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Controlling the Chen attractor using linear feedback based on parameter identification*

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In this paper we investigate the method for controlling the chaotic Chen system with unknown parameters using linear feedback functions based on parameter identification. The observer is applied to the identification of the unknown parameters of Chen system. Then an efficient feedback function is designed for controlling the Chen system. Numerical simulations are provided to show the effectiveness and feasibility of the developed controller.

Keywords: feedback control, Chen attractor, parameter identification
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1. Introduction

Chaos control, in a broader sense, can be divided into two categories:[1] one is to suppress the chaotic dynamical behaviour and the other is to generate or enhance chaos in nonlinear systems (known as chaotification or anti-control of chaos).[2−4] Nowadays, different techniques and methods[1−15] have been proposed to achieve chaos control: for example, the OGY method,[5] the differential geometric method,[6] linear state space feedback,[7] inverse optimal control[8] and output feedback control,[9] among many others.[1]

However, for some systems which have unknown parameters, many of the aforementioned methods will fail. An important problem in chaos control is how to achieve linear control of complex dynamical systems with unknown parameters. This problem involves the identification of the unknown parameters and the approach of controlling chaos. In this paper, the observer is applied to the identification of the unknown parameters of the Chen system.[3] Then an efficient feedback function is designed for controlling the system. The suggested tool enables the controlled chaotic Chen system to approach any desired periodic orbits or points. A computer simulation is given for illustration and verification.

2.System description

Recently, in a research on anti-controlling chaos, Chen[3] has developed a new chaotic system, bearing the name of the Chen system, which was derived from the classical Lorenz system.[4] The nonlinear differential equations that describe the system are

$$\begin{cases} \dot{x} = a(y - x)\,, \\ \dot{y} = (c - a)x - xz + cy\,, \\ \dot{z} = xy - bz\,, \end{cases} \tag{1}$$

which have a chaotic attractor, as shown in Fig.1, when $a = 35$, $b = 3$, $c = 28$.

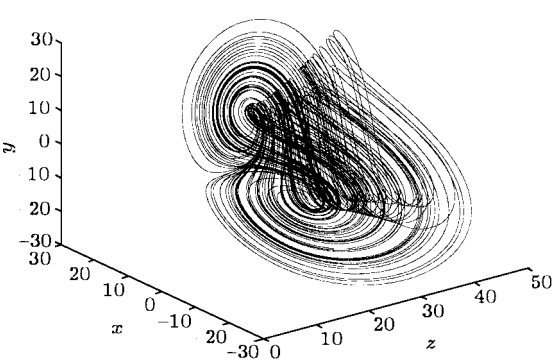


Fig.1. The chaotic Chen attractor.

From the point of view of control engineering, the chaotic Chen attractor is relatively difficult to control as compared with the Lorenz system due to the prominent three-dimensional and complex topological

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features of the attractor, especially its rapid change in velocity in the z-direction.[9]

We are interested in developing control techniques which can drive a strange attractor with unknown parameters not only to a periodic orbit but also to a steady state. For simplification, we assume that only the parameter b of system (1) is unknown. Moreover, we add two control inputs to the first and second equations, respectively. Thus the controlled system becomes

$$\begin{cases} \dot{x} = a(y-x) + u_1\,, \\ \dot{y} = (c-a)x - xz + cy + u_2\,, \\ \dot{z} = xy - bz\,. \end{cases} \tag{2}$$

3. Identification of the unknown parameters

In this section, we provide an observer that can identify the unknown parameter b of the chaotic Chen system.

Since the dynamical information of system (1) about the parameter b is hard to obtain, we design an observer which can identify the parameter b. Note that b is a constant of system (1), so

$$\dot{b} = 0\,. \tag{3}$$

The unknown parameter b can act as a state variable. Then system (1) can be augmented by Eq.(3). Furthermore, we imagine that all scalar variables x, y, z are measured as the system output.

According to system (1), we have

$$bz = xy - \dot{z},$$

and the observer is

$$\dot{\hat{b}} = -G(z)z\hat{b} + G(z)(xy - \dot{z}) \tag{4}$$

where $G(z)$ is a gain function. Letting

$$e = b - \hat{b},$$

then, using Eqs.(3) and (4), we have

$$\dot{e}(t) = \dot{b} - \dot{\hat{b}} = -G(z)ze(t)\,.$$

We can choose the gain function $G(z)$ such that the system

$$\dot{e}(t) + G(z)ze(t) = 0$$

is exponentially asymptotically stable for all z; that is, when $t \to \infty$, $\hat{b}(t)$ converges to $b(t)$ with an exponential rate. Obviously, the gain function $G(z)$ can be chosen as k/z. Hence we obtain

