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A design of novel type superconducting magnet for super-high field functional magnetic resonance imaging by using the harmonic analysis method of magnetic vector potentials*

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The approach of expanding the magnetic scalar potential in a series of Legendre polynomials is suitable for designing a conventional superconducting magnetic resonance imaging magnet of distributed solenoidal configuration. Whereas the approach of expanding the magnetic vector potential in associated Legendre harmonics is suitable for designing a single-solenoid magnet that has multiple tiers, in which each tier may have multiple layers with different winding lengths. A set of three equations to suppress some of the lowest higher-order harmonics is found. As an example, a 4T single-solenoid magnetic resonance imaging magnet with 4×6 layers of superconducting wires is designed. The degree of homogeneity in the 0.5m diameter sphere volume is better than 5.8 ppm. The same degree of homogeneity is retained after optimal integralization of turns in each correction layer. The ratio $B_{\rm m}/B_0$ in the single-solenoid magnet is 30% lower than that in the conventional six-solenoid magnet. This tolerates higher rated superconducting current in the coil. The Lorentz force of the coil in the single-solenoid system is also much lower than in the six-solenoid system. This novel type of magnet possesses significant advantage over conventional magnets, especially when used as a super-high field functional magnetic resonance imaging magnet.

Keywords: super-high field superconducting magnet for functional magnetic resonance imaging, high homogeneity functional magnetic resonance imaging magnet, single-solenoid type functional magnetic resonance imaging magnet, magnetic vector potential, harmonic analysis approach, end correction

PACC: 0755, 8760I, 4110F, 7470P

1. Introduction

The magnetic resonance imaging (MRI) scanner is a powerful non-invasive imaging tool that produces cross-sectional tomographic and three-dimensional images. It provides more information than other imaging tools since the magnetic resonance (MR) signal depends on a multitude of tissue parameters such as proton density ρ, longitudinal relaxation time $T1$, transverse relaxation time $T2$, chemical shift δ, diffusion coefficient D, diffusion tensor $\mathbf{D}$, etc. Novel MRI parameters such as single quantum coherences (SQCs), double quantum coherences (DQCs), multiple quantum coherences (MQCs), intermolecular multiple quantum coherences (IMQCs),[1] etc, have been proposed.

Clinical MRI scanners typically have field strengths no stronger than 2.0T (open to 3.0T now according to FDA), but those used for research, such as human brain functional magnetic resonance imaging (FMRI), require higher field strengths in order to obtain a better signal-to-noise ratio (SNR) and better resolution. FMRI scanners with 4, 7 and 8T field strengths have already been used, while the 10T system is in planning. The superconducting (SC) main magnet is a key component of an FMRI scanner. It costs are over half the costs of an FMRI scanner.

The conventional MRI SC magnet has distributed-solenoid structure[2,3] such as the six-coil system.[4] The magnetic field in the central zone of the magnet is analysed by means of magnetic scalar potentials[5–7] expanded in spherical harmonics. We have proposed a single-solenoid structure with four

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tiers. There are six layers in each tier, as shown in Fig.1. The residual field error reached over the ninth harmonics. Our strategy is to expand the magnetic vector potential instead of the magnetic scalar potential.[8]

2.Formulation

The magnetic vector potential[9,10] produced by the current carrying solenoid is

$$A_\varphi = \mu_0 n I_0 \sum_{q=0}^{\infty} \frac{1}{(2q+1)(2q+2)} \frac{r^{2q+1}}{R^{2q}} \mathrm{P}_{2q}^1(\cos\theta) \times \int_{\alpha_0}^{\pi/2} \mathrm{P}_{2q+1}^1(\cos\alpha) \sin^{2q}\alpha \mathrm{d}\alpha, \quad (1)$$

where μ_0 is the permeability in vacuum, n is the turn density, I_0 is the current in the solenoid winding, $\mathrm{P}_{2q+1}^1(x)$ is the associated Legendre polynomial, R and L are the inner radius and the half-length of the main solenoid, respectively, and $\alpha_0 = \arctan(R/L)$. For convenience, we use a function $F^{(q)}(\alpha)$ to represent the integral item in Eq.(1):

$$F^{(q)}(\alpha) = \int_{\alpha_0}^{\pi/2} \mathrm{P}_{2q+1}^1(\cos\alpha) \sin^{2q}\alpha \mathrm{d}\alpha. \quad (2)$$

