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Resonance modes in optical fibres

Yu Shou-Mian(余寿绵)[a] and Yu Tian(余 恬)[b]

[a] *Department of Physics, Shandong University, Jinan* 250100, *China*

[b] *School of Information Science and Engineering, Shandong University, Jinan* 250100, *China*

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The weakly nonlinear boundary value problem of wave propagation in an optical fibre (for the transverse electric mode, for example) is formulated and a modified linear solution is obtained. It is shown that a self-consistent theory of fibre optics should be weakly nonlinear. The mode of critical refraction that does not exist in the linear theory is obtained, showing that it is a mode consisting of resonance modes. It is shown that the signal carriers in a long fibre are of resonance modes, not normal modes. Some experimental data are given for comparison with the theoretical predictions, and the agreement seems satisfactory.

Keywords: optical fibre, weak nonlinearity, critical refraction, resonance modes
PACC: 0290, 0340, 4245S, 4281

1. Introduction

Recently, the boundary value problem of fibre optics has been solved for the transverse electric (TE) mode by using the covariant form of Sommerfeld's formula[1] for a spherical wave. An integral solution is obtained which gives the well-known normal modes and the radiation mode in closed analytic forms. The mode of critical refraction is missing in the integral solution, it must be the particular solution at the branch point corresponding to the critical angle of incidence. However, it is well known in the theory of guided waves that the particular solution at this point does not exist since the corresponding beam intensity diverges, and the principle of energy conservation is violated. So the linear theory of fibre optics is not self-consistent.

The motivation of this paper is to overcome this difficulty by introducing correction terms into Maxwell's equations, assuming that natural dielectrics are weakly nonlinear in general.

In the following sections, we construct a boundary value problem of weakly nonlinear fibre optics and obtain a modified linear solution with self-consistency.

2.Conventions and fundamental assumptions

2.1. The normalized unit system

Physical relations should be scaling independent. Hence it is convenient to use the normalized unit system in fibre optics, in which the radius of the fibre core, the light velocity in vacuum, and the vacuum magnetic permeability are all equal to unity. In this unit system, all space–time variables are dimensionless, and the dimensions of the field strengths $\boldsymbol{E}$ and $\boldsymbol{H}$ are the same. This system will be used hereafter.

2.2. The weak field assumption

We assume that the field strengths are much smaller than the magnitude of the breakdown electric field strength $E_{\max}$ of the dielectric. Let $\boldsymbol{E}$ ($\boldsymbol{H}$) denote the electric (magnetic) field strength divided by $E_{\max}$, then $\boldsymbol{E}$ and $\boldsymbol{H}$ are dimensionless small quantities of the first order.

2.3. The assumption of weak nonlinearity

For a uniform isotropic and non-dissipative dielectric, the dielectric constant ε should be an analytic function of $\boldsymbol{E}$ (and $\boldsymbol{E}^*$) such that: firstly, it is regular in the neighbourhood of the point $\boldsymbol{E}$=0;[2] secondly, it is independent of the spatial direction of $\boldsymbol{E}$; and thirdly, it does not depend on space–time coordinates explicitly.

Hence we assume that

$$\varepsilon = \varepsilon_0(1 - \alpha^2 u^2) \quad (u^2 = \boldsymbol{E}^* \cdot \boldsymbol{E}, \quad \alpha^2 > 0), \qquad (1)$$

(retaining terms up to the first power of u^2) where $\varepsilon_0 = n^2$ (n is the refractive index, which has no intrinsic dispersion by assumption) and α^2 is a real parameter.

Substituting the above equation into Maxwell's equations

$$\dot{\boldsymbol{D}} = \nabla \times \boldsymbol{H}, \quad \boldsymbol{H} = -\nabla \times \boldsymbol{E},$$

$$\nabla \cdot \boldsymbol{D} = \nabla \cdot \boldsymbol{H} = 0, \quad (\boldsymbol{D} = \varepsilon \boldsymbol{E}), \tag{2}$$

we obtain, retaining terms up to the first power of u^2, the wave equation for $\boldsymbol{E}$ in the following form

$$\Delta \boldsymbol{E} - n^2 \boldsymbol{E} = \sigma[\nabla(\boldsymbol{E} \cdot \nabla u^2) - u^2 \Delta \boldsymbol{E}], \tag{3}$$

where $\sigma = n^2\alpha^2$. The magnetic field strength $\boldsymbol{H}$ can be obtained by using the second formula of Eq.(2). Equation (3) is in the form of a weakly nonlinear equation given in Ref.[3], which can be solved by a certain method of perturbation.