$$\dot{e}(t) + ke(t) = 0\,,$$

where $k > 0$ and determines the convergence rate. Since $\dot{z}$ is hard to observe, the observer (4) is not an applicable value. We introduce an auxiliary variable

$$\delta = \hat{b} + Q(z)\,, \tag{5}$$

where $Q(z)$ is a design function that satisfies the condition

$$G(z) = \frac{\mathrm{d}Q(z)}{\mathrm{d}z}\,.$$

According to Eq.(5), we have

$$\begin{aligned} \dot{\delta} = & \dot{\hat{b}} + \frac{\mathrm{d}Q(z)}{\mathrm{d}z}z \\ = & -G(z)z\delta + G(z)(zQ(z) + xy) \end{aligned} \tag{6}$$

and

$$\hat{b} = \delta - Q(z)\,. \tag{7}$$

If the design function $Q(z)$ makes

$$\dot{e}(t) + \frac{\mathrm{d}Q(z)}{\mathrm{d}z}ze(t) = 0$$

exponentially asymptotically stable, then, when $t \to \infty$, $\hat{b}(t)$ will converge to $b(t)$ with an exponential rate. However, the observer Eqs.(6) and (7) are the identification observers of the unknown parameter b of system (1), where $G(z)$ is a gain function and $Q(z)$ is a design function satisfying $\mathrm{d}Q(z)/\mathrm{d}z = G(z)$.

Note that the observer Eqs.(6) and (7) rely only on the third equation of system (1). That is, when the structure of the first and second equations of system (1) or the parameters a and b are changed, the results of the identification are not influenced. So, the observers have strong robustness. Letting $Q(z) = k\ln z$, the observers become

$$\begin{cases} \dot{\delta} = -k\delta - k^2\ln z + \dfrac{kxy}{z}, \\ \hat{b} = \delta - k\ln z. \end{cases} \tag{8}$$

4. Nonlinear feedback control

In this section, we give our main results concerning a new and simple control law, which can make the trajectories of the controlled chaotic Chen system with unknown parameters approach any target periodic orbits or points.

Firstly, we assume that the parameter b of controlled system (2) has been identified, and $b = \hat{b}$. According to the ergodic property of chaotic orbits, for any x_0, the Chen system can reach the point $x = x_0$ in a certain time, but is not stable at $x = x_0$. In order to make the Chen system stabilize at $x = x_0$, the control inputs must satisfy the stable condition

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}z}{\mathrm{d}t} = 0\,.$$

Letting the control inputs u_1 and u_2 be

$$\begin{cases} u_1 = -ay - a|x_0|(x - x_0 - \mathrm{sign}\,(ax_0))\,, \\ u_2 = ax - (c+1)y\,, \end{cases} \tag{9}$$

the controlled system (2) becomes

$$\begin{cases} \dot{x} = -ax - a|x_0|(x - x_0 - \mathrm{sign}\,(ax_0))\,, \\ \dot{y} = cx - xz - y\,, \\ \dot{z} = xy - \hat{b}z\,. \end{cases} \tag{10}$$

Obviously, system (10) has a unique equilibrium point $S_0\left(x_0,\ \dfrac{\hat{b}cx_0}{\hat{b}+x_0^2},\ \dfrac{cx_0^2}{\hat{b}+x_0^2}\right)$. In addition, the Jacobian matrix $A(S_0)$ of system (10) is

$$A(S_0) = \begin{pmatrix} -a - a|x_0| & 0 & 0 \\ \dfrac{\hat{b}c}{\hat{b}+x_0^2} & -1 & -x_0 \\ \dfrac{\hat{b}cx_0}{\hat{b}+x_0^2} & x_0 & -\hat{b} \end{pmatrix}, \tag{11}$$

whose characteristic equation is

$$(\lambda + a + a|x_0|)[\lambda^2 + (\hat{b}+1)\lambda + \hat{b} + x_0^2] = 0.$$

So the three characteristic roots have negative real parts. This indicates that the unique steady state S_0 is stable.

According to Eq.(8), we can obtain the control law of the controlled system (2) with the unknown parameter b

$$\begin{cases} u_1 = -ay - a|x_0|(x - x_0 - \mathrm{sign}\,(ax_0))\,, \\ u_2 = ax - (c+1)y\,, \\ \dot{\delta} = -k\delta - k^2\ln z + \dfrac{kxy}{z}\,, \\ \hat{b} = \delta - k\ln z\,, \end{cases} \tag{12}$$

where $k > 0$ is a control constant.

Therefore, under the action of feedback control inputs (12), the Chen system with unknown parameters can quickly approach target point S_0.