Thus Eq.(1) can be rewritten as

$$A_\varphi = \mu_0 n I_0 \sum_{q=0}^{\infty} \frac{1}{(2q+1)(2q+2)} \frac{r^{2q+1}}{R^{2q}} \times \mathrm{P}_{2q}^1(\cos\theta) F^{(q)}(\alpha). \quad (3)$$

To obtain each order of harmonics of the magnetic vector potentials, $A_{\varphi-}^{(q)}$, we only need to calculate the integral $F^{(q)}(\alpha)$ for each tier. Then Eq. (2) should be substituted as

$$F^{(q)}(\alpha) = \int_{\alpha_{0i}}^{\alpha_i} \mathrm{P}_{2q+1}^1(\cos\alpha) \sin^{2q}\alpha \mathrm{d}\alpha = F^{(q)}(\alpha_i) - F^{(q)}(\alpha_{0i}). \quad (4)$$

Here the subscript 'i' indicates the tier number, i=0 means the main tier of the solenoid, and i=1,2,3 denote the successive correction tiers. The central magnetic field B_0 is the zeroth-order harmonic

$$B_0 = B_z^{(0)} = \mu_0 n I_0 \left(\sum_{i=0}^{3} \cos\alpha_{0i} - \sum_{i=1}^{3} \cos\alpha_i \right). \quad (5)$$

The other higher-order harmonics are as follows

$$B_z^{(1)} = \frac{\mu_0 n I_0}{2} r^2 [3\cos\theta \mathrm{P}_3(\cos\theta) + \sin\theta \mathrm{P}_3^1(\cos\theta)] \times \left[\sum_{i=1}^{3} R_i^{-2} F^{(1)}(\alpha_i) - \sum_{i=0}^{3} R_i^{-2} F^{(1)}(\alpha_{0i}) \right], \quad (6)$$

$$B_z^{(2)} = -\frac{3\mu_0 n I_0}{8} r^4 [5\cos\theta \mathrm{P}_5(\cos\theta) + \sin\theta \mathrm{P}_5^1(\cos\theta)] \times \left[\sum_{i=1}^{3} R_i^{-4} F^{(2)}(\alpha_i) - \sum_{i=0}^{3} R_i^{-4} F^{(2)}(\alpha_{0i}) \right], \quad (7)$$

$$B_z^{(3)} = \frac{\mu_0 n I_0}{16} r^6 [7\cos\theta \mathrm{P}_7(\cos\theta) + \sin\theta \mathrm{P}_7^1(\cos\theta)] \times \left[\sum_{i=1}^{3} R_i^{-6} F^{(3)}(\alpha_i) - \sum_{i=0}^{3} R_i^{-6} F^{(3)}(\alpha_{0i}) \right] \quad (8)$$

where R_i is the radius of the ith tier.

From Eqs. (6)–(8) it is easy to see that in order to nullify the third, fifth and seventh harmonics, we have to solve the set of three equations as below

$$\begin{aligned} &\sum_{i=1}^{3} R_i^{-2} F^{(1)}(\alpha_i) - \sum_{i=0}^{3} R_i^{-2} F^{(1)}(\alpha_{0i}) = 0, \\ &\sum_{i=1}^{3} R_i^{-4} F^{(2)}(\alpha_i) - \sum_{i=0}^{3} R_i^{-4} F^{(2)}(\alpha_{0i}) = 0, \\ &\sum_{i=1}^{3} R_i^{-6} F^{(3)}(\alpha_i) - \sum_{i=0}^{3} R_i^{-6} F^{(3)}(\alpha_{0i}) = 0, \end{aligned} \quad (9)$$

where α_i are the unknown variables to be found. We can determine precisely the length of the ith correction tier by α_i values. If there is only one layer in each tier, we call it a 'single-layer' solenoid magnet, which is only capable of generating 1.0T or so field strength. In order to obtain higher field strength such as 2–10T, the 'multi-layer' solenoid magnet structure must be taken into account. If there are N layers in each tier, Eq.(9) should be replaced by

