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Sum and two-atom dipole squeezing in a system of a two-mode vacuum field interacting with two coupled atoms*

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We discuss the effects of the initial atomic coherent factors (including the phase and the distribution parameter) and the dipole–dipole interaction on the sum and two-atom dipole squeezing in the system. The results show that the maximum squeezing in both cases is determined by the phase and the distribution parameter, and the duration of squeezing becomes longer by strengthening the dipole–dipole interaction. In addition, the comparison of the two types of squeezing shows that one type of squeezing is obtained at the expense of increased fluctuations in the other squeezing function.

Keywords: two-mode vacuum field, sum squeezing, two-atom dipole squeezing, dipole-dipole interaction

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1. Introduction

The squeezing of a radiation field,[1] due to its potential applications,[2] attracts extensive attention. The field squeezing of a two-mode field interacting with a two-level atom has been discussed in a number of papers.[3−5] Hillery[6] has generalized sum squeezing as a version of higher-order squeezing of the two-mode field. On the other hand, since atomic squeezing was first investigated by Wolls and Zoller,[7] much attention has been paid to atomic dipole squeezing in a variety of Jaynes–Cummings models (JCM) under different initial conditions.[8−14] Barnett and Dupertuis[15] have defined two-atom dipole squeezing (TADS). The relation between a field and the atomic dipole squeezing in the JCM has been pointed out in some papers.[8,16,17] In this paper, we discuss sum squeezing and TADS, as well as their relation, in a system of a two-mode vacuum field interacting with two coupled atoms.

2. Model and solution

We consider the nondegenerate two-photon interaction of the two-mode field with two identical two-level atoms involving the emission or absorption of two photons from two different modes per transition, and assume that the distance between the two atoms is so short that the dipole–dipole interaction cannot be neglected. The effective Hamiltonian for the system in the rotating-wave approximation reads[18,19]

$$\hat{H} = \hat{H}_0 + \hat{H}_1 + \hat{H}_2, \tag{1}$$

where

$$\hat{H}_0 = \omega_1 \hat{a}_1^+ \hat{a}_1 + \omega_2 \hat{a}_2 \hat{a}_2 + \omega_0 \sum_{i=1}^{2} \hat{s}_z^{(i)}, \tag{2}$$

$$\hat{H}_1 = \sum_{i=1}^{2} g(\hat{a}_1^+ \hat{a}_2^+ \hat{s}_-^{(i)} + \hat{a}_1 \hat{a}_2 \hat{s}_+^{(i)}), \tag{3}$$

and

$$\hat{H}_2 = g_{\rm A}(\hat{s}_+^{(1)} \hat{s}_-^{(2)} + \hat{s}_-^{(1)} \hat{s}_+^{(2)}), \tag{4}$$

where $\hat{s}_-^{(i)}(\hat{s}_+^{(i)})$ and $\hat{s}_z^{(i)}$ are the lowering (raising) and inversion operators for the ith atom, $\hat{a}_i^+$ $(\hat{a}_i)$ is the creation (annihilation) operator for the field, g is the atom–field coupling, and $g_{\rm A}$ is the coupling between the two atoms. For simplicity, we take the resonant case, i.e. $\omega_0 = \omega_1 + \omega_2$.

By assuming that the two atoms are initially in an atomic coherent state[20]

$$|\psi(0)\rangle_{\rm A} = \cos\theta |g,g\rangle - \sin\theta {\rm e}^{{\rm i}\phi} |e,e\rangle, \tag{5}$$

and the field in a two-mode vacuum state

$$|\psi(0)\rangle_{\rm F} = |0,0\rangle, \tag{6}$$

i.e. the initial state for the total system is

$$|\psi(0)\rangle = \cos\theta |g,g;0,0\rangle - \sin\theta {\rm e}^{{\rm i}\phi} |e,e;0,0\rangle, \tag{7}$$

where θ denotes the distribution of the initial atoms ranging from zero to π, and ϕ is the phase of the

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atomic coherent state. In the interaction picture, the state at a later time t can be written as

$$\begin{aligned}|\psi^{\mathrm{I}}(t)\rangle =& \cos\theta|g,g;0,0\rangle - \sin\theta\exp[\mathrm{i}(\phi-\delta\tau)]\\ &\times[c_{ge}(t)|g,e;1,1\rangle + c_{gg}(t)|e,g;1,1\rangle\\ &+ c_{ee}(t)|e,e;0,0\rangle + c_{gg}(t)|g,g;2,2\rangle]. \quad (8)\end{aligned}$$

