects in the mixed system. $D$ is the magnitude of the single-ion anisotropy, assumed to be negative. The first summation is carried out only over all nearest-neighbour pairs of spins. The second summation extends over all sites of sublattice $A$. The third and the fourth summations involve all sites of sublattice $B$. $J_{ij}$ is the nearest-neighbour exchange interaction parameter and has an independent bond-dilution probability distribution.

$$P(J_{ij}) = p\delta(J_{ij} - J) + (1-p)\delta(J_{ij}). \tag{2}$$

The starting point of the statistics of the present system is the relation proposed by Sa Barreto *et al.*[24] Then the thermal averages of spin operators $\sigma_i^z, S_j^z$ and $(S_j^z)^2$ at sites $i$ and $j$ are generally given by

$$\sigma = \langle\langle\sigma_i^z\rangle\rangle_r = \left\langle\left\langle \frac{\mathrm{Tr}_i\sigma_i^z \mathrm{e}^{-\beta H_i}}{\mathrm{Tr}_i \mathrm{e}^{-\beta H_i}}\right\rangle\right\rangle_r, \tag{3}$$

$$m = \langle\langle S_j^z\rangle\rangle_r = \left\langle\left\langle \frac{\mathrm{Tr}_j S_j^z \mathrm{e}^{-\beta H_j}}{\mathrm{Tr}_j \mathrm{e}^{-\beta H_j}}\right\rangle\right\rangle_r \tag{4}$$

and

$$q = \langle\langle (S_j^z)^2\rangle\rangle_r = \left\langle\left\langle \frac{\mathrm{Tr}_j (S_j^z)^2 \mathrm{e}^{-\beta H_j}}{\mathrm{Tr}_j \mathrm{e}^{-\beta H_j}}\right\rangle\right\rangle_r, \tag{5}$$

where $\beta = 1/k_{\mathrm{B}}T$, the inner $\langle\cdots\rangle$ indicates the canonical thermal average, and the outer $\langle\cdots\rangle_r$ denotes the bond-diluted average for Eq.(2). $H_i$ and $H_j$ include all parts of Hamiltonian (1) associated with the $i$th site of sublattice $A$ and the $j$th site of sublattice $B$, while $\mathrm{Tr}_i$ and $\mathrm{Tr}_j$ are the partial traces with respect to the $i$th site of sublattice $A$ and the $j$th site of sublattice $B$, respectively. Thus, the partial partition functions of different sublattices are given by

$$Z_i = \mathrm{Tr}_i \mathrm{e}^{-\beta H_i} = \sum_{m=1}^{2} \exp\left[(-1)^m - \frac{\beta}{2}(E_i^2 + \Omega_{1/2})^{1/2}\right], \tag{6}$$

$$Z_j = \mathrm{Tr}_j \mathrm{e}^{-\beta H_j} = \sum_{n=1}^{3} \exp\left[2\beta C\cos\frac{(n-1)2\pi + \theta}{3} + \frac{2\beta D}{3}\right], \tag{7}$$

where

$$\theta = \arccos(A/C^3), \tag{8}$$

$$C^6 = A^2 + B^2, \tag{9}$$

with

$$A = -\frac{1}{27}D^3 + \frac{1}{3}DE_i^2 - \frac{1}{6}D\Omega_1^2, \tag{10}$$

$$B=\frac{1}{9}\left[3(D^2E_j-E_j^3)^2+\frac{3}{4}D^2\Omega_1^4+15D^2\Omega_1^2E_j^2+9\Omega_1^2E_j^4+9\Omega_1^4E_j^2+3\Omega_1^6\right]. \tag{11}$$

