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Modulation instability of femtosecond pulses in fibres with slowly decreasing dispersion*

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In this paper, we investigate the modulation instability for generating femtosecond pulses in fibres with slowly decreasing dispersion. Higher-order dispersion and higher-order nonlinear effects are taken into account when the continuous wave or quasi-continuous wave evolves into sub-picosecond and femtosecond pulses by modulation instability in the optical fibres. Our research results show that the gain spectrum of the dispersion-decreasing fibres is much wider than that in conventional fibres. The third-order dispersion effect has no contribution to gain spectrum, while the self-steepening effect reduces the maximum value and gain bandwidth, and the Raman self-scattering effect widens the extent to which the modulation instability occurs.

Keywords: modulation instability, ultrashort pulses generation, fibres with slowly decreasing dispersion, third-order dispersion
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1. Introduction

The continuous wave (cw) or quasi-continuous wave (quasi-cw) exhibits an exponential growth due to the interaction of nonlinear effects and dispersion effects in optical fibres. This phenomenon is referred to as modulation instability (MI).[1] MI was first theoretically analysed[2] by Hasegawa and Breinkman and first verified experimentally by Tai *et al.*[3] Because of its various applications in frequency generation, optical switching and laser techniques, MI has received much attention recently. It is verified that picosecond–femtosecond soliton pulses with high repetition rates can be generated by MI in conventional fibres. However, experimental results have shown that the generated pulses have broad pedestals.[4] Pulse pedestals not only lead to a deterioration in the quality of pulse trains, but also make the compressed pulse unstable due to the nonlinear interaction of pedestals with the compressed pulses. Recently, a novel approach has been used to compress and generate high-quality ultrashort pulses (pedestal free) in fibres with slowly decreasing dispersion (FSDD) by adiabatic amplification. This is regarded as a promising method for the generation of ultrashort pulses.[5–9] In this paper, we analyse the properties of MI in the FSDD: the gain spectrum of MI, and the influences of higher-order dispersion and nonlinear effects on MI.

2. Theoretical model

With the break-up of cw or quasi-cw radiation into a train of sub-picosecond and femtosecond pulses in the FSDD, the fibre loss, the higher-order dispersion, the nonlinear dispersion and the Raman self-scattering effects are taken into account. We consider the propagation of a light beam inside the FSDD, which is governed by the generalized nonlinear Schrödinger equation[9,10]

$$\frac{\partial q}{\partial z}+\beta_1\frac{\partial q}{\partial t}+\mathrm{i}\frac{\beta_2}{2}\frac{\partial^2 q}{\partial t^2}+\frac{\alpha}{2}q$$
$$=\mathrm{i}\gamma|q|^2q+\frac{1}{6}\beta_3\frac{\partial^3 q}{\partial t^3}-\frac{\gamma}{\omega_0}\frac{\partial}{\partial t}(|q|^2q)-\mathrm{i}\gamma T_{\mathrm{R}}q\frac{\partial|q|^2}{\partial t}, \quad (1)$$

where z and t are the propagation distance and time, respectively, q is a slowly varying envelope, β_1 is the inverse of the group velocity, β_2 is the coefficient of group velocity dispersion (GVD), β_3 is the coefficient of third-order dispersion, α is the loss coefficient of fibres, γ is the nonlinearity coefficient, ω_0 is the signal frequency, and T_{R} is the Raman response time.

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The third and fourth terms on the left-hand side of Eq.(1) represent the GVD and fibre loss, respectively. The terms on the right-hand side of Eq.(1) are the self-phase modulation (SPM), third-order dispersion, self-steepening, and Raman self-scattering effects, respectively.

We take the following transform

$$T = t - \beta_1 z, \quad \xi = \frac{1}{\alpha}[1 - \exp(-\alpha z)],$$
$$A = \exp\left(\frac{\alpha}{2} z\right) q. \tag{2}$$

Then the normalized form of Eq.(1) can be obtained as

$$\begin{aligned} &\frac{\partial A}{\partial \xi} + \frac{i}{2}\beta_2 \exp(\alpha z)\frac{\partial^2 A}{\partial T^2} \\ =&\mathrm{i}\gamma |A|^2 A + \frac{1}{6}\beta_3 \frac{\partial^3 A}{\partial T^3}\exp(\alpha z) \\ &- \frac{\gamma}{\omega_0}\frac{\partial}{\partial T}(|A|^2 A) - \mathrm{i}\gamma T_\mathrm{R} A \frac{\partial |A|^2}{\partial T}. \end{aligned} \tag{3}$$

