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UNCORRELATED TWO-MODE $SU(1,1)$ COHERENT STATES

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We have constructed a new kind of two-mode bosonic realization of $SU(1,1)$ Lie algebra, on the basis of which the $SU(1,1)$ generalized coherent states in the two-mode Fock space are derived. These two-mode $SU(1,1)$ coherent states, which are called uncorrelated two-mode $SU(1,1)$ coherent states, include three special cases. For these states, we study the mean photon number distribution and their non-classical properties, which are photon anti-bunching, violations of Cauchy–Schwarz inequality and two-mode squeezing.

Keywords: uncorrelated two-mode $SU(1,1)$ coherent states, non-classical properties
PACC: 4250, 0365, 0220

I. INTRODUCTION

It is well known that the Lie group of $SU(1,1)$ is of great interest in many branches of physics as it plays the role of a spectrum-generating dynamical group for a large number of systems, for example, the square-well potential, the harmonic oscillator, the hydrogen atom, the Morse oscillator and the many-body system of superfluid helium.[1−3] In particular, the Lie algebra of the $SU(1,1)$ group is widely used in quantum optics.[4−7] For example, many authors have examined that