The Poynting flux $\boldsymbol{J} = \mathrm{Re}(\boldsymbol{E}^* \times \boldsymbol{H})$ and the continuity condition for the field strength across an interface between two different media remain the same as those in the linear theory.

3. Boundary value problem of fibre optics

3.1. The axisymmetrical case

The configuration of the fibre and the cylindrical coordinate system are shown in Fig.1.

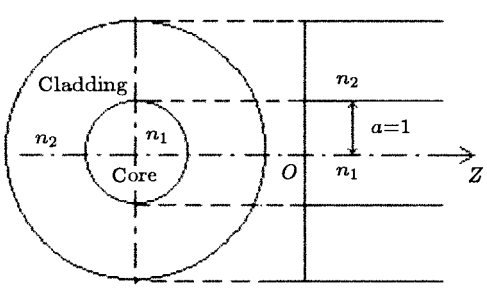


Fig.1. Configuration of the fibre.

We assume that the material outside the cladding is a black body that cannot affect the field inside the fibre. So the cladding can be considered as unbounded also in the radial direction.

The system is conservative and uniform along the z-axis, so a steady state of plane wave motion can be established in the fibre, i.e. for cases with axial symmetry, we have

$$\boldsymbol{E}(r, z, \omega, t) = \boldsymbol{E}(r, q, \omega)\mathrm{e}^{\mathrm{i}(qz - \omega t)}. \tag{4}$$

The amplitude $\boldsymbol{E}(r, q, \omega)$ satisfies the following equation

$$(\nabla^2_{r,\varphi} - \mu^2)\boldsymbol{E} = \sigma\left[\nabla\left(E_r \frac{\partial u^2}{\partial r}\right) - u^2 \nabla^2 \boldsymbol{E}\right], \tag{5}$$

by using Eq.(3), where u^2 is now a function of r only, and $\mu^2 = q^2 - \omega^2 n^2$.

3.2. The boundary conditions

Let the subscript ν(=1 and 2) label quantities pertaining to the core and the cladding, respectively. The angular frequency ω and the wavenumber q are the same in both regions.

The boundary conditions are the same as in the linear theory, which are given in Ref.[1]. Let $\boldsymbol{F}$ denote $\boldsymbol{E}$ and $\boldsymbol{H}$, collectively, we have:

(i) $F_z = 0$ at $r = \infty$.

(ii) $F_{1z} = F_{2z}$, $F_{1\varphi} = F_{2\varphi}$ at the interface r=1.

(iii) The field is produced by the source, which is the illuminated entrance end of the core—a circular disc with the radius being unity. In other words, the incident wave is the diffraction field of the illuminated disc.

It is important to note that the continuity condition (ii) given above is not valid across the entrance aperture due to the presence of diffraction.

3.3. The linear case

In the case $\mu_\nu \neq 0$ (ν=1, 2), the nonlinear term on the right-hand side of Eq.(5) can be safely neglected in comparison with the linear term $\mu_\nu^2 \boldsymbol{E}$. The equation then becomes linear. The boundary value problem has been solved (for the TE mode, for example) in Ref.[1]. The result is a contour integral in the complex q plane. It consists of a mode of radiation and a series of normal modes. The mode of critical refraction (and reflection) is missing in the integral solution. This is the particular solution at the branch point corresponding to critical incidence, i.e. $q = \omega n_2$, which is a zero of the double valued function $\mu_2 = (q^2 - \omega^2 n_2^2)^{1/2}$.

We now turn to this solution in the next section.

4. The mode of critical refraction in linear theory

4.1. The particular solution at $q = \omega n_2$ within the core

According to Ref.[1], the non-vanishing components of field strength in the TE mode within the core are given by

$$H_{1z} = \mu_1^2 \phi_1, \quad H_{1r} = -\mathrm{i}q\frac{\partial \phi_1}{\partial r}, \quad E_{1\varphi} = \mathrm{i}\omega\frac{\partial \phi_1}{\partial r}, \tag{6}$$

where ϕ_1 is given by Eq.(25) of Ref.[1], i.e.

$$\phi_1(r,z,\omega,t) = \frac{\eta}{\pi}\oint_C \mathrm{d}q[\mathrm{K}_0(\mu_1 r) + A(q)\mathrm{I}_0(\mu_1 r)]\mathrm{e}^{\mathrm{i}(qz-\omega t)}, \quad (7)$$

where η is a first-order small quantity newly added to show explicitly that ϕ_1 and the field strength are small quantities of the first order. The form factor $F(q)$ is temporarily dropped (equivalent to a point source at r=0).

