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Global adaptive synchronization of chaotic systems with uncertain parameters*

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We propose a novel adaptive synchronization method for a class of nonlinear chaotic systems with uncertain parameters. Using the chaos control method, we derive a synchronizer, which can make the states of the driven system globally track the states of the drive system asymptotically. The advantage of our method is that our problem setting is more general than those that already exist, and the synchronizer is simply constructed by an analytic formula, without knowledge in advance of the unknown bounds of the uncertain parameters. A computer simulation example is given to validate the proposed approach.

Keywords: chaotic systems, chaos control, adaptive chaos synchronization, uncertain parameters
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1. Introduction

Since the paper by Pecora and Carroll[1], synchronization in chaotic systems has received increasing attention. Usually, chaos synchronization has the following feature: the trajectories of a driven system must track the trajectories of the drive system. Since chaos synchronization is an important problem in nonlinear science, much effort has been devoted to investigating it, and numerous achievements[2−9] have been made. In this paper, the main purpose is to derive an adaptive synchronizer by using chaos control methods[10,11] which can make the states of the driven system globally track the states of the drive system asymptotically in the presence of the unknown parameters of the system.

2.Problem formulation

The drive nonlinear chaotic system considered in this paper is assumed to be

$$\dot{x} = f(x) + F(x)\theta, \tag{1}$$

where the states of the system are $x \in R^n$, and the uncertain parameters are $\theta \in R^p$.

We construct the synchronizer, called the driven system, for Eq.(1)

$$\dot{\hat{x}} = f(\hat{x}) + F(\hat{x})\hat{\theta} - u(x, \hat{x}, \hat{\theta}), \tag{2}$$

where $u(x, \hat{x}, \hat{\theta})$ is a controller to be designed, and $\hat{\theta} \in R^p$ and $\hat{x} \in R^n$ are estimated values of the unknown parameter vector $\theta \in R^p$ and the states of the drive system, respectively.

Subtracting Eq.(2) from Eq.(1) gives the synchronization error dynamical system

$$\dot{e} = \tilde{f}(e, \hat{x}) + \tilde{F}(e, \hat{x})\theta + F(\hat{x})\tilde{\theta} + \overline{u}(e, \hat{x}, \hat{\theta}), \tag{3}$$

where

$$\tilde{f}(e, \hat{x}) = f(x) - f(\hat{x}),\ \ \overline{u}(e, \hat{x}, \hat{\theta}) = u(x, \hat{x}, \hat{\theta}),$$

$$\tilde{F}(e, \hat{x}) = F(x) - F(\hat{x}),\ \ \tilde{\theta}(t) = \theta - \hat{\theta}(t),\ \ e = x - \hat{x}.$$

The goal of control is to find a controller $\overline{u}(e, \hat{x}, \hat{\theta})$ and a parameter estimation update law $\dot{\hat{\theta}}$ for Eq.(3) that makes the states of driven system (2) globally track the states of drive system (1) asymptotically, i.e.

$$\lim_{t\to\infty} \| e(t) \| = \lim_{t\to\infty} \| x(t) - \hat{x}(t) \| = 0. \tag{4}$$

3. Design of synchronizer

Assumption 1: For an arbitrarily given $\hat{x}, \dot{e} = \tilde{f}(e, \hat{x})$ being asymptotically stable at e=0, i.e. $\forall (e, t) \in R^n \times R^+$, there is a positive number $\gamma > 0$ and a scalar function with well-defined continuous first derivatives

$$V : R^n \times R^+ \to R^+,$$

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such that

1) $\lambda_{\min}||e||^2 \le V(e,t) \le \lambda_{\max}||e||^2$, (5)

2) $$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\partial V(e,t)}{\partial t} + (\mathrm{grad}\, V, \tilde{f}(e,\hat{x})) \le -\gamma||e||^2. \quad (6)$$

Theorem 1: If Assumption 1 holds for Eq.(3), then a controller $\overline{u}(e,\hat{x},\hat{\theta})$ and a parameter estimation update law $\dot{\hat{\theta}}$ exist such that

$$\lim_{t\to\infty} \| e(t) \| = \lim_{t\to\infty} \| x(t) - \hat{x}(t) \| = 0,$$

where

$$\overline{u}(e,\hat{x},\hat{\theta}) = -\tilde{F}(e,\hat{x})\hat{\theta}, \quad (7)$$

$$\dot{\hat{\theta}} = (\tilde{F}(e,\hat{x}) + F(\hat{x}))^{\mathrm{T}}\mathrm{grad}\, V. \quad (8)$$