Inserting Eq.(8) into the Schrödinger equation, we obtain the coefficients

$$c_{ee}(t) = \frac{1}{5}[\cos\Omega + \mathrm{i}\delta\sin\Omega/\Theta + 4\exp(\mathrm{i}\delta\tau/2)], \quad (9)$$

$$c_{ge}(t) = c_{eg}(t) = -2\mathrm{i}\sin\Omega/\Theta, \quad (10)$$

and

$$c_{gg}(t) = \frac{2}{5}[\cos\Omega + \mathrm{i}\delta\sin\Omega/\Theta - \exp(\mathrm{i}\delta\tau/2)], \quad (11)$$

where $\tau = gt, \Omega = \sqrt{\delta^2+40\tau}/2$ and $\Theta = \sqrt{\delta^2+40}$, and $\delta = g_{\mathrm{A}}/g$ is the relative value of the atom–field and dipole–dipole coupling.

3. Theory

To consider the field squeezing, we use two slowly-varying Hermitian operators defined as[5]

$$\hat{X}_1 = 2^{-3/2}\sum_{i=1}^{2}[\hat{a}_i^+\exp(-\mathrm{i}\omega_i t) + \hat{a}_i\exp(\mathrm{i}\omega_i t)], \quad (12)$$

$$\hat{X}_2 = \mathrm{i}2^{-3/2}\sum_{i=1}^{2}[\hat{a}_i^+\exp(-\mathrm{i}\omega_i t) - \hat{a}_i\exp(\mathrm{i}\omega_i t)], \quad (13)$$

with the commutation relation

$$[\hat{X}_1, \hat{X}_2] = \frac{i}{2}. \quad (14)$$

The variances $(\Delta\hat{X}_i)^2 = \langle\hat{X}_i^2\rangle - \langle\hat{X}_i\rangle^2$ satisfy the uncertainty relation $(\Delta\hat{X}_1)^2(\Delta\hat{X}_2)^2 \geq 1/16$, then the fluctuations in $\hat{X}_i$ are said to be squeezed, if

$$(\Delta\hat{X}_i)^2 < \frac{1}{4}, \quad (15)$$

or

$$F_i = (\Delta\hat{X}_i)^2 - \frac{1}{4} < 0. \quad (16)$$

We call this squeezing two-mode normal squeezing (TMNS).

For sum squeezing, the corresponding Hermitian operators are given by[6]

$$\begin{aligned}\hat{V}_1 =& \frac{1}{2}\{\hat{a}_1^+\hat{a}_2^+\exp[-\mathrm{i}(\omega_1+\omega_2)t]\\ &+ \hat{a}_1\hat{a}_2\exp[\mathrm{i}(\omega_1+\omega_2)t]\}, \quad (17)\end{aligned}$$

$$\begin{aligned}\hat{V}_2 =& \frac{i}{2}\{\hat{a}_1^+\hat{a}_2^+\exp[-\mathrm{i}(\omega_1+\omega_2)t]\\ &- \hat{a}_1\hat{a}_2\exp[\mathrm{i}(\omega_1+\omega_2)t]\}, \quad (18)\end{aligned}$$

and they satisfy the commutation relation

$$[\hat{V}_1, \hat{V}_2] = \frac{i}{2}(\hat{a}_1^+\hat{a}_1 + \hat{a}_2^+\hat{a}_2 + 1). \quad (19)$$

Then the uncertainty relation is $(\Delta\hat{V}_1)^2(\Delta V_2)^2 \geq \frac{1}{16}\langle\hat{a}_1^+\hat{a}_1 + \hat{a}_2^+\hat{a}_2 + 1\rangle^2$, where the variances are $(\Delta\hat{V}_i)^2 = \langle\hat{V}_i^2\rangle - \langle\hat{V}_i\rangle^2$. Sum squeezing exists if

$$(\Delta\hat{V}_i)^2 < \frac{1}{4}\langle\hat{a}_1^+\hat{a}_1 + \hat{a}_2^+\hat{a}_2 + 1\rangle, \quad (20)$$

or

$$R_i = (\Delta\hat{V}_i)^2 - \frac{1}{4}\langle\hat{a}_1^+\hat{a}_1 + \hat{a}_2^+\hat{a}_2 + 1\rangle < 0. \quad (21)$$