In the case of cw radiation, $A(z,t)$ remains time-independent during propagation in the FSDD. Assuming that A is independent of T at the input end of the fibre at z=0, then Eq.(3) has a steady-state solution

$$A_\mathrm{s} = \sqrt{P_0}\exp(\mathrm{i}\gamma P_0 \xi), \tag{4}$$

where P_0 is the initial input power. Equation (4) indicates that the cw beam remains unchanged except that its phase increases with ξ. In order to study the solution stability under a perturbation, we introduce a small perturbation b, then the steady-state solution has the following form

$$A = (\sqrt{P_0} + b)\exp(\mathrm{i}\gamma P_0 \xi), \quad |b| \ll \sqrt{P_0}. \tag{5}$$

By substituting Eq.(5) into Eq.(3) and linearizing in b, we have

$$\begin{aligned} &\frac{\partial b}{\partial \xi} + \frac{i}{2}\beta_2 \exp(\alpha z)\frac{\partial^2 b}{\partial T^2} \\ =&\mathrm{i}\gamma P_0 (b + b^*) + \frac{1}{6}\beta_3 \exp(\alpha z)\frac{\partial^3 b}{\partial T^3} \\ &- \frac{\gamma}{\omega_0} P_0 \left(2\frac{\partial b}{\partial T} + \frac{\partial b^*}{\partial T}\right) - \mathrm{i}\gamma T_\mathrm{R} P_0 \left(\frac{\partial b}{\partial T} + \frac{\partial b^*}{\partial T}\right). \end{aligned} \tag{6}$$

Here, b^* is the conjugate complex of b. We assume that the solution of Eq.(6) can be expressed as follows

$$b(\xi, T) = U(\xi, T) + \mathrm{i}V(\xi, T), \tag{7}$$

where

$$U = \mathrm{Re}(U_0 \exp[\mathrm{i}(k\xi - \omega T)]),$$
$$V = \mathrm{Im}(V_0 \exp[\mathrm{i}(k\xi - \omega T)]), \tag{8}$$

and k and ω represent the wavenumber and the shifted frequency of the perturbation, respectively. U_0 and V_0 are the amplitudes of the perturbation wave. Substituting Eq.(7) into Eq.(6) and separating Eq.(6) into real and imaginary parts, we obtain two coupled equations

$$\begin{aligned} &\frac{\partial U}{\partial \xi} - \frac{1}{2}\beta_2 \exp(\alpha z)\frac{\partial^2 V}{\partial T^2} \\ =&\frac{1}{6}\beta_3 \exp(\alpha z)\frac{\partial^3 U}{\partial T^3} - 3\frac{\gamma}{\omega_0} P_0 \frac{\partial U}{\partial T}, \end{aligned} \tag{9a}$$

$$\begin{aligned} &\frac{\partial V}{\partial \xi} + \frac{1}{2}\beta_2 \exp(\alpha z)\frac{\partial^2 U}{\partial T^2} \\ =&2\gamma P_0 U + \frac{1}{6}\beta_3 \exp(\alpha z)\frac{\partial^3 V}{\partial T^3} \\ &- \frac{\gamma}{\omega_0} P_0 \frac{\partial V}{\partial T} - 2\gamma T_\mathrm{R} P_0 \frac{\partial U}{\partial T}. \end{aligned} \tag{9b}$$

Equations (9) provide a set of two homogeneous equations for U and V. This set of equations has a nontrivial solution only when k and ω satisfy the dispersion relation

$$\begin{aligned} k =&\frac{1}{6}\beta_3 \exp(\alpha z)\omega^3 + 2\frac{\gamma}{\omega_0} P_0 \omega \\ &\pm \frac{1}{2}\omega |\beta_2| \exp(\alpha z)\left[\omega^2 + \mathrm{sgn}(\beta_2)\omega_\mathrm{c}^2 \right. \\ &\left. + \frac{4(\gamma/\omega_0)^2 P_0^2}{\beta_2^2 \exp(2\alpha z)} + \mathrm{i}\frac{\mathrm{sgn}(\beta_2) 4\gamma T_\mathrm{R} P_0 \omega}{|\beta_2|\exp(\alpha z)}\right]^{1/2}, \end{aligned} \tag{10}$$