It should be noted here that $E_i$ and $E_j$ in the above expressions are the local fields on sites $i$ and $j$, respectively. Thus, expressions of the averaged different sublattice magnetizations of the present system are given by using effective field theory:

$$\sigma=\langle\langle\sigma_i^z\rangle\rangle_r=\left\langle\left\langle\prod_{j=1}^{z}\left[(S_j^z)^2\cosh(J_{ij}\nabla)+S_j^z\sinh(J_{ij}\nabla)+1-(S_j^z)^2\right]\right\rangle\right\rangle_r F(x)|_{x=0}, \tag{12}$$

$$m=\langle\langle S_j^z\rangle\rangle_r=\left\langle\left\langle\prod_{i=1}^{z}\left[\cosh\left(\frac{1}{2}J_{ij}\nabla\right)+2\sigma_i^z\sinh\left(\frac{1}{2}J_{ij}\nabla\right)\right]\right\rangle\right\rangle_r G(x)|_{x=0}. \tag{13}$$

The parameter $q$ in Eq.(5) is defined by

$$q=\langle\langle(S_j^z)^2\rangle\rangle_r=\left\langle\left\langle\prod_{i=1}^{z}\left[\cosh\left(\frac{1}{2}J_{ij}\nabla\right)+2\sigma_i^z\sinh\left(\frac{1}{2}J_{ij}\nabla\right)\right]\right\rangle\right\rangle_r H(x)|_{x=0}, \tag{14}$$

where $z$ is a coordination number, while $\nabla$ is a differential operator. Functions $F(x)$, $G(x)$ and $H(x)$ are defined as

$$F(x)=\frac{1}{2}\frac{x}{(\Omega_{1/2}^2+x^2)}\tanh\left(\frac{\beta}{2}(\Omega_{1/2}^2+x^2)^{1/2}\right), \tag{15}$$

$$G(x)=\sum_{n=1}^{3}\left[Z_j\left[\frac{2x}{3C}\cos\frac{(n-1)2\pi+\theta}{3}+\frac{2}{27}\frac{D^3x-Dx^3+(7/2)D\Omega_1^2x}{BC}\sin\frac{(n-1)2\pi+\theta}{3}\right]\right]Z_j^{-1} \tag{16}$$

and

$$H(x)=\sum_{n=1}^{3}\left[Z_j\left[\frac{2D}{9C}\cos\frac{(n-1)2\pi+\theta}{3}+\frac{2}{27}\frac{(1/2)\Omega_1^2x^2-D^2x^2+x^4-(1/2)\Omega_1^4}{BC}\sin\frac{(n-1)2\pi+\theta}{3}+\frac{2}{3}\right]\right]Z_j^{-1}. \tag{17}$$

Since Eqs.(12)–(14) deal with multi-spin correlations, we cannot treat exactly all the spin-spin correlations. Therefore, we adopt a simple decoupling approximation

$$\langle\langle\sigma_i^z\sigma_j^z\cdots\sigma_l^z\rangle\rangle_r\approx\langle\langle\sigma_i^z\rangle\rangle_r\langle\langle\sigma_j^z\rangle\rangle_r\cdots\langle\langle\sigma_l^z\rangle\rangle_r$$

and

$$\langle\langle S_j^z(S_k^z)^2\cdots S_m^z\rangle\rangle_r\approx\langle\langle S_j^z\rangle\rangle_r\langle\langle(S_k^z)^2\rangle\rangle_r\cdots\langle\langle S_m^z\rangle\rangle_r,$$

for $i\neq j\neq k\neq\cdots$. Now applying the above decoupling approximation, the different sublattice magnetizations $\sigma$ and $m$, and parameter $q$, can be evaluated from the following coupled equations

$$\sigma=\left[q\langle\cosh(J_{ij}\nabla)\rangle_r+m\langle\sinh(J_{ij}\nabla)\rangle_r+1-q\right]^z F(x)|_{x=0}, \tag{18}$$

$$m=\left[\left\langle\cosh\left(\frac{1}{2}J_{ij}\nabla\right)\right\rangle_r+2\sigma\left\langle\sinh\left(\frac{1}{2}J_{ij}\nabla\right)\right\rangle_r\right]^z G(x)|_{x=0}, \tag{19}$$

$$q=\left[\left\langle\cosh\left(\frac{1}{2}J_{ij}\nabla\right)\right\rangle_r+2\sigma\left\langle\sinh\left(\frac{1}{2}J_{ij}\nabla\right)\right\rangle_r\right]^z H(x)|_{x=0}. \tag{20}$$