Proof: For Eq.(3), we choose the Lyapunov function $W(e,\tilde{\theta},t) = V(e,t) + \frac{1}{2}\tilde{\theta}^{\mathrm{T}}\tilde{\theta}$ and $V(e,t)$ satisfies assumption 1. We have

$$\begin{aligned}\frac{\mathrm{d}W}{\mathrm{d}t} =& \frac{\partial V(e,t)}{\partial t} + (\mathrm{grad}\, V, \tilde{f}(e,\hat{x})) \\ &+ (\mathrm{grad}\, V, \tilde{F}(e,\hat{x})\theta + F(\hat{x})\tilde{\theta}) \\ &+ (\mathrm{grad}\, V, \overline{u}(e,\hat{x},\hat{\theta})) + \dot{\tilde{\theta}}^{\mathrm{T}}\tilde{\theta} \\ \le& -\gamma \| e \|^2 + (\mathrm{grad}\, V, (\tilde{F}(e,\hat{x}) \\ &+ F(\hat{x}))\tilde{\theta}) + \dot{\tilde{\theta}}^{\mathrm{T}}\tilde{\theta} \\ =& -\gamma \| e \|^2 .\end{aligned}$$

This completes the proof.

4. Simulation

For example, we let the Rössler system with unknown parameters be the drive system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ x_3 & 0 & -c \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix}. \quad (9)$$

We first rewrite Eq.(9) in the form of Eq.(1)

$$\dot{x} = f(x) + F(x)\theta, \quad (10)$$

where

$$f(x) = \begin{bmatrix} -x_2 - x_3 \\ x_1 \\ x_1 x_3 \end{bmatrix}, \quad F(x) = \begin{bmatrix} 0 & 0 & 0 \\ x_2 & 0 & 0 \\ 0 & 1 & -x_3 \end{bmatrix},$$

and $\theta = [a \;\; b \;\; c]^{\mathrm{T}}$ is the unknown parameter vector of the system.

It has been proven that the Rössler system presents chaos when a=b=0.2, c=5.7. According to Eq.(2), we construct the synchronizer which is the driven system for Eq.(10) as follows

$$\dot{\hat{x}} = f(\hat{x}) + F(\hat{x})\hat{\theta} - u(x,\hat{x},\hat{\theta}).$$

The synchronization error dynamical system is described by

$$\dot{e} = \tilde{f}(e,\hat{x}) + \tilde{F}(e,\hat{x})\theta + F(\hat{x})\tilde{\theta} + \overline{u}(e,\hat{x},\hat{\theta}). \quad (11)$$

The true value of the "unknown" parameter vector, the initial value of the error state vector, the initial value of the estimate for the "unknown" parameter vector and the initial value of the estimate for the system state vector determined by Eq.(10) are taken to be $\theta = [0.2 \;\; 0.2 \;\; 5.7]^{\mathrm{T}}$, $e(0) = [6 \;\; 6 \;\; 6]$, $\hat{\theta}(0) = [0.1 \;\; 0.1 \;\; 2]^{\mathrm{T}}$ and $\hat{x}(0) = [10 \;\; 10 \;\; 10]$, respectively.

From theorem 1, we construct the controller $\overline{u}(e,\hat{x},\hat{\theta})$ and parameter estimation update law $\dot{\hat{\theta}}$ according to Eqs.(7) and (8), respectively, as follows

$$\overline{u}(e,\hat{x},\hat{\theta}) = \overline{u}_r(e,\hat{x}) - \tilde{F}(e,\hat{x})\hat{\theta},$$

$$\dot{\hat{\theta}} = (\tilde{F}(e,\hat{x}) + F(\hat{x}))^{\mathrm{T}}\mathrm{grad}\, V.$$

We choose

$$V(e,t) = e^{\mathrm{T}}Pe,$$

$$u_r(e,\hat{x}) = \begin{bmatrix} -2 & 1 & 1 \\ -1 & -2 & 0 \\ -e_3 - \hat{x}_3 & 0 & -2 - \hat{x}_1 \end{bmatrix} e,$$

where

$$P = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}, \quad \lambda_{\min} = \lambda_{\max} = 5, \quad \gamma = 20.$$

Using theorem 1, we conclude that there is an adaptive controller, such that

$$\lim_{t\to\infty} \| e(t) \| = \lim_{t\to\infty} \| x(t) - \hat{x}(t) \| = 0.$$

The time response of the drive system and, after adding $\overline{u}(e,\hat{x},\hat{\theta})$ to the dynamical error system (11), the time response of estimates for the unknown state determined by drive system (10), and the time response of the error state between the drive system and driven system, $x_i, \hat{x}_i$ and e_i for i=1,2,3 are shown in Figs.1, 2 and 3, respectively.