For a particular solution, ϕ_1 is given by the integrand of Eq.(7) at $q = \omega n_2$, i.e.

$$\phi_1(r,z,\omega,t) = \frac{\eta}{\pi}[\mathrm{K}_0(\mu_{12} r) + A(q)\mathrm{I}_0(\mu_{12} r)]\mathrm{e}^{-\mathrm{i}\omega\tau_2}, \quad (8)$$

where $\mu_{12} = \mu_1(q = \omega n_2) = -\mathrm{i}V$ ($V = |\omega|\sqrt{n_1^2 - n_2^2}$, the normalized frequency) for the outgoing solution, $\tau_2 = t - n_2 z$, and I_0 and K_0 are Bessel and Hankel functions with an imaginary argument, respectively.

4.2. The corresponding particular solution in the cladding

At $q = \omega n_2$, μ_2=0, the linearized equation (5) for ν=2 becomes

$$\nabla^2_{r\varphi}\boldsymbol{E}_2 = 0. \quad (9)$$

In terms of components, we have

$$\frac{\partial^2 E_{2z}}{\partial r^2} + \frac{1}{r}\frac{\partial E_{2z}}{\partial r} = 0, \quad (10)$$

$$\frac{\partial}{\partial r}\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)\begin{pmatrix} E_{2r} \\ E_{2\varphi} \end{pmatrix} = 0. \quad (11)$$

Equation (10) gives E_{2z}=0, and Eq.(11) together with $\nabla \cdot \boldsymbol{E}$=0 gives E_{2r}=0 and

$$E_{2\varphi} = \frac{\eta B(q)}{\pi r}\mathrm{e}^{-\mathrm{i}\omega\tau_2}. \quad (12)$$

The non-vanishing component of $\boldsymbol{H}_2$ is

$$H_{2r} = -n_2 E_{2\varphi}. \quad (13)$$

4.3. The resonance

The coefficients $A(q)$ and $B(q)$ in Eqs.(8) and (12) at $q = \omega n_2$ can be determined from the boundary condition (ii) given in section 3. We have

$$A(\omega n_2) = -\frac{\mathrm{K}_0(\mu_{12})}{\mathrm{I}_0(\mu_{12})}, \quad B(\omega n_2) = -\frac{\mathrm{i}\omega}{\mathrm{I}_0(\mu_{12})}. \quad (14)$$

Note that in determining B, the identity

$$\mu[\mathrm{K}_0(\mu)\mathrm{I}_1(\mu) + \mathrm{K}_1(\mu)\mathrm{I}_0(\mu)] = 1 \quad (15)$$

has been used.

Substituting Eq.(14) into Eq.(8) we obtain

$$\phi_1(r,z,\omega,t) = \frac{\eta}{\pi}[\mathrm{K}_0(\mu_{12} r) - \frac{\mathrm{K}_0(\mu_{12})}{\mathrm{I}_0(\mu_{12})}\mathrm{I}_0(\mu_{12} r)]\mathrm{e}^{-\mathrm{i}\omega\tau_2}, \quad (16)$$

$$E_{2\varphi} = -\frac{1}{n_2}H_{2r} = -\mathrm{i}\frac{\omega\eta}{\pi r}\frac{1}{\mathrm{I}_0(\mu_{12})}\mathrm{e}^{-\mathrm{i}\omega\tau_2}. \quad (17)$$

The singularity at r=0 in Eq.(16) can be removed by an integral with respect to ω.