In the vicinity of the second-order phase transition line, by expanding the right-hand side of Eqs.(18)–(20) and combining them, the self-consistent equation of the magnetization $\sigma$ in sublattice $A$ is given by

$$\sigma=a\sigma+b\sigma^3+\cdots. \tag{21}$$

Since $\sigma$ is small enough near the second-order phase transition line, we retain only the first and second terms to obtain an explicit solution of the magnetization in sublattice $A$. Then its magnetization $\sigma$ can be obtained as

$$\sigma^2=(1-a)/b. \tag{22}$$

Now, we are interested in the study of magnetic ordering of the present system. We have to determine

the equation of the second-order phase transition line, which is given by $a = 1$ and $b < 0$, i.e.

$$a = 2z^2 L_1 \langle \sinh(J_{ij}\nabla)\rangle_r [Q_1 \langle \cosh(J_{ij}\nabla)\rangle_r + 1 - Q_1]^{z-1} F(x)|_{x=0} = 1. \quad (23)$$

Here $L_1$ and $Q_1$ are defined as

$$L_1 = \left\langle \sinh\left(\frac{1}{2}J_{ij}\nabla\right)\right\rangle_r \times \left[\left\langle \cosh\left(\frac{1}{2}J_{ij}\nabla\right)\right\rangle_r\right]^{z-1} G(x)|_{x=0}, \quad (24)$$

$$Q_1 = \left[\left\langle \cosh\left(\frac{1}{2}J_{ij}\nabla\right)\right\rangle_r\right]^{z} H(x)|_{x=0}. \quad (25)$$

For simplicity, we only discuss the honeycomb lattice case, although any two-sublattice system with an arbitrary coordination number $z$ can be discussed in the same way. This is because the magnetic ordering of a square lattice in a uniform transverse field has been analysed.[22] Therefore, we believe that the lattice with a higher coordination number can also show a similar phenomenon. Of course, it is well known that the honeycomb lattice with $z$=3 does not have a tricritical point on the phase diagrams.[25] In other words, the phase transition lines are always of the second-order for the honeycomb lattice. Thus, we have

$$\begin{aligned} b =& 24 L_2 \langle \sinh(J_{ij}\nabla)\rangle_r \\ & \times [Q_1 \langle \cosh(J_{ij}\nabla)\rangle_r + 1 - Q_1]^2 F(x)|_{x=0} \\ & + 216 L_1^3 [\langle \sinh(J_{ij}\nabla)\rangle_r]^3 F(x)|_{x=0} \\ & + 432 L_1 Q_2 \langle \sinh(J_{ij}\nabla)\rangle_r \{Q_1 [\langle \cosh(J_{ij}\nabla)\rangle_r]^2 \\ & + (1 - 2Q_1)\langle \cosh(J_{ij}\nabla)\rangle_r \\ & - (1 - Q_1)\} F(x)|_{x=0} < 0, \end{aligned} \quad (26)$$

where $L_2$ and $Q_2$ are in the following form

$$L_2 = \left[\left\langle \sinh\left(\frac{1}{2}J_{ij}\nabla\right)\right\rangle_r\right]^3 G(x)|_{x=0}, \quad (27)$$

$$Q_2 = \left[\left\langle \sinh\left(\frac{1}{2}J_{ij}\nabla\right)\right\rangle_r\right]^2 \times \left\langle \cosh\left(\frac{1}{2}J_{ij}\nabla\right)\right\rangle_r H(x)|_{x=0}. \quad (28)$$

The exchange interaction $J_{ij}$ in the above expressions is given by the independent random variable with bond-dilution probability distribution i.e. Eq.(2). Up to now, we have derived the expressions for the bond-diluted mixed spin-1/2 and spin-1 Ising system with a single-ion anisotropy as well as the different transverse fields. The next step is to solve Eq.(23) numerically, and to discuss the magnetic ordering of the present system. We find that the present system can show very rich magnetic ordering phenomena, which depend on the transverse fields of different sublattices, the single-ion anisotropy and the bond concentration. Hence, in the following section, we examine the influence of the transverse fields of different sublattices on the magnetic ordering of the system and pay much attention to the new magnetic ordering phenomena induced. On the other hand, it is worth mentioning that in the above derivations we have not touched the sign of $J_{ij}$. Although the ground state of the mixed spin system with $J_{ij} > 0$ is ferromagnetic, and that with $J_{ij} < 0$ is ferrimagnetic, the phase diagrams or the magnetic ordering obtained in the present system are the same at all. Certainly, if the magnetization behaviours are discussed, ferromagnetic and ferromagnetic systems are very different.