$$\begin{aligned} &\sum_{i=1}^{3} \sum_{j=0}^{N-1} R_{ij}^{-2} F^{(1)}(\alpha_i) - \sum_{i=0}^{3} \sum_{j=0}^{N-1} R_{ij}^{-2} F^{(1)}(\alpha_{0ij}) = 0, \\ &\sum_{i=1}^{3} \sum_{j=0}^{N-1} R_{ij}^{-4} F^{(2)}(\alpha_i) - \sum_{i=0}^{3} \sum_{j=0}^{N-1} R_{ij}^{-4} F^{(2)}(\alpha_{0ij}) = 0, \\ &\sum_{i=1}^{3} \sum_{j=0}^{N-1} R_{ij}^{-6} F^{(3)}(\alpha_i) - \sum_{i=0}^{3} \sum_{j=0}^{N-1} R_{ij}^{-6} F^{(3)}(\alpha_{0ij}) = 0, \end{aligned} \quad (10)$$

where $\alpha_{0ij} = \arctan(R_{ij}/L)$, $\alpha_{ij} = \arctan R_{ij}/(L - L_{ij})$, R_{ij} and L_{ij} are the radius and the length of the jth layer (j=0 to N-1) in the ith tier (i=0,1,2,3), respectively. As soon as α_i is found through solving

Eqs.(10), the central field strength B_0 can be calculated using

$$B_0 = \mu_0 n I_0 \left(\sum_{i=0}^{3} \sum_{j=0}^{N-1} \cos\alpha_{0ij} - N \sum_{i=1}^{3} \cos\alpha_i \right). \quad (11)$$

The residual field error is dominated by the lowest nonzero higher-order harmonics (HOHs) $B_z^{(4)}$ and $B_z^{(5)}$:

$$B_z^{(4)} = -\frac{\mu_0 n I_0}{128} r^8 [9\cos\theta \mathrm{P}_9(\cos\theta) + \sin\theta \mathrm{P}_9^1(\cos\theta)] \times \left[\sum_{i=1}^{3} \sum_{j=0}^{N-1} R_{ij}^{-8} F^{(4)}(\alpha_i) - \sum_{i=0}^{3} \sum_{j=0}^{N-1} R_{ij}^{-8} F^{(4)}(\alpha_{0ij}) \right], \quad (12)$$

$$B_z^{(5)} = \frac{\mu_0 n I_0}{256} r^{10} [11\cos\theta \mathrm{P}_{11}(\cos\theta) + \sin\theta \mathrm{P}_{11}^1(\cos\theta)] \times \left[\sum_{i=1}^{3} \sum_{j=0}^{N-1} R_{ij}^{-10} F^{(5)}(\alpha_i) - \sum_{i=0}^{3} \sum_{j=0}^{N-1} R_{ij}^{-10} F^{(5)}(\alpha_{0ij}) \right]. \quad (13)$$

3. Design

We have designed a 4T single solenoid magnet as shown in Fig.1. It consists of a main tier and three correction tiers.[11] In each tier there are six layers of conducting wire (see Fig.1). The conducting wire is 1.07mm diameter insulated SC wire. There is a 3mm thick insulating sheet with bores in it for liquid helium to pass

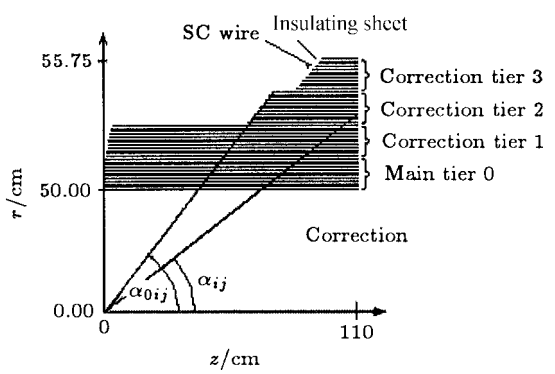


Fig.1. The structure of the 4T single-solenoid FMRI magnet. only one quadrant of the section is shown.