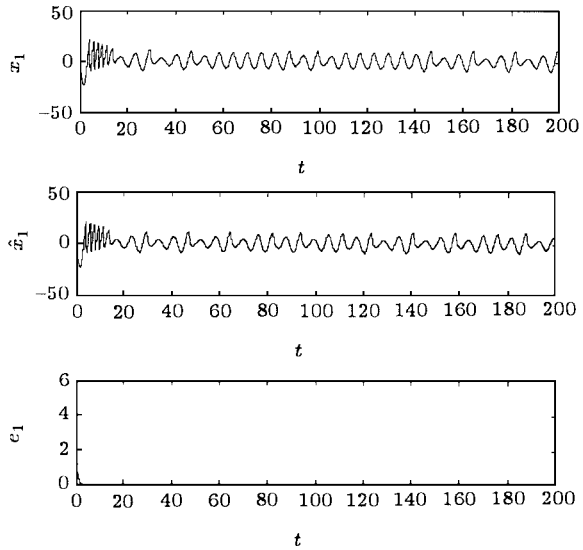


Fig.1. Time response of system state, x_1, estimation $\hat{x}_1$ for x_1 and error e_1.

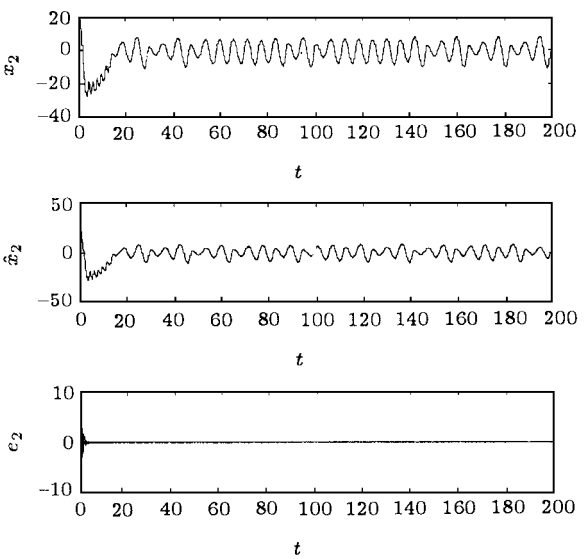


Fig.2. Time response of system state x_2, estimation $\hat{x}_2$ for x_2 and error e_2.

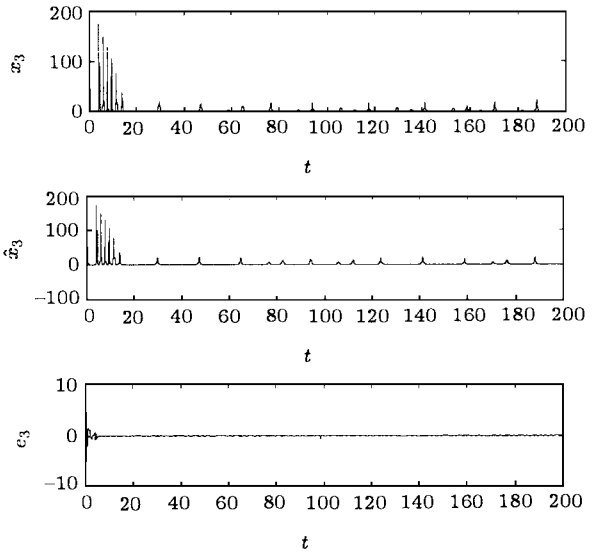


Fig.3. Time response of system state x_3, estimation $\hat{x}_3$ for x_3 and error e_3.

5. Conclusion

Based on the chaos control method,[10,11] a novel adaptive synchronization method is proposed for a class of chaotic systems with uncertain parameters. Using the present method, an adaptive synchronizer is developed, which can make the states of the driven system globally track the states of the drive system asymptotically. The advantage of our method is that our problem setting is more general than those that already exist,[6−9] and the synchronizer can be simply constructed by an analytic formula without knowing in advance the unknown bounds about the uncertain parameters. A computer simulation example is given to validate the proposed approach.

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