## 3. Numerical results and discussion

In this section, the phase diagrams or the magnetic ordering of the bond-diluted mixed spin-1/2 and spin-1 system with a single-ion anisotropy as well as different transverse fields on the honeycomb lattice are determined numerically in the $(T, D)$ plane from Eq.(23) for various cases. Results are discussed in terms of transverse fields of different sublattices and bond-diluted concentrations.

Figure 1 illustrates the phase diagrams for the absence of the transverse field of sublattice $B$, for two values of the transverse field of sublattice $A$ ($\Omega_{1/2}$/J=0.5 in Fig.1(a) and $\Omega_{1/2}$/J=1.0 in Fig.1(b)), when values of bond concentration $p$ are changed. It is very clear that the phase transition is always of second order for a honeycomb lattice. Some studies have learned that the negative single-ion anisotropy takes three fixed values at absolute zero temperature and the re-entrant phenomena of the second-order phase transition can be observed, when the transverse field is absent and the bond concentration varies in a certain range for the honeycomb lattice. On the other hand, when a uniform transverse field is present, the three fixed values of the single-ion anisotropy no longer appear and the re-entrant phenomenon is depressed until its disappearance with increasing uniform transverse field.[21] However, if the transverse fields of different sublattices replace the uniform transverse field, the phase diagrams will undergo an apparent change. As shown in Figs.1(a) and 1(b), the present system may

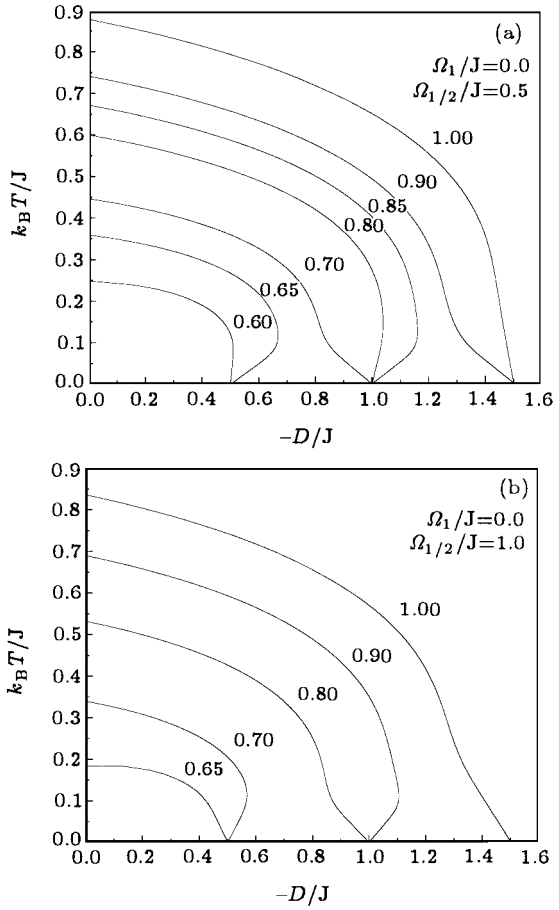



**Fig.1.** Curie temperature depending on the single-ion anisotropy for $B$ $\Omega_1$/J=0 sublattice and $A$ $\Omega_{1/2}$/J sublattice values of (a) 0.5 and (b) 1.0. The numbers on the curves are the values of bond concentration $p$.