Now $\mathrm{I}_0(\mu_{12}) = J_0(V)$ is the Bessel function of V of the zeroth order, which has a set of zeros at $V = \alpha_m$ (m= 1,2,...). When the frequency coincides with one of the zeros, the field becomes infinity, so the mode of critical refraction consists of resonance modes. The mth mode has an eigenfrequency

$$\omega_m = \frac{\alpha_m}{\sqrt{n_1^2 - n_2^2}} \quad (\omega_{-m} = -\omega_m). \quad (18)$$

It is interesting to note that ω_m is just the cut-off frequency of the mth normal mode.[1]

The amplitude of resonance depends upon the frequency spectrum $f(\omega)$ of the source. The fundamental solution (the Green's function) is the solution produced by a transient source $\delta(t)$ ($f(\omega) = 1$). The solution for the outgoing wave is then given by multiplying Eqs.(16) and (17) with $\frac{\mathrm{d}\omega}{2\pi}$ and integrating with respect to ω along the upper part of the real ω axis. We have for the field inside the core

$$\phi_1^{(\delta)}(r,z,t) = \frac{\eta}{2\pi^2}\int_{-\infty+\mathrm{i}\varepsilon}^{+\infty+\mathrm{i}\varepsilon}\mathrm{d}\omega\left[\mathrm{K}_0(\mu_{12} r) - \frac{\mathrm{K}_0(\mu_{12})}{\mathrm{I}_0(\mu_{12})}\mathrm{I}_0(\mu_{12} r)\right]\mathrm{e}^{-\mathrm{i}\omega\tau_2}, \quad (19)$$

where the subscript (δ) denotes the $\delta(t)$ source.

The path of integration can be changed into a great circular arc enclosing the whole lower half space including the real ω axis.

The first term $\mathrm{K}_0(\mu_{12} r)$ in the square brackets in Eq.(19) has a singularity at $\omega = 0$ like $\ln|\omega|$, but it does not contribute to the integral because

$$\oint_{\omega=0}\mathrm{d}\omega\mathrm{K}_0(\mu_{12} r)\mathrm{e}^{-\mathrm{i}\omega\tau_2} = \oint_{\omega=0}\mathrm{d}\omega\mathrm{K}_0(\mu_{12} r) \propto \oint_{\omega=0}\mathrm{d}\omega\ln|\omega| = 0. \quad (20)$$

The second term following $\mathrm{K}_0(\mu_{12}r)$ in Eq.(19) has a series of simple poles at $\omega = \omega_m$ $(m = \pm1, \pm2, \cdots)$. Applying the method of residues to the pole integration, and using the identity, Eq.(15), i.e. $\mathrm{K}_0(\mu_{12}r) = -\dfrac{1}{\mu_{12}\mathrm{I}_1(\mu_{12})}$ at the poles, we obtain $\phi_1^{(\delta)}(r,z,t) = \sum_{m=1}^{\infty}\phi_{1m}^{(R)}(r,z,t)$+c.c., with

$$\phi_{1m}^{(R)}(r,z,t) = \frac{\mathrm{i}\eta J_0(\alpha_m r)\mathrm{e}^{-\mathrm{i}\omega_m\tau_2}}{\pi\alpha_m J_1^2(\alpha_m)\sqrt{n_1^2-n_2^2}}. \tag{21}$$

The superscript (R) denotes a resonance.

For the field in the cladding, we have

$$\begin{aligned} E_{2\varphi m}^{(R)}(r,z,t) &= -\frac{\mathrm{H}_{2rm}^{(R)}(r,z,t)}{n_2} \\ &= -\frac{\mathrm{i}\omega_m\eta\mathrm{e}^{-\mathrm{i}\omega_m\tau_2}}{\pi r\mathrm{J}_1(\alpha_m)\sqrt{n_1^2-n_2^2}}. \end{aligned} \tag{22}$$

5. The resonance mode in the weak nonlinear theory

5.1. The difficulty of the linear theory

Equation (22) shows that the Poynting flux outside the core is inversely proportional to r^2, so the beam intensity of the critically refracted wave would diverge like $\ln r$ when $r \to \infty$. This difficulty is well known.[4]

In this section we prove that this difficulty can be removed by taking into account the weak nonlinearity, and an approximate solution for this mode is obtained.

