Vol 11 No 10, October 2002
1009-1963/2002/11(10)/0996-03
Chinese Physics
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Probabilistic teleportation of an unknown atomic state*

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(Received 20 September 2001; revised manuscript received 29 May 2002)

We propose two schemes for teleporting an unknown state. In our schemes, a three-particle Greenberger–Horne–Zeilinger state is used as a quantum channel. We show that the probabilistic teleportation of an unknown quantum state can be realized.

Keywords: teleportation, Greenberger–Horne–Zeilinger state, atom–cavity interaction
PACC: 0365, 4250

1. Introduction

Bennett *et al*[1] have proposed a scheme for the teleportation of an unknown quantum state from one observer to another. This can be accomplished in two steps. Firstly, the sender Alice disassembles information of a quantum state into two parts, one of which is sent through a quantum channel and the other is sent through a classical channel. Secondly, the receiver Bob reconstructs the state by making a unitary transformation on the basis of information from both the quantum and classical channels. In this scheme, such a quantum channel is presented by a maximally entangled Bell state. Based on the quantum teleportation protocol, Bouwmeester *et al*[2] have realized experimentally quantum teleportation for a photon in an unknown polarization state. Davidovich *et al*[3] have presented a scheme for the teleportation of an unknown atomic state between two high-Q cavities initially prepared in an entangled photon-number state. Cirac *et al*[4] have made another cavity quantum electrodynamics (QED) proposal for the quantum teleportation of an atomic state. A number of authors[5−13] have also proposed schemes for teleporting an unknown atomic state and the field states. In most of these schemes, the quantum channel is usually an entangled Einstein–Podolsky–Rosen (EPR) pair. In this paper, we propose two other schemes for the teleportation of an unknown atomic state using three partly entangled states. The teleportation can be realized with certain probability.

2. Teleportation of an unknown state by three-particle entangled states

Consider that the sender Alice has an unknown state of a particle labelled 1. She wants to teleport the unknown state $|\psi\rangle_1$ to a receiver Bob who is at a remote station. The state $|\psi\rangle_1$ may be expressed as

$$|\psi\rangle_1 = a|g\rangle_1 + b|e\rangle_1, \tag{1}$$

where the coefficients a and b are the arbitrary constants and satisfy the relation $|a|^2 + |b|^2 = 1$.

Alice and Bob are supposed to share a partly entangled state which is in the following state

$$|\phi\rangle_{234} = \cos\theta|g\rangle_2|g\rangle_3|g\rangle_4 + \sin\theta|e\rangle_2|e\rangle_3|e\rangle_4. \tag{2}$$

The particles 2 and 3 of the state $|\phi\rangle_{234}$ belong to the sender Alice. The other particle 4 belongs to the receiver Bob. The total state can be written as

$$\begin{aligned}|\psi\rangle_{1234} =& |\psi\rangle_1 \otimes |\phi\rangle_{234} \\ =& a\cos\theta|g\rangle_1|g\rangle_2|g\rangle_3|g\rangle_4 \\ &+ a\sin\theta|g\rangle_1|e\rangle_2|e\rangle_3|e\rangle_4 \\ &+ b\cos\theta|e\rangle_1|g\rangle_2|g\rangle_3|g\rangle_4 \\ &+ b\sin\theta|e\rangle_1|e\rangle_2|e\rangle_3|e\rangle_4. \end{aligned} \tag{3}$$

*Project supported by the Natural Science Foundation of Anhui Province, China (Grant No 2000j1027).

If Alice makes a Bell state measurement on particles 1 and 2, the state of particles 3 and 4 will collapse into one of the following states

$$\langle \Phi^{\pm}|_{12}\psi\rangle_{1234} = \frac{a\cos\theta}{\sqrt{2}}|g\rangle_3|g\rangle_4 \pm \frac{b\sin\theta}{\sqrt{2}}|e\rangle_3|e\rangle_4, \quad (4)$$

$$\langle \Psi^{\pm}|_{12}\psi\rangle_{1234} = \frac{a\sin\theta}{\sqrt{2}}|e\rangle_3|e\rangle_4 \pm \frac{b\cos\theta}{\sqrt{2}}|g\rangle_3|g\rangle_4, \quad (5)$$

$$|\Phi^{\pm}\rangle_{12} = \frac{1}{\sqrt{2}}(|g\rangle_1|g\rangle_2 \pm |e\rangle_1|e\rangle_2),$$

where

$$|\Psi^{\pm}\rangle_{12} = \frac{1}{\sqrt{2}}(|g\rangle_1|e\rangle_2 \pm |e\rangle_1|g\rangle_2).$$

Now we perform a rotation operation $\hat{R}(\theta)$ on particle 3, which leads to

$$\hat{R}(\theta)|e\rangle_3 = \cos\theta|g\rangle_3 - \sin\theta|e\rangle_3,$$

$$\hat{R}(\theta)|g\rangle_3 = \sin\theta|g\rangle_3 + \cos\theta|e\rangle_3,$$

which will transform Eqs.(4) and (5) into

$$\frac{1}{\sqrt{2}}\cos\theta\sin\theta(a|g\rangle_4 \pm b|e\rangle_4)|g\rangle_3 + \frac{1}{\sqrt{2}}(a\cos^2\theta|g\rangle_4 \mp b\sin^2\theta|e\rangle_4)|e\rangle_3, \quad (6)$$

$$\frac{\cos\theta\sin\theta}{\sqrt{2}}(\pm b|g\rangle_4 + a|e\rangle_4)|g\rangle_3 + \frac{1}{\sqrt{2}}(\pm b\cos^2\theta\sin\theta|g\rangle_4 - a\sin^2\theta|e\rangle_4)|e\rangle_3. \quad (7)$$