Now, we turn to TADS. The Hermitian operators of the atoms are defined as[15]

$$\hat{S}_1 = \frac{1}{2}\sum_{i=1}^{2}[\hat{s}_+^{(i)}\exp(-\mathrm{i}\omega_i t) + \hat{s}_-^{(i)}\exp(\mathrm{i}\omega_i t)], \quad (22)$$

$$\hat{S}_2 = \frac{1}{2\mathrm{i}}\sum_{i=1}^{2}[\hat{s}_+^{(i)}\exp(-\mathrm{i}\omega_i t) - \hat{s}_-^{(i)}\exp(\mathrm{i}\omega_i t)]. \quad (23)$$

They satisfy the commutation relation

$$[\hat{S}_1, \hat{S}_2] = \mathrm{i}\hat{S}_z, \quad (24)$$

where $\hat{S}_z = \sum_{i=1}^{2}\hat{s}_z^{(i)}$. The uncertainty relation is

$$(\Delta\hat{S}_1)^2(\Delta\hat{S}_2)^2 \geq \frac{1}{4}\langle\hat{S}_z\rangle^2, \quad (25)$$

where $(\Delta\hat{S}_i)^2 = \langle\hat{S}_i^2\rangle - \langle\hat{S}_i\rangle^2$ are the variances in $\hat{S}_i$. The fluctuations in $\hat{S}_i$ are said to be squeezed if

$$(\Delta\hat{S}_i)^2 < \frac{1}{2}|\langle\hat{S}_z\rangle|, \quad (26)$$

or

$$Q_i = (\Delta\hat{S}_i)^2 - \frac{1}{2}|\langle\hat{S}_z\rangle| < 0. \quad (27)$$

After rigorous calculations, we obtain some quantities as follows

$$F_1 = F_2 = \sin^2\theta[|c_{ge}(t)|^2 + |c_{gg}(t)|^2], \quad (28)$$

$$\begin{aligned}R_j =& \sin^2\theta[|c_{ge}(t)|^2 + 2|c_{gg}(t)|^2] + (-1)^{\mathrm{j}}\frac{1}{2}\sin\theta\cos\theta\\ &\times[\exp[-\mathrm{i}(\phi-\delta\tau/2)]c_{gg}^*(t)\\ &+ \exp[\mathrm{i}(\phi-\delta\tau/2)]c_{gg}(t)], \quad (29)\end{aligned}$$

and

$$\begin{aligned}Q_j =& \frac{1}{2}\{\sin^2\theta[|c_{ee}(t)|^2 + 4|c_{ge}(t)|^2 + |c_{gg}(t)|^2]\\ &+ \cos^2\theta + (-1)^{\mathrm{j}}\sin\theta\cos\theta\\ &\times[\exp[-\mathrm{i}(\phi-\delta\tau/2)]c_{ee}^*(t)\\ &+ \exp[\mathrm{i}(\phi-\delta\tau/2)]c_{ee}(t)]\}\\ &- \frac{1}{2}|\sin^2\theta[|c_{ee}(t)|^2 - |c_{gg}(t)|^2] - \cos^2\theta|. \quad (30)\end{aligned}$$

4. Results and discussion

Firstly, we consider the squeezing of the field. According to Eq.(28), the functions F_1 and F_2 cannot be negative, i.e. the TMNS does not exist. The term $\hat{a}_1^+\hat{a}_2^+\hat{s}_-^{(i)} + \hat{a}_1\hat{a}_2\hat{s}_+^{(i)}$ in Eq.(1) and the resonant condition ($w_0 = w_1 + w_2$) allow only two photons from two different modes to be produced or destroyed at the same time per transition, implying $\langle\hat{a}^{+2}\rangle = \langle\hat{a}^2\rangle$=0. That is why the TMNS does not exist.