5.2. The lowest-order correction due to weak nonlinearity

Let us turn to Eq.(5) for $\nu = 2$. The quantities on the right-hand side of Eq.(5) can be replaced by their linear approximation obtained in the last section, i.e. those for the mth resonance mode.

$$u_m^2 = \eta^2\frac{C_m^2}{r^2}, \quad C_m = \frac{\omega_m}{\pi\mathrm{J}_1(\alpha_m)\sqrt{n_1^2-n_2^2}}, \quad (u^2 = \boldsymbol{E}^*\cdot\boldsymbol{E}), \tag{23}$$

and E_{2rm} (on the right-hand side) $= 0$, so Eq.(5) for ν=2 gives

$$\frac{\partial^2 E_{2zm}^{(R)}}{\partial r^2} + \frac{1}{r}\frac{\partial E_{2zm}^{(R)}}{\partial r} = 0, \tag{24}$$

$$\frac{\partial}{\partial r}\left(\frac{\partial E_{2rm}^{(R)}}{\partial r} + \frac{E_{2rm}^{(R)}}{r}\right) = 0, \tag{25}$$

$$\frac{\partial}{\partial r}\left(\frac{\partial E_{2\varphi m}^{(R)}}{\partial r} + \frac{E_{2\varphi m}^{(R)}}{r}\right) = \sigma\eta^2\frac{C_m^2\omega_m^2 n_2^2}{r^2}E_{2\varphi m}^{(R)}. \tag{26}$$

Equations (24) and (25) together with $\nabla\cdot\boldsymbol{D} = 0$ lead to $E_{2zm}^{(R)} = E_{2rm}^{(R)} = 0$.

Equation (26) reduces to the following form of Euler's equation

$$\left[r^2\frac{\partial^2}{\partial r^2} + r\frac{\partial}{\partial r} - (1+\sigma\eta^2\omega_m^2 n_2^2)\right]E_{2\varphi m}^{(R)} = 0, \tag{27}$$

which gives

$$E_{2\varphi m}^{(R)} = -\mathrm{i}\eta C_m r^{-(1+\frac{1}{2}\sigma\eta^2\omega_m^2 n_2^2 C_m^2)} \tag{28}$$

and

$$\mathrm{H}_{2rm}^{(R)} = \mathrm{i}\eta n_2 C_m r^{-(1+\frac{1}{2}\sigma\eta^2\omega_m^2 n_2^2 C_m^2)}. \tag{29}$$

The Poynting flux then becomes inversely proportional to $r^{(2+\sigma\eta^2\omega_m^2 n_2^2 C_m^2)}$. Consequently, the beam intensity converges when $r \to \infty$.

5.3. The linear approximation with a cut-off

The approximation solution obtained above is not very useful because the parameters η and σ are not known.

Knowing that the resonance mode of the critical refraction does converge in the far field, the linear solution obtained in the last section can be used as an approximate solution of the weakly nonlinear equation provided a cut-off is made at a certain distance r_m in the radial direction for each mode m, i.e. Eq.(21) remains unaltered, while Eq.(22) is multiplied by a unit step function

$$h\left(\frac{r}{r_m}\right) = \begin{cases} 1 & r \le r_m; \\ 0 & r > r_m. \end{cases} \tag{30}$$

The cut-off radius r_m has the meaning of an effective beam radius of the mth resonance mode, the value of which will be determined in the next section.

6. The cut-off radius r_m determined by the law of energy conservation

6.1. The excitation and maintenance of a resonance mode

Before determining the magnitude of r_m, let us consider the physical mechanism of the resonance mode more closely.

A resonance mode m is a steady state of plane wave motion—the propagation of a radial eigen vibration with eigenfrequency ω_m along the fibre.

The eigen vibration is excited by the transient point source $\delta(t) = \int_{-\infty}^{\infty} \mathrm{d}\omega \mathrm{e}^{-\mathrm{i}\omega t}/2\pi$. Comparing the phase factor $\mathrm{e}^{-\mathrm{i}(\omega_m \tau_2 - \pi/2)}$ of $\phi_{1m}^{(R)}$ given by Eq.(21) at $z = 0$ with that of the partial vibration $\mathrm{e}^{-\mathrm{i}\omega_m t}$ of the source $\delta(t)$, we see that the resonance mode m lags behind the source by a time interval $\Delta t = \pi/2\omega_m$. This time lag must be the time needed for establishing the steady state of resonance mode m.

After the mode m is established, the point source, as a part of the whole system, should vibrate in tune with the mode, i.e. with the same phase factor $\mathrm{e}^{-\mathrm{i}(\omega_m \tau_2 - \pi/2)}$. This steady state of source vibration brings energy into the fibre to maintain the propagation of the resonance.