Similarly, in order to make the controlled system (2) reach any known point $x = x_1$, $y = y_1$, we can obtain the control law of the controlled system (2) with the unknown parameter b

$$\begin{cases} u_1 = -ay - a|x_1|(x - x_1 - \mathrm{sign}(ax_1))\,, \\ u_2 = ax - (c+1)y - |d|(y - y_1 - \mathrm{sign}(d))\,, \\ \dot{\delta} = -k\delta - k^2\ln z + \dfrac{kxy}{z}\,, \\ \hat{b} = \delta - k\ln z\,, \end{cases} \tag{13}$$

where $k > 0$ is a control constant.

Therefore, under the action of feedback control inputs (13), the Chen system with unknown parameters can quickly approach target point S_1.

Next, in order to make the trajectories of the controlled Chen system with unknown parameters approach any target periodic orbits, we assume that x_1 and y_1 are periodic functions.

We let

$$x_1 = r\cos\omega t,\ y_1 = r\sin\omega t \tag{14}$$

and

$$\begin{cases} u_1 = -ay - ar|\cos\omega t|(x - r\cos\omega t \\ \qquad -\mathrm{sign}\,(ar\cos\omega t))\,, \\ u_2 = ax - (c+1)y - |d|(y - r\sin\omega t - \mathrm{sign}\,(d))\,, \\ \dot{\delta} = -k\delta - k^2\ln z + \dfrac{kxy}{z}\,, \\ \hat{b} = \delta - k\ln z\,, \end{cases} \tag{15}$$

where $d = \dfrac{r^3\sin 2\omega t\cos\omega t}{2b} + r\sin\omega t - cr\cos\omega t$, and $k > 0$ is a control constant.

Analogously, under the action of feedback control inputs (15), we can prove that the Chen system with unknown parameters can quickly approach the target periodic orbit

$$x_1 = r\cos\omega t,\ y_1 = r\sin\omega t,\ z = \frac{r^2\sin 2\omega t}{2b}\,. \tag{16}$$

In fact, the feedback controller (15) can easily be realized in practical applications. We can design an external oscillator which produces a periodic sine signal. By using the periodic signal and systematic output signal as control input signals, we can make the Chen system with unknown parameters stable at the periodic orbit (16).

5. Simulation results

In order to verify the effectiveness of the observer (8) and the applicability of the proposed control laws (12), (13) and (15), we assume $b = 2.655$ and initial

conditions $x(0) = 2.0$, $y(0) = -1.5$, $z(0) = 0.8$. The fourth-order Runge–Kutta method is used to solve differential equations, such as (12) and (13), with the same time step size, 0.001 s, in all numerical simulations.

Letting $k = 0.8$, Fig.2 shows the effectiveness of the observer (8). We investigate the effectiveness of observers under the conditions of no external force and external force $f = 3$.

With the control law (12) applied and different x_0, the results are shown in Figs.3(a) and 3(b). These indicate that the system can reach target point $\left(x_0, \dfrac{\hat{b}cx_0}{\hat{b}+x_0^2}, \dfrac{cx_0^2}{\hat{b}+x_0^2}\right)$ within 1.6 s for all x_0.

With the control law (13) applied and different x_1, y_1, the results are shown in Figs.4(a) and 4(b). These indicate that the system can reach target point $\left(x_1, y_1, \dfrac{x_1 y_1}{\hat{b}}\right)$ within 2.1 s for all x_1, y_1.

With the control law (15) applied and different r, ω, the results are shown in Figs.5(a) and 5(b). These indicate the following. (i) For different r, ω, the system can stably reach periodic orbits within $2\pi/\omega$ s. The period is $T = 2\pi/\omega$, and there is no distinct correlation between the period and orbits. (ii) When $0.3 < r < 1$, the controlled system is stable at periodic orbit, but the shape and period of the orbit are different from the theoretical results. When $r \to 0$, the system is unstable at periodic orbit. We think that it may be possible that the feedback signal becomes relatively weaker. (iii) When $r > 1$ and $0.05 < \omega < 120$, the projected orbits on the x–y plane evolve from similar roundness to ellipse. These results confirm our theoretical analysis. Some differences between the numerical simulation and theoretical analysis, we think, may be caused by the relatively stronger feedback signal.