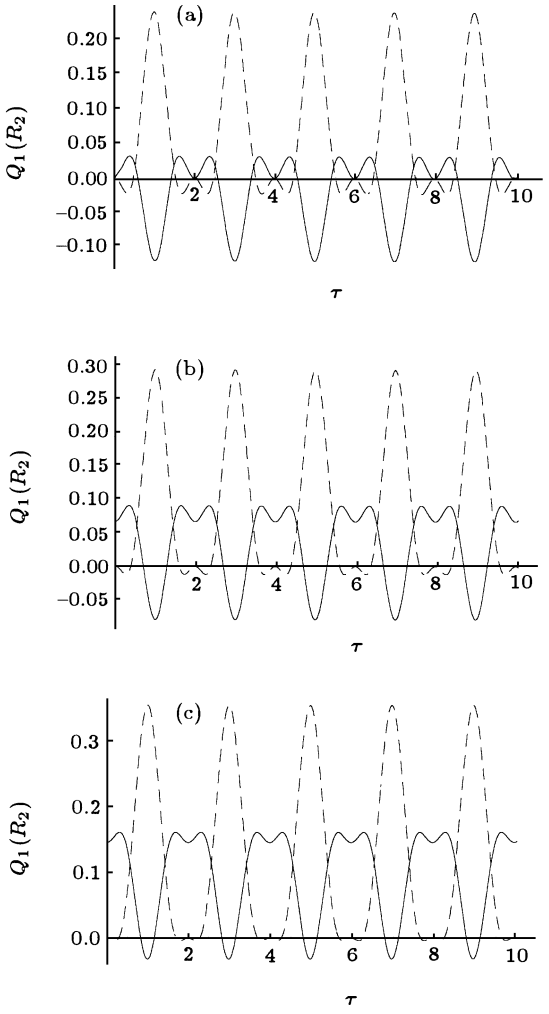


Fig.1. Time evolutions of R_2 (dashed line) and Q_1 (solid line) as functions of τ=gt for δ=0, θ=π/4: (a) ϕ=0; (b) ϕ=π/6; (c) ϕ=π/4.

In the following, we discuss another field squeezing, i.e. sum squeezing. The results for the squeezing function R_2 in Eq.(29) for different ϕ, with the same θ (=π/4), when neglecting the dipole–dipole interaction (δ=0), are shown in Fig.1. The effect of ϕ on the squeezing in R_2 is that the maximum squeezing decreases and the duration of squeezing becomes shorter with the increase in ϕ, and the peak position of squeezing does not change.

Choosing δ=0 and ϕ=0, we consider the influence of θ on the squeezing in R_2. It is seen in Fig.2 that the increase of θ causes the maximum squeezing to decrease quickly. In particular, the squeezing, as shown in Fig.2(a), appears almost all the time in the internal [0,10] of $\tau = gt$ for $\theta = \pi/6$; its maximum squeezing is about 7%, but it disappears for $\theta = \pi/3$. In both cases, the squeezing occurs periodically.

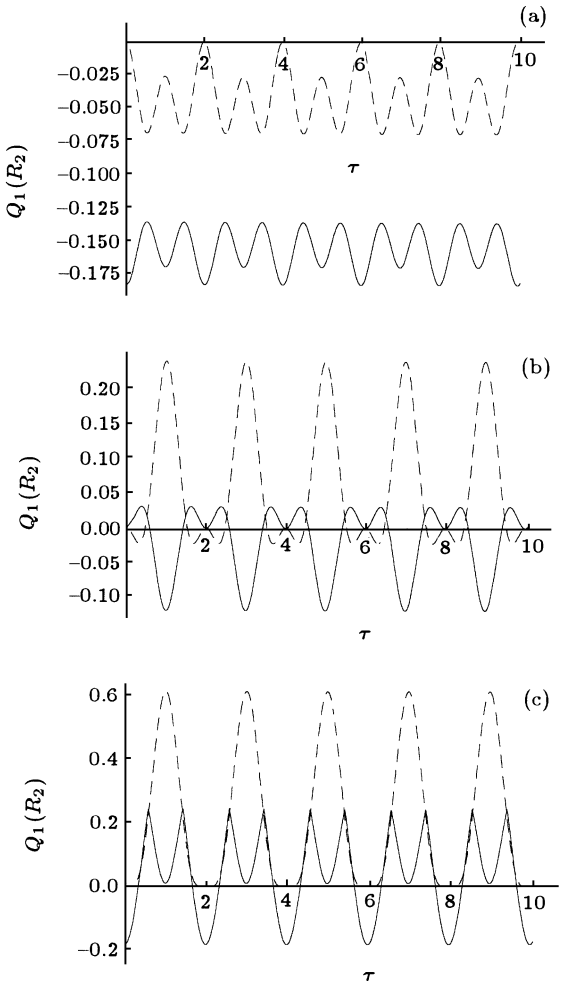


Fig.2. Time evolutions of R_2 (dashed line) and Q_1 (solid line) as functions of τ=gt for δ=0, ϕ=0: (a) θ=π/6; (b) θ=π/4; (c) θ=π/3.