also show three fixed single-ion anisotropy values at zero temperature and re-entrant phenomenon, even though there exists the transverse field of sublattice $A$ $\Omega_{1/2}$. This is a very unusual feature. This means that the existence of the transverse field of sublattice $A$ $\Omega_{1/2}$ does not affect the basic structure of the phase diagrams. For instance, the phase transition lines at bond concentrations $p$=0.9 and 0.7 with $\Omega_{1/2}$/J=0.5 take fixed single-ion anisotropy values $D$/J=–1.5 and –1.0 at $T$=0K, while they take $D$/J=–1.0 and –0.5 with $\Omega_{1/2}$/J=1.0. Hence, no matter how the transverse field of sublattice $A$ $\Omega_{1/2}$ and the bond concentration $p$ vary in the phase diagrams, we can infer that the single-ion anisotropy parameter can only take three fixed values at $T$=0K, i.e. $D$/J=–1.5, –1.0 and –0.5. Of course, with a further increase of $\Omega_{1/2}$, the single-ion anisotropy parameter may take two fixed values of $D$/J=–1.0 and –0.5 or one fixed value of $D$/J=–0.5 at $T$=0K. On the other hand, the re-entrant phenomenon is another important problem in phase transition. We can observe re-entrant phenomenon in some appropriate ranges of parameters from Figs.1(a) and 1(b). From a traditional point of view, the transverse field would depress and destroy the re-entrant phenomenon. But, in the absence of the transverse field of sublattice $B$ $\Omega_1$, the role of the transverse field of sublattice $A$ $\Omega_{1/2}$ is either to depress or to assist the re-entrant phenomenon under certain conditions. This result indicates that it is difficult to determine the role of $\Omega_{1/2}$ in the re-entrant phase transition. Thus, the role of $\Omega_{1/2}$ becomes complicated.

Figure 2 shows two phase diagrams in the ($T$, $D$) plane, for two values of the transverse field of sublattice $B$ ($\Omega_1$/J=0.2 in Fig.2(a) and $\Omega_1$/J=0.5 in Fig.2(b)) in the absence of $\Omega_{1/2}$, when the values of bond concentration $p$ are changed. From Fig.2(a) we can see that the three fixed values of the single-ion anisotropy parameter at $T$=0K have disappeared and that the re-entrant phenomenon is depressed rapidly, although the phase transition line with $p$=0.8 still shows a weak re-entrant phenomenon. In this sense, the phase diagram is similar to that of Fig.1(b) in Ref.[21]. In Fig.2(b), we find that the re-entrant phenomenon has disappeared in the phase diagram. Therefore, it is obvious that there is a critical value $\Omega_1$, and surpassing this critical value will lead to the disappearance of the re-entrant phenomenon in the system. From our calculation, the critical value is $\Omega_1$/J=0.3386. Then we can see that the role of $\Omega_1$ is to destroy the re-entrant phase transition. In this aspect, the influences of $\Omega_{1/2}$ and $\Omega_1$ on the phase transition are in opposite directions. This characteristic can be understood by a simple physical interpretation. The re-entrant phenomenon in the present system results mainly from both the single-ion anisotropy in spin-1 $B$ sublattice and the bond dilution. The existence of $\Omega_1$ seems to depress the sublattice $B$ spin state easily, i.e. most of the $S_j^z$ spins take the $S_j^z$=0 state steadily and act like nonmagnetic atoms, because they belong to the same sublattice. On the other hand, the single-ion anisotropy fails to contribute to the spin-1/2 sublattice $A$. Then it is hard to judge whether the transverse field of sublattice $A$ $\Omega_{1/2}$ is to depress or to assist the re-entrant phenomenon. In addition, we also notice that the critical value $\Omega_1$/J=0.3386 is smaller than that reported in Ref.[21]. This is reasonable since the transverse field of sublattice $A$ $\Omega_{1/2}$ has a tendency to assist the re-entrant phase transition. Thus, the critical value of the transverse field of sublattice $B$ $\Omega_1$ can take a smaller value when the transverse field of sublattice $A$ $\Omega_{1/2}$ is absent. In fact, the

existence of $\Omega_1$ influences the structure of the phase diagram, observed by comparing Figs.2(a) and 2(b).