where

$$\omega_\mathrm{c}^2 = \frac{4\gamma P_0}{|\beta_2|\exp(\alpha z)}. \tag{11}$$

Equation (10) indicates that the MI occurs only in the region of anomalous group-velocity dispersion. For conventional fibres, the dispersion parameter of the fibre is constant for a certain wavelength, that is $\beta_2(z) = \beta_2(0)$. If the higher-order nonlinear effect, higher-order dispersion and fibre loss are negligible, k and ω_c can easily be obtained. This is in agreement with the result in Ref.[1]. For the FSDD, the dispersion parameter decreases with the fibre distance at an exponential growth rate, that is, $\beta_2(z) = \beta_2(0)\exp(-\mu z)$. Then Eq.(10) should be modified

$$\begin{aligned} k =&\frac{1}{6}\beta_3 \exp(\alpha z)\omega^3 + 2\frac{\gamma}{\omega_0} P_0 \omega \\ &\pm \frac{1}{2}\omega |\beta_2(0)\exp(-\mu z)| \exp(\alpha z)\left[\omega^2 + \mathrm{sgn}(\beta_2)\omega_\mathrm{c}^2 \right. \\ &+ \frac{4(\gamma/\omega_0)^2 P_0^2}{[\beta_2(0)\exp(-\mu z)]^2 \exp(2\alpha z)} \\ &\left. + \mathrm{i}\frac{\mathrm{sgn}(\beta_2) 4\gamma T_\mathrm{R} P_0 \omega}{|\beta_2(0)\exp(-\mu z)|\exp(\alpha z)}\right]^{1/2}, \end{aligned} \tag{12}$$

where

$$\omega_c^2 = \frac{4\gamma P_0}{|\beta_2(0)\exp(-\mu z)|\exp(\alpha z)}. \quad (13)$$

We now introduce the gain spectrum in order to study the MI in the FSDD. The gain spectrum is defined as

$$\begin{aligned} G(\omega) =& 2\mathrm{Im}(k) \\ =& \frac{\sqrt{2}}{2}\{B + [B^2 + 16\beta_2 \\ & \times \exp(2\alpha z)\omega^6(\gamma T_R)^2 P_0^2]^{1/2}\}^{1/2}, \end{aligned} \quad (14)$$

where

$$B = \omega^2\beta_2^2\exp(2\alpha z)\left[\omega_c^2 - \omega^2 - \frac{4(\gamma/\omega_0)^2 P_0^2}{\beta_2^2\exp(2\alpha z)}\right]. \quad (15)$$

From Eqs.(14) and (15), we can clearly see that the pump power, fibre dispersion parameter, fibre loss, higher-order nonlinear effect (including the self-steepening and Raman self-scattering effects) contribute to the MI, but the third-order dispersion does not influence it. This is in agreement with the results in Refs.[11]–[13].

3. Results and discussion

3.1. The MI of picosecond pulse in FSDD

We neglect the higher-order nonlinear and higher-order dispersion effects on picosecond pulses. Then the dispersion relation and gain spectrum from Eqs.(12) and (14) are

$$k = \pm\frac{1}{2}\omega|\beta_2(0)|\exp(\alpha z)\exp(-\mu z)(\omega^2 - \omega_c^2)^{1/2}, \quad (16)$$

$$G(\omega) = |\omega\beta_2(0)|\exp(\alpha z)\exp(-\mu z)(\omega_c^2 - \omega^2)^{1/2}, \quad (17)$$

where

$$\omega_c^2 = \frac{4\gamma P_0\exp(\mu z)}{|\beta_2(0)|\exp(\alpha z)}. \quad (18)$$

Equation (17) shows that the gain spectrum of MI exists only in the case of small shifted frequency perturbation ω that satisfies the relation $|\omega| \le \omega_c$. When $\omega_{max} = \pm\frac{\omega_c}{\sqrt{2}} = \pm\sqrt{\frac{2\gamma P_0}{|\beta_2(0)|}}\exp\left[\frac{(\mu-\alpha)z}{2}\right]$, the gain spectrum $G(\omega)$ has a maximum value

$$\begin{aligned} G_{max} =& G(\omega_{max}) \\ =& \frac{1}{2}|\beta_0(0)|\exp(-\mu z)\exp(\alpha z)\omega_c^2 = 2\gamma P_0. \end{aligned} \quad (19)$$

Equation (19) indicates that the maximum value of the gain spectrum is independent of the dispersion parameter μ, fibre loss α, and increases linearly with the initial power P_0.