In the former cases, we discuss only the influence of the initial atomic conditions. In these results, it is natural to wonder if the important quantity δ affects the sum squeezing. We illustrate the results in Fig.3 for θ=$\pi/4$ and ϕ=$\pi/6$. It is shown that, when the quantity δ is relatively small (=0.2), the squeezing in R_2, as shown in Fig.3(a), occurs only at the initial moments, and then the squeezing is revoked permanently. However, in Fig.3(b), the squeezing recurs after some time with a larger value of δ (=6). A comparison between Figs.3(b) and 3(c) shows that the duration of squeezing becomes longer with the increase of δ, but the frequency of the squeezing becomes smaller in the same interval [0,10] of τ. Comparing Fig.1(b) with Fig.3, it is easy to see that the dipole–dipole interaction destroys the periodicity of squeezing.

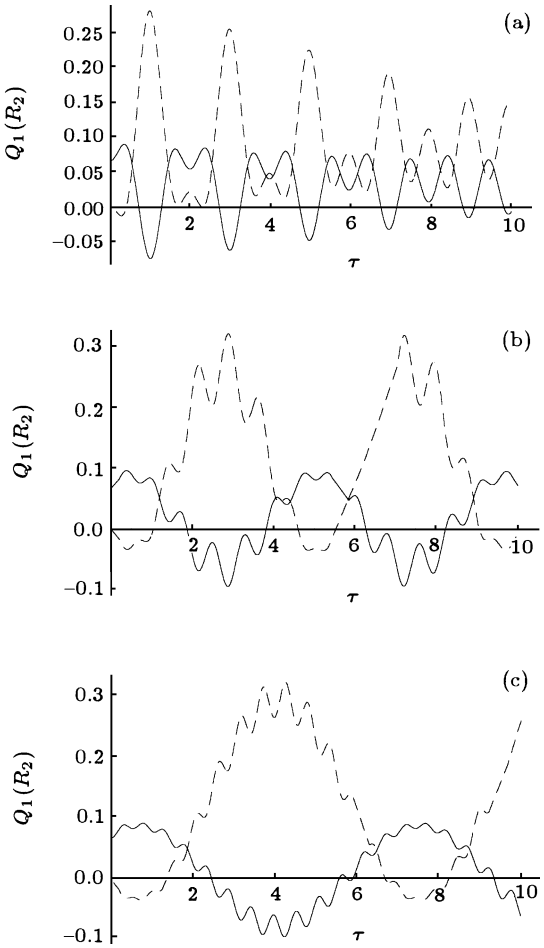


Fig.3. Time evolutions of R_2 (dashed line) and Q_1 (solid line) as functions of τ=gt for θ=$\pi/4$, ϕ=$\pi/6$: (a) δ=0.2; (b) δ=6; (c) δ=10.

Secondly, we focus on the TADS. The influences of ϕ and δ on the atomic squeezing in Q_1 are similar to the case in R_2, but the squeezing magnitudes are larger than that of the sum squeezing. For δ=0, ϕ=0 and θ=$\pi/4$, the maximum squeezing in Q_1 (see Fig.1(a)) is about 12%. However, the increase of θ (see Fig.2) causes a larger maximum of squeezing in Q_1. At this time, the maximum squeezing is approximately 18%.

Now, we turn to the relation between the sum squeezing and TADS. Figures 1–3 show that one type of squeezing is obtained at the expense of increased fluctuations in the other. This demonstrates that we can hardly obtain both types of squeezing at the same time. It is noticeable that both types of squeezing cannot appear in spontaneous radiation for which $\theta = (k + 1/2)\pi$.

5. Conclusion

We have discussed the atomic dipole and field squeezing in a system of a two-mode vacuum field interacting with two coupled atoms. The maximum squeezing in both cases is determined by the phase ϕ and distribution θ: increases of these quantities will decrease the maximum squeezing except that the maximum squeezing in Q_1 becomes larger with the increase of θ. The dipole–dipole interaction disorders the periodicity of oscillation in both R_2 and Q_1. A larger δ results in a longer duration and a smaller frequency of the squeezing. The atomic (field) squeezing is obtained at the expense of the field (atomic) noise.

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