6.2. Equation of energy conservation and the determination of beam width

In a resonant steady state m, the spherical wave emitted from the point source should have the same wavenumber and frequency with the resonance. Inside the core we have $R \cong z$ when $R \to \infty$, hence in the radiation region ($R \to \infty$), the potential of the source $\phi_{1m}^{(0)}(R, t)$ should be given by

$$\phi_{1m}^{(0)}(R, t) = \frac{\eta}{2\pi R} \mathrm{e}^{-\mathrm{i}\omega(t - n_2 R - \pi/2)}. \tag{31}$$

The Poynting flux along R is given by (see Fig.2 and Eq.(6))

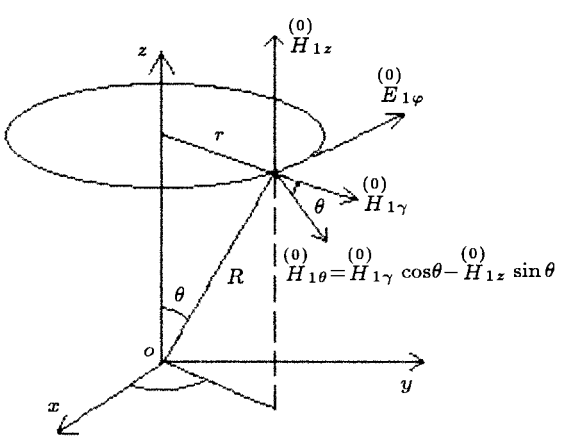


Fig.2. Components of field strength for the spherical wave.

$$-\overset{(0)}{E}{}^{*}_{1\varphi} \overset{(0)}{H}_{1\theta} = \frac{\eta^2 \omega_m^4 n_2}{4\pi^2 R^2} \times \sin^2\theta [n_2^2 \cos\theta + (n_1^2 - n_2^2)]. \tag{32}$$

So the intensity emitted upwardly from the source is

$$\overset{(0)}{I}_m = 2\pi R^2 \int_0^{\pi/2} \mathrm{d}\theta \sin\theta \left(-\overset{(0)}{E}{}^{*}_{1\varphi} \overset{(0)}{H}_{1\theta}\right) = \frac{\eta^2 \omega_m^4 n_2}{2\pi} \left(\frac{1}{4} n_1^2 + \frac{5}{6} n_1^2 \Delta\right) \underset{\Delta=0}{\cong} \frac{\eta^2 \omega_m^4 n_2 n_1^2}{8\pi}. \tag{33}$$

Similarly, the beam intensity of the resonance mode m can be obtained from Eqs.(21) and (25) with a cut-off factor $h(r/r_m)$. We have

$$I_{1m} = \frac{2\omega_m^2 n_2 \eta^2}{\pi \mathrm{J}_1^2(\alpha_m)(n_1^2 - n_2^2)}, \tag{34}$$

$$I_{2m} = \frac{2\omega_m^2 n_2 \eta^2 \ln r_m}{\pi \mathrm{J}_1^2(\alpha_m)(n_1^2 - n_2^2)}. \tag{35}$$

Energy conservation requires that $\overset{(0)}{I}_m = I_{1m} + I_{2m}$, from which we obtain

$$\ln r_m \cong \frac{n_1^2}{16} \alpha_m^2 \mathrm{J}_1^2(\alpha_m) - 1. \tag{36}$$

7. Properties of resonance modes and experimental verifications

7.1. Resonance modes are 'first arrival' at the receiving terminal of the fibre

Each resonance mode propagates with the same light velocity in the cladding ($n = n_2$), which is greater than the group velocities of all normal modes. So it is the 'first arrival' at the receiving terminal, and should be detected as the first.

7.2. The resonance modes, not the normal modes, are responsible for signal transmission in long optical fibres

The fibre has no intrinsic dispersion and is not dissipative by assumption. The normal modes are dispersive due to the waveguide effect, while the resonance modes are not. The former propagates in the form of a wave packet which will gradually diffuse away in a long fibre. The latter, however, dose not attenuate and change shape along with propagation. Furthermore, the amplitude of a wave packet consists of a group of normal modes within the frequency range $\omega_0 - \Delta\omega/2 \to \omega_0 + \Delta\omega/2$ and is proportional to $\Delta\omega$. Hence it is much smaller than the amplitude of a resonance mode. Consequently, it is the resonance modes,

not the normal modes that are responsible for carrying signals in optical fibres.