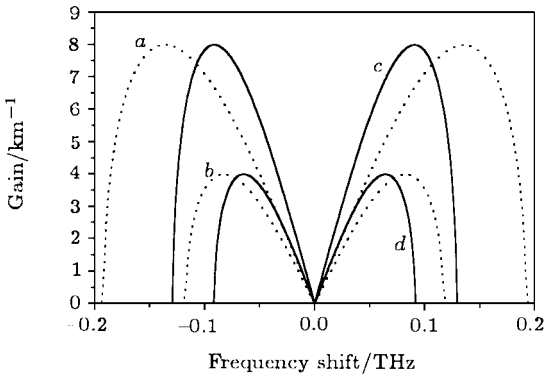


Fig.1. The MI gain spectra with the following parameters: curve *a*, FSDD, P_0=2W; curve *b*, FSDD, P_0=1W; curve *c*, CF, P_0=2W; curve *d*, CF, P_0=1W. The other parameters for all the curves are as follows: $|\beta_2(0)|$=20ps^2/km; γ=2.0/W·km; μ=0.5dB/km; α=0.2dB/km; z=1km.

Figure 1 shows the results of the gain spectra of the MI in conventional fibres (CF) and the FSDD with the initial pump powers of 1 and 2 W, respectively. The parameters are selected to be: $|\beta_2(0)|$=20ps^2/km; γ=2.0/W·km; μ=0.5dB/km; α=0.2dB/km; z=1km. The similarity of the two types of fibres are as follows. (1) The maximum values of the gain spectra are equal under the same power. For both CF and FSDD samples, the maximum gains are 4/km for P_0=1W. (2) The fibre loss reduces the bandwidth of the gain spectrum. Although the gain spectrum becomes broader with increasing initial pump power and propagation distance in both fibres, the bandwidth of the gain spectrum in the FSDD is wider than that in CF at the same initial pump power. The bandwidth ratio of gain spectra in the two fibres is obtained from Eq.(17)

$$\frac{\Delta\omega_{FSDD}}{\Delta\omega_{CF}} = \exp\left(\frac{\mu z}{2}\right)\exp\left(-\frac{\alpha z}{2}\right). \quad (20)$$

As $\mu - \alpha$=0, the ranges of the gain spectra are the same. As $\mu - \alpha > 0$, Eq.(20) shows that the range of the gain spectrum in FSDD increases at an exponential growth rate, compared with that for CF. As $\mu - \alpha < 0$, the FSDD loss is so large that the range of FSDD is smaller than that of CF. So, in order to acquire broader range of MI to generate ultrashort pulses, we should produce FSDD with a larger dispersion parameter μ and a lower loss α.

3.2. The influence of the self-steepening effect on MI

When T_R=0 (omitting the Raman self-scattering effect), the dispersion relation and the gain spectrum

can be written in the following forms from Eqs.(12) and (14)

$$k = \frac{1}{6}\beta_3 \exp(\alpha z)\omega^3 + 2\frac{\gamma}{\omega_0}P_0\omega \pm \frac{1}{2}\omega|\beta_2(0)|\exp(-\mu z)\exp(\alpha z) \times \left[\omega^2 - \omega_c^2 + \frac{4(\gamma/\omega_0)^2 P_0^2}{\beta_2^2(0)\exp(-2\mu z)\exp(2\alpha z)}\right]^{1/2}, \quad (21)$$

$$G(\omega) = |\omega\beta_2(0)|\exp(-\mu z)\exp(\alpha z) \times \left[\omega_c^2 - \omega^2 - \frac{4(\gamma/\omega_0)^2 P_0^2}{\beta_2^2(0)\exp(-2\mu z)\exp(2\alpha z)}\right]^{1/2}. \quad (22)$$