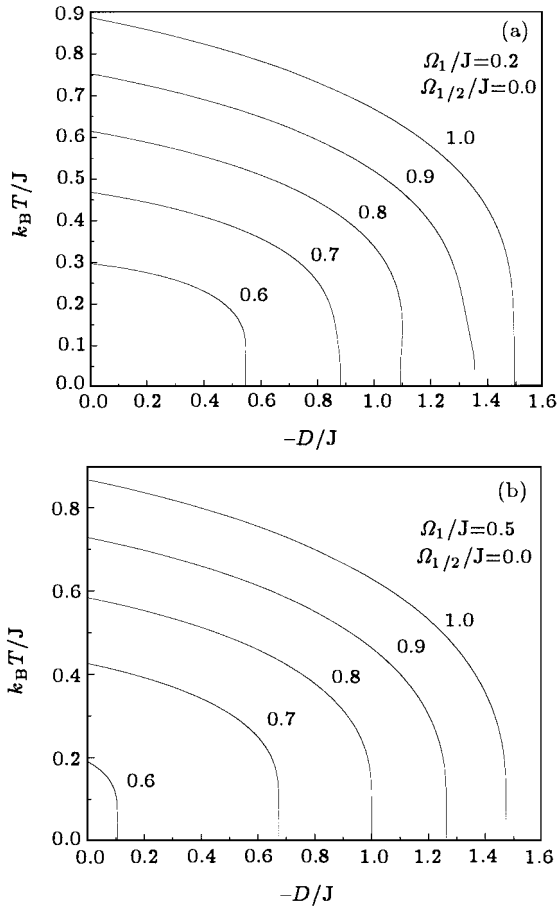



**Fig.2.** Curie temperature depending on the single-ion anisotropy for sublattice $A$ $\Omega_{1/2}$/J=0 and sublattice $B$ $\Omega_1$/J values of (a) 0.2 and (b) 0.5. The numbers on the curves are the values of bond concentration $p$.

Due to the dependence of magnetic ordering or phase diagram on the bond concentration, it is necessary to give the bond percolation threshold. When the Curie temperature reaches zero, the bond percolation threshold is $p_c$=0.5378.[26] Usually, one considers the system to be in an ordering state at $p > p_c$, and to be in a disordering state at $p \leq p_c$. However, we find that the present system still shows magnetic ordering even though the bond concentration equals its percolation threshold or is below the threshold. In other words, the Curie temperature still takes finite values, i.e. the system maintains a ferromagnetic (or ferrimagnetic) phase. Figure 3 shows the critical temperature dependences of the single-ion anisotropy at the threshold $p_c$=0.5378 with $\Omega_1$=0, for various values of the transverse field of sublattice $A$ $\Omega_{1/2}$. From Fig.3 we see that the critical temperature takes a series of finite values for $-0.5 < D/\mathrm{J} < 0$ although the bond concentration equals the percolation threshold. On increasing the transverse field of sublattice $A$ $\Omega_{1/2}$, the magnetic ordering is depressed. Thus, there is a critical value $\Omega_{1/2k}$. From our calculation, $\Omega_{1/2k}$=0.5482J. The magnetic ordering will disappear when $\Omega \geq \Omega_{1/2k}$. On the other hand, Fig.4 shows the critical temperature dependences of the single-ion anisotropy at the threshold $p_c$=0.5378 with $\Omega_{1/2}$=0, for various values of the transverse field of sublattice $B$ $\Omega_1$. We observed that the magnetic ordering induced by the single-ion anisotropy decreases rapidly with the increasing transverse field of sublattice $B$ $\Omega_1$. Thus, we can also obtain a critical value $\Omega_{1k}$, which leads to the disappearance of induced magnetic ordering. According to our numerical result, $\Omega_{1k}$=0.2040J. These results indicate that the single-ion anisotropy can induce new magnetic ordering in a small range of $\Omega_{1/2}$ or $\Omega_1$. The occurrence of the induced magnetic ordering comes from the mixed spin system because this phenomenon is not found in a single spin system. Secondly, the single-ion anisotropy can stimulate some sites of sublattice $B$ in the states $S = \pm 1$ even if the bond concentration equals the percolation threshold. On the other hand, we wish to point out the distinction between Figs.3 and 4. The induced magnetic ordering with $\Omega_{1/2} < \Omega_{1/2k}$ is always for $-0.5 < D/\mathrm{J} < 0$ in Fig.3, while in Fig.4 the existence of the induced magnetic ordering with $\Omega_1 < \Omega_{1k}$ corresponds to various variation regions of the single-ion anisotropy. The reason that such a distinction exists is also due to the fact that the single-ion anisotropy contributes only to the spin-1 sublattice $B$. Finally, it is worth mentioning that, if $\Omega_{1/2} = \Omega_1 = \Omega$ in the present system, we will have results consistent with those in Ref.[21].