From the above analysis, we can find that if the result of the measurement on particle 3 is $|e\rangle$ in Eqs.(6) and (7), the probable outcomes of the states of particle 4 are the states $a\cos^2\theta|g\rangle_4 \mp b\sin^2\theta|e\rangle_4$ and $\pm b\cos^2\theta|g\rangle_4 - a\sin^2\theta|e\rangle_4$, which cannot be rotated back to the desired state without having any knowledge of the state parameters a and b. The teleportation fails. If the result is $|g\rangle_3$, the states of particle 4 collapse to $\frac{1}{\sqrt{2}}\cos\theta\sin\theta(a|g\rangle_4 \pm b|e\rangle_4)$ or $\frac{\cos\theta\sin\theta}{\sqrt{2}}(\pm b|g\rangle_4 + a|e\rangle_4)$, a unitary operator performs on particle 4, then perfect teleportation can be achieved. So, the total probability of successful teleportation is $2\cos^2\theta\sin^2\theta$.

Next, we give another scheme using cavity QED. Similarly, a Bell state measurement is performed by Alice on particles 1 and 2 of the state $|\psi\rangle_{1234}$; we obtain Eqs.(4) and (5).

Now let atom 3 interact with a single-mode cavity field. Suppose that the atomic transition is resonant with the cavity field. Then, in the interaction picture, the atom–cavity interaction is described by the Jaynes–Cummings Hamiltonian

$$H_{\rm I} = \lambda(a^+|g\rangle\langle e| + a|e\rangle\langle g|).$$

We assume that a cavity field is initially prepared in the superposition of the vacuum and one-photon states

$$|\psi(0)\rangle_{\rm f} = \frac{1}{\sqrt{2}}(|0\rangle + {\rm i}|1\rangle).$$

After the interaction, the state of Eq.(4) is given by

$$\begin{aligned}&\frac{1}{2}[a\cos\theta|g\rangle_3|g\rangle_4|0\rangle + (a\cos\theta\sin\lambda\tau|g\rangle_4\\ &\pm b\sin\theta\cos\lambda\tau|e\rangle_4)|e\rangle_3|0\rangle\\ &+ {\rm i}(a\cos\theta\cos\lambda\tau|g\rangle_4\\ &\mp b\sin\theta\sin\lambda\tau|e\rangle_4)|g\rangle_3|1\rangle\\ &\pm {\rm i}b\sin\theta\cos\lambda\tau|e\rangle_3|e\rangle_4|1\rangle\\ &\pm b\sin\theta\sin\lambda\tau|g\rangle_3|e\rangle_4|2\rangle],\end{aligned} \quad (8)$$

where τ is the interaction time. We choose the length of the cavity carefully so that $\lambda\tau = 0$ is fulfilled. If the measured results are $|0\rangle$ and $|e\rangle_3$, the teleportation is successfully accessed. If we select the condition $\lambda\tau = \pi/2 - \theta$, while the measured results are $|1\rangle$ and $|g\rangle_3$, the teleportation can be achieved. So, the probability of successful teleportation in Eq.(8) is $2\cos^2\theta\sin^2\theta$.

For Eq.(5), after interaction, we can obtain

$$\begin{aligned}&\frac{1}{2}[a\sin\theta\sin\lambda\tau|g\rangle_3|e\rangle_4|2\rangle + (a\sin\theta\cos\lambda\tau|e\rangle_4\\ &\pm b\cos\theta\sin\lambda\tau|g\rangle_4)|e\rangle_3|0\rangle\\ &- {\rm i}(a\sin\theta\sin\lambda\tau|e\rangle_4\\ &\mp b\cos\theta\cos\lambda\tau|g\rangle_4)|g\rangle_3|1\rangle\\ &+ {\rm i}a\sin\theta\cos\lambda\tau|e\rangle_3|e\rangle_4|1\rangle\\ &\pm b\cos\theta|g\rangle_3|g\rangle_4|0\rangle].\end{aligned} \quad (9)$$

Similarly, if we can choose appropriately the length of the cavity and make the measurement on particle 3 and the cavity, perfect teleportation can be achieved. So, using this scheme, successful teleportation can be obtained with a total probability $2\cos^2\theta\sin^2\theta$.

3. Conclusion

We have proposed two schemes for the teleportation of an unknown state via three partly entangled particles. Comparing the two schemes, although the successful teleportation probability is the same, the second scheme is more complicated; it is necessary to prepare the cavity field initially in the superposition of the vacuum and one-photon states. Furthermore, Song[14] has proposed a method for generating three-particle entangled states through the interaction of a two-level atom with a coherent state cavity field. So the quantum teleportation of an unknown state can be realized with certain probability by using the three-particle state as quantum channel.

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