When

$$\omega = \omega_{\max} = \frac{1}{\sqrt{2}}\left[\omega_c^2 - \frac{4(\gamma/\omega_0)^2 P_0^2}{\beta_2^2(0)\exp(-2\mu z)\exp(2\alpha z)}\right]^{1/2},$$

the gain spectrum also has the maximum value

$$G_{\max}(\omega) = \frac{1}{2}|\beta_2(0)|\exp(-\mu z)\exp(\alpha z) \times \left[\omega_c^2 - \frac{4(\gamma/\omega_0)^2 P_0^2}{\beta_2^2(0)\exp(-2\mu z)\exp(2\alpha z)}\right]^{1/2}, \quad (23)$$

where $\omega_c^2 = \dfrac{4\gamma P_0 \exp(\mu z)}{|\beta_2(0)|\exp(\alpha z)}$. From Eqs.(22) and (23), we can see that the self-steepening effect not only makes the range of gain spectrum narrower, but also reduces the maximum value of the gain spectrum. The bandwidth of the gain spectrum is

$$\Delta\omega_{sse} = 2\left[\frac{4\gamma P_0 \exp(\mu z)}{|\beta_2(0)|\exp(\alpha z)} - \frac{4(\gamma/\omega_0)^2 P_0^2}{\beta_2^2(0)\exp(-2\mu z)\exp(2\alpha z)}\right]^{1/2}. \quad (24)$$

Figure 2 shows the influence of the self-steepening effect on the MI gain spectrum bandwidth. The self-steepening effect is referred to as γ/ω_0. With the self-steepening effect strengthening, the bandwidth becomes narrower for a certain dispersion parameter μ. Equation (24) also shows that the largest bandwidth exists under a certain condition which must be satisfied, as follows

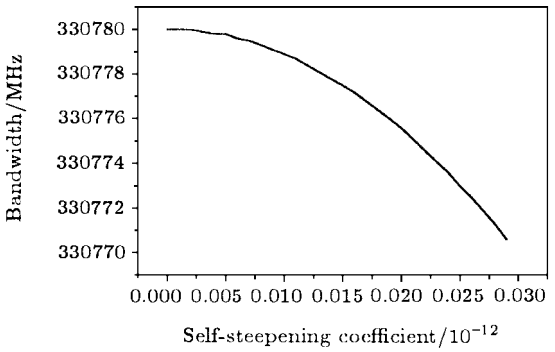


Fig.2. Influence of the self-steepening effect on MI gain spectrum bandwidth in FSDD. The self-steepening coefficient is equal to γ/ω_0. The parameters used are: $|\beta_2(0)|$=20ps^2/km; γ=2.0/W·km; μ=0.5dB/km; α=0.2dB/km; z=1km; P_0=2W.

$$|\beta_2(0)|\exp[(\alpha - \mu)z] = \frac{2(\gamma/\omega_0)^2 P_0}{\gamma}. \quad (25)$$

The largest bandwidth is $\Delta f_{\max} = 2\omega_0/(2\pi)$. For a sample, if the initial power P_0 is 1 W, the fibre nonlinear coefficient γ=2.0/W·km, the light wavelength λ=1.55μm, fibre loss α=0.2dB/km and the propagation distance z=10km, then we should have μ=1.782dB/km in order to satisfy Eq.(25), and the largest bandwidth is $\Delta f_{\max}$=387THz. Figure 3 shows that there is actually an optimal value of the dispersion parameter μ when the MI bandwidth is maximum.

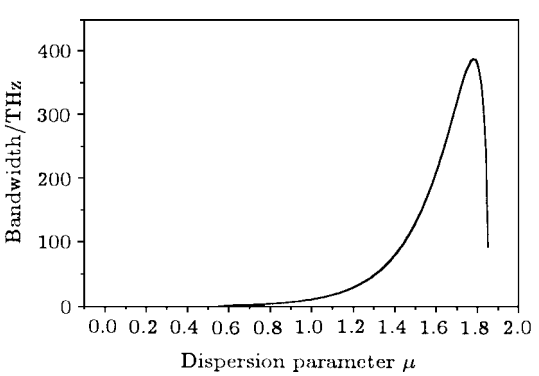


Fig.3. Different bandwidths induced by MI with different values of dispersion parameter μ for $|\beta_2(0)|$=20ps^2/km, γ=2.0/W·km, α=0.2dB/km, z=10km, and λ=1.55μm.