7.3. The frequency range of the resonance modes in the case of axial symmetry

According to Eq.(36), the resonance mode could only be exited at high frequencies, since r_m should be greater than unity, i.e.,

$$\alpha_m^2 \mathrm{J}_1^2(\alpha_m) > 16/1.5^2 = 7.11$$

$$(n_1 = 1.5) \quad \text{or} \quad \alpha_m > 11.5. \tag{37}$$

Thus we obtain a minimum value of the normalized frequency V for the resonance modes

$$V_{\min} = 11.5. \tag{38}$$

7.4. Experimental verifications

The above results are given for the case of axial symmetry, i.e. the angular wavenumber l is equal to zero. In this case, the field inside the core is proportional to $\mathrm{J}_0(\alpha_m r)$. For $l > 0$, the amplitude is proportional to $\mathrm{J}_l(V)$, which is much smaller than $\mathrm{J}_0(\alpha_m r)$ inside the core in the high-frequency range (say $V > 10$). Hence, in the high-frequency range, only the axisymmetrical modes could exist.

According to Snyder and Love,[5] the multimode fibre is used in the high-frequency range. Figure 3 (taken from Ref.[5]) shows the configuration of the multimode optical fibre. From the data listed in the figure, we obtain the experimental lowest working frequency

$$V_{\min} = \frac{2\pi a_{\min}}{\lambda_{\max}} n_1 \sqrt{2\Delta_{\min}} = \frac{2\pi \times 12.5}{1.6} \times 1.5 \times \sqrt{0.02} = 10.4, \tag{39}$$

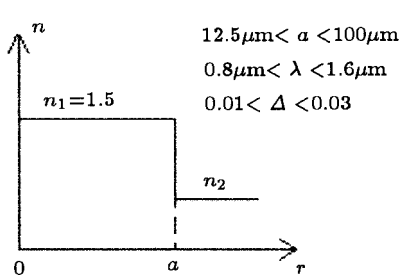


Fig.3. Configuration of a multimode optical fibre.

which agrees with the theoretical prediction, Eq.(38), fairly well.

Furthermore, there are three available sets of experimental data:

(i) $a = 25\mu$m, $\lambda = 1.3\mu$m, Δ=0.005[6];

(ii) $a = 25\mu$m, $\lambda = 0.8\mu$m, Δ=0.005[7];

(iii) $a = 25\mu$m, $\lambda = 0.85\mu$m, n_2=1.475[8].

Taking n_1=1.5, we obtain V(i)=18.1, V(ii)=29.4, V(iii)=50.4. On the other hand, we have α_6=18.1, α_{10}=30.6, α_{16}=49.5. The coincidence of V(i), V(ii) and V(iii) respectively with the theoretical predictions α_6, α_{10} and α_{16} is satisfactory.

These three sets of data can serve as a strong verification of the theory, because the resonance frequency is equal to the cut-off frequency of a normal mode, at which the normal mode vanishes. Hence the observed modes must be resonance modes.

8. Summary and discussion

The most important contribution of this paper to the theory of fibre optics can be summarized as follows.

(i) In the conventional theory of wave guides, the solution is given by a contour integration in the wavenumber plane; the branch point corresponding to the critical reflection is indented from the integration path.[1] So in that theory, the critical reflected mode is missing, and the observed critical refraction is described as a nearly axial radiation mode (or the leaky mode) which is given by a complex root of the security equation.[5] It is well known that such modes do not vanish when $(r, z) \to \infty$ along arbitrary directions, hence they are non-physical.

In the present theory, the critical refraction mode is given by the particular solution at the branch point, and it is proven that the well-known difficulty of the divergence of power flow, when $r \to \infty$, can be removed by taking into account the weak nonlinearity of the dielectric.

(ii) It is proven that the critical refraction is a resonance, because the amplitude becomes infinitely large at the eigenfrequency before an integration with respect to ω.

The resonance modes of wave propagation have been predicted earlier in the theory of elastic wave guides.[9-11] It is now clear that such modes also exist in electromagnetic wave guides.

The most interesting and remarkable result, which is unexpected in the conventional theory, is that the modes responsible for the signal transmission in a long fibre are the resonance modes rather than the normal modes. The experimental data seem to con-

firm the theoretical predictions satisfactorily.

Finally, we should point out that only the axisymmetrical case is treated in this paper. For the non-axisymmetrical cases, it is proven in our recent paper[12] that the guided modes are all modes of critical refraction. Whether the conclusions obtained in this paper are valid for the latter cases will be clarified in another work.

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