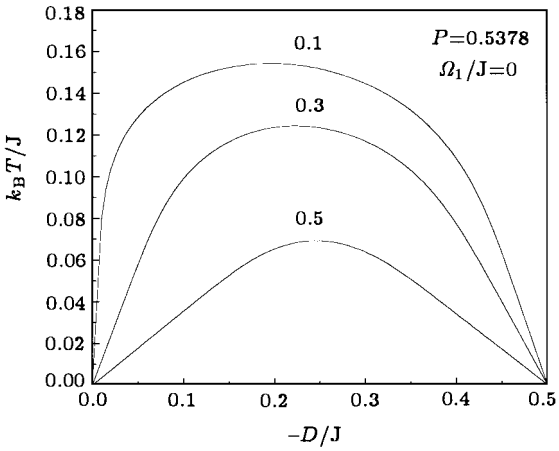



**Fig.3.** Curie temperature depending on the single-ion anisotropy for sublattice $B$ $\Omega_1$/J=0 and bond percolation threshold $p_c$=0.5378. The numbers on the curves are the values of sublattice $A$ $\Omega_{1/2}$/J.

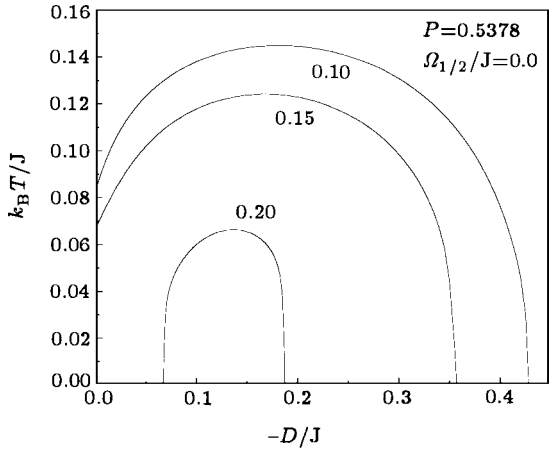



**Fig.4.** Curie temperature depending on the single-ion anisotropy for sublattice $A$ $\Omega_{1/2}/\mathrm{J}=0$ and bond percolation threshold $p_c=0.5378$. The numbers on the curves are the values of sublattice $B$ $\Omega_1/\mathrm{J}$.

# 4. Conclusion

In this paper, the normal magnetic ordering and the magnetic ordering induced by the single-ion anisotropy have been studied by using the EFT. We have discussed in detail the influence of the transverse fields of different sublattices on the normal and induced magnetic ordering and have discovered that the transverse fields $\Omega_{1/2}$ and $\Omega_1$ play very different roles. The role of $\Omega_{1/2}$ is either to depress or to assist the re-entrant phenomenon according to bond concentrations, while the role of $\Omega_1$ destroys the second-order re-entrant phase transition. Although $\Omega_{1/2}$ and $\Omega_1$ can depress the magnetic ordering induced by the single-ion anisotropy, however they show different behaviours, as depicted in Figs.3 and 4. Also, some factors that affect the structure of the phase diagrams have been analysed. We have also given some critical transverse fields values, at which the re-entrant phenomenon and the induced magnetic ordering can no longer occur. Of course, these results are obtained within the framework of the EFT. Finally, it is necessary to indicate that, if some more accurate methods such as series expansions or Monte Carlo simulations were adopted, it would lead to somewhat better results.

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