3.3. The influence of the self-steepening and Raman self-scattering effects on MI

When the MI-induced femtosecond pulses propagate in the FSDD, higher-order dispersion and higher-order nonlinear effects must be taken into account. From the above discussion, we know that the third-order dispersion effect does not contribute to the gain and bandwidth that are induced by MI. The higher-order nonlinear effects (including the self-steepening

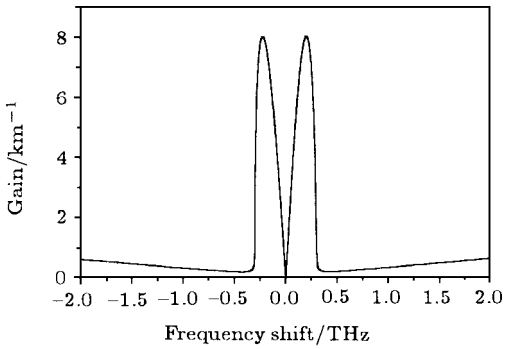


Fig.4. Influence of higher-order nonlinear effects (including the self-steepening and the Raman self-scattering effects) on MI gain spectrum. The parameters are: $|\beta_2(0)|$=20ps^2/km; γ=2.0/W·km; μ=0.5dB/km; α=0.2dB/km; z=5km; T_R=6fs; P_0=2W.

and the Raman self-scattering effects) play a strong role in MI. We have solved Eq.(14) numerically, and the parameters are selected to be: wavelength λ=1.55μm; pump power P_0=2W; initial dispersion coefficient $\beta_2(0) \approx$ 20ps^2/km; nonlinear coefficient γ=2.0/W·km; Raman response time T_R=6.0fs; fibre loss α=0.2dB/km; and dispersion slowly varying parameter μ=0.5dB/km. The gain spectrum curve is shown in Fig.4 after the pulses propagate over a distance of 10km. From Fig.4, we know that when the perturbation frequency is less than 0.3 THz, the GVD, SPM and self-steepening effect dominate the MI. When the perturbation frequency is larger than 0.3 THz, the Raman self-scattering effect becomes major in the gain spectrum. The gain increases linearly with the perturbation frequency that represents the difference between FSDD and CF. This is because the gain spectrum $G(\omega)$ increases with $\omega^{3/2}$ when $\omega >$0.3THz, according to Eq.(14). As the perturbation frequency ω increases, the self-scattering effect dominates MI and results in a new region of gain spectrum where MI occurs in the optical fibres. Therefore the Raman self-scattering effect widens the extent of MI.

4. Conclusion

The MI of the femtosecond pulses in FSDD has been investigated. We have found that the bandwidth of the MI gain spectrum, induced by a lower pumping power in FSDD, is much wider than that in conventional fibres. When the cw break-up evolves into a train of sub-picosecond or femtosecond pulses, the higher-order dispersion and the higher nonlinear effects strongly affect the pulse evolution in the optical fibres. We also find that the third-order dispersion effect does not contribute to MI, whereas the self-steepening effect reduces the maximum value of the gain and bandwidth, and the Raman self-scattering effect widens the region of MI.

References

[1] Agrawal G P 1995 *Nonlinear Fiber Optics* 2nd edn (New York: Academic) ch 5

[2] Hasegawa A 1984 *Opt. Lett.* **9** 288

[3] Tai K, Hasegawa A and Tomita A 1986 *Phys. Rev. Lett.* **56** 135

[4] Taylor J R 1992 *Optical Solitons-Theory and Experiment* (Cambridge: Cambridge University Press) ch 8

[5] Xu W C, Luo A P, Guo Q and Liu S H 2000 *Acta Opt. Sin.* **20** 1435 (in Chinese)

[6] Xu W C, Zhang S M, Luo A P, Chen W C, Guo Q and Liu S H 2000 *Acta Photon. Sin.* **29** 1005

[7] Gouveia-Neto A S, Faldon M E and Sombra A S B 1988 *Opt. Lett.* **13** 901

[8] Xu W C, Guo Q, Liao C J and Liu S H 1994 *Acta Phys. Sin.* **43** 734 (in Chinese)

[9] Xu W C, Wen S C, Liu S H, Guo Q and Liao C J 1997 *Chin. Phys. Lett.* **14** 470

[10] Potasek M J 1987 *Opt. Lett.* **12** 932

[11] Su W H 1998 *J. Optoelectron. · Laser* **9** 76 (in Chinese)

[12] Cavalcanti S B, Cressoni J C, Cruz H R and Gouveia-Neto A S 1991 *Phys. Rev.* A **43** 6162

[13] Vysloukh V A and Sukhotskova N A 1987 *Sov. J. Quantum Electron.* **17** 1509