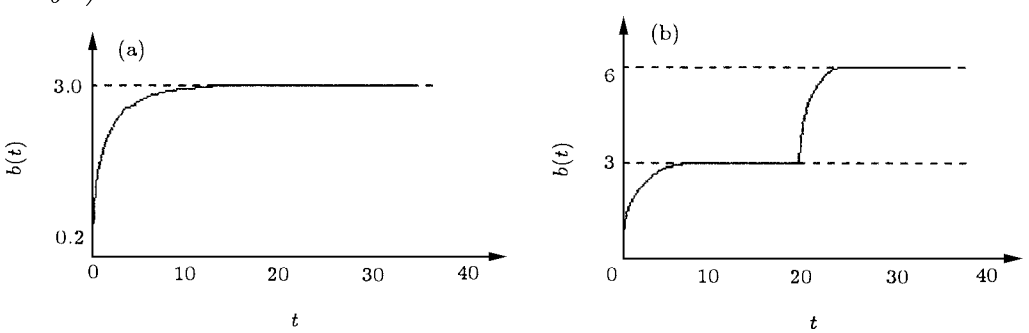


Fig.2. The identification results of the Chen system: (a) without external force; (b) under the action of external force $f = 3$.

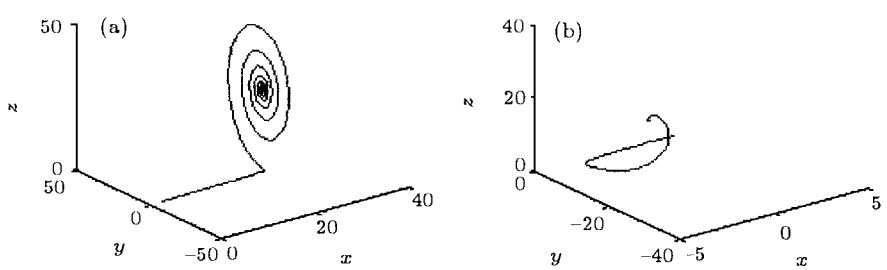


Fig.3. Approaching target points: (a) $x_0 = 25$, $S_0(25, 3.3439, 27.8662)$; (b) $x_0 = -3$, $S_0(-3, -21, 21)$.

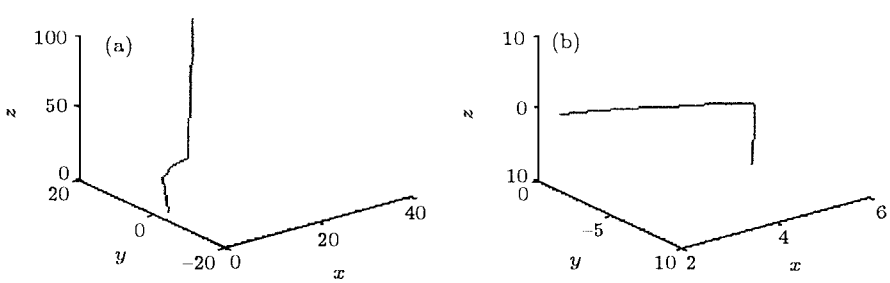


Fig.4. Approaching target points: (a) $x_1 = 20$, $y_1 = 15$, $S_1(20, 15, 100)$; (b) $x_1 = 5$, $y_1 = -5$, $S_1(5, -5, -8.3333)$.

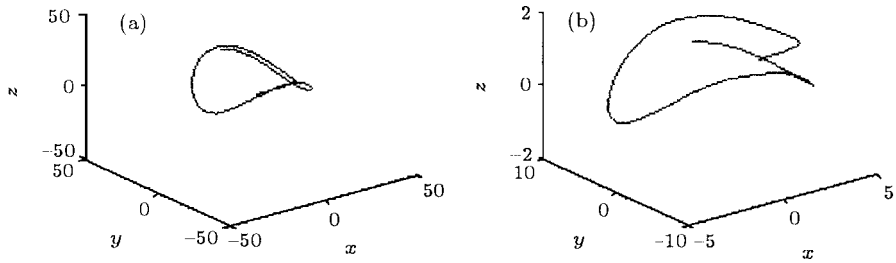


Fig.5. Approaching target periodic orbits: (a) $r = 25$, $\omega = 15$; (b) $r = 5$, $\omega = 10$.

6. Conclusion

We have presented a new and effective control law to make the trajectories of the chaotic Chen system with unknown parameters approach any target periodic orbits or points. Compared to other existing chaos control methods, this controller can reach any desired targets in the least time. At the same time, the results of controlling chaos can be deduced beforehand. Especially, this method combines the identification of unknown parameters with a feedback function to control the chaotic system with unknown parameters.

It should be noted that although this paper focuses mainly on the control of the Chen attractor with unknown parameters, we believe that the higher nonlinearity and complexity of the chaotic Chen attractor can justify the practical application of the proposed method to some other complex dynamical systems with unknown parameters.

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