through between adjacent layers. The SC wire selected is standard and commercially available, such as C84-3.3C NbTi filamentary copper matrix wire provided by GEC Alsthom. Each layer of the main tier has 2056 turns. So the total length of the magnet is $2L$=2.19992m. The radius of each layer in each tier can be expressed as

$$R_{ij} = [50 + 1.542i + j(\text{even}) \times 0.407/2]\text{cm}, \quad (14a)$$

$$R_{\mathrm{ij}} = \{50 + 1.542i + [j(\text{odd}) - 1] \times 0.407/2\}\text{cm}, \quad (14b)$$

where j(even) means j=0,2,4; j(odd) means j=1,3,5. Substituting these values of R_{ij} and α_{0ij} into Eqs.(10) and taking N=6 there, then solving the set of the three equations, we obtain the three real roots α_1, α_2 and α_3. Thus, the lengths of each layer in each correction tier can be precisely determined. The data for details are listed in Table 1.

4. Results

The values of α_i obtained through solving Eqs.(10) are listed in the fourth column of Table 1. The central magnetic induction B_0=4T, the theoretical homogeneity dominated by the ninth and the eleventh order of harmonics is better than 5.8ppm. However, the ideal number of turns in each layer (see the fifth column of Table 1) corresponding to the ideal angle α_i is not an integer in general. In manufacturing a magnet, it is necessary to make the number of turns an integer. Thus an 'integralization of turns' treatment is required. The integralization of turns in each layer results in a corresponding inner-end-angle α_{ij} instead of α_i. But α_{ij} no longer satisfies exactly the equation set (10) in general. Thus the third, fifth and seventh harmonics reappear and thereby make a significant contribution to the inhomogeneity. Therefore, the field homogeneity is determined by

$$\frac{\Delta B}{B_0} = \sum_{q=1}^{5} B_z^{(q)} / B_0. \quad (15)$$

To calculate the qth harmonic component $B_z^{(q)}$, the corresponding factors $F^{(q)}(\alpha)$ in Eqs.(6), (7), (8), (12) and (13) should be replaced by

$$\sum_{i=1}^{3} \sum_{j=0}^{5} R_{ij}^{-2q} F^{(q)}(\alpha_{ij}) - \sum_{i=0}^{3} \sum_{j=0}^{5} R_{ij}^{-2q} F^{(q)}(\alpha_{0ij}). \quad (16)$$

If integralizing the turns of each layer N_{ij} to the adjacent integer, the homogeneity deteriorates as much as 50ppm (see the sixth column of Table 1). By making Eq.(15) extremely minimum using a mathematical program, a set of optimal integral numbers of turns N_{ij} (the seventh column of Table 1) which corresponds to a set of α_{ij} (the eighth column of Table 1) can be found, such that the homogeneity or uniformity is still

remains better than 5.8ppm. The comparison between the uniformity after integralization and the ideal homogeneity is shown in Fig.2.

As is well known, the allowed current of a SC solenoid coil, i.e. the rated SC current, depends on the maximum magnetic field $B_{\rm m}$ in its winding. The ratio $B_{\rm m}/B_0$ is related to the geometry of a magnet. For the single-solenoid magnet designed here, $B_{\rm m}/B_0$ <1.39. However, $B_{\rm m}/B_0$ is greater than 2.0 in a conventional distributed-solenoid magnet such as the six-coil system.

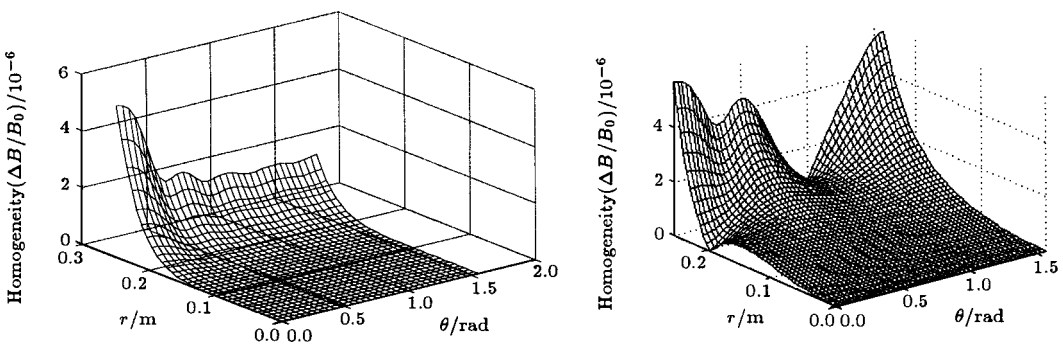


Fig.2. For the 4-T single-solenoid magnet designed for FMRI, the comparison between the ideal homogeneity (a) and the homogeneity determined by the optimal integralization of turn numbers (b).

Table 1. The data of the 4T single solenoid magnet designed for FMRI.

No. of tier	No. of Layer	Outer-end angle α_{0ij}/rad	Theoretical inner-end angle α_i/rad	Ideal turn Number N_{ij} (half length)	To the nearest whole number N_{ij}(half length)	Optimal N_{ij}(half length)	Corresp. optimal α_{ij}/rad
	0	0.426641		1028.00	1028	1028	1.570796
Main	1	0.427447		1028.00	1028	1028	1.570796
tier	2	0.430507		1028.00	1028	1028	1.570796
(0)	3	0.431310	1.57079633	1028.00	1028	1028	1.570796
	4	0.434359		1028.00	1028	1028	1.570796
	5	0.435159		1028.00	1028	1028	1.570796
	0	0.438198		1027.40	1028	1028	1.570796
First	1	0.438995		1027.40	1027	1028	1.570796
corr.	2	0.442022		1027.40	1027	1028	1.570796
tier(1)	3	0.442817	1.56955581	1027.40	1027	1028	1.570796
	4	0.445833		1027.39	1027	1026	1.566726
	5	0.446625		1027.39	1027	1026	1.566734
	0	0.449630		263.94	264	264	0.575926
Second	1	0.450419		262.40	262	262	0.575652
corr.	2	0.453414		256.54	257	257	0.576162
tier	3	0.454199	0.57588856	255.00	255	255	0.575889
(2)	4	0.457183		249.14	249	249	0.575806
	5	0.457966		247.60	248	246	0.574953
	0	0.460938		170.32	170	170	0.536746
Third	1	0.461718		168.64	169	170	0.537607
corr.	2	0.464680		162.25	162	162	0.536784
tier	3	0.465457	0.53691000	160.57	161	161	0.537128
(3)	4	0.468407		154.18	154	154	0.536820
	5	0.469181		152.50	153	151	0.536159
Homogeneity				5.8ppm	50ppm	5.8ppm	

5. Discussion

It is feasible that our single-solenoid FMRI magnet structure uses three correction tiers each including multiple layers to annul the third, fifth and seventh HOHs, and to develop a highly homogeneous field. An optimal set of integers of turns which keeps the homogeneity within the limit of theoretical homogeneity can be found. If using 4×15 layers instead of 4×6 layers here, the overall field strength will reach 10T while the same homogeneity can be kept. Furthermore, an active shielded FMRI magnet system can be constructed using two proposed single-solenoid coils, one inside the other coaxially. The outer coil is used as the bucking coil which satisfies the requirement that the net magnetic moment of the composite equals zero when viewed from a distance much greater than the radius of the magnet bore.[12]

In summary, the single-solenoid structure proposed here possesses two outstanding advantages over the conventional distributed-solenoid structure. Firstly it removes the tremendous Lorentz force between the separated windings. The huge force can cause the coil windings to be deformed resulting in a bad deviation from the theoretical homogeneity unless using particularly heavy supports. Secondly, the ratio $B_{\rm m}/B_0$ in a single-solenoid magnet is significantly lower than that in a distributed-solenoid magnet.

Thereby relatively larger rated current is allowed in the former. The higher the working magnetic field, the more serious the problems of both the Lorentz force and the ratio $B_{\rm m}/B_0$. The conventional FMRI magnet will drastically increase in cost with increasing field strength. Therefore the great potential of the single-solenoid FMRI magnet will make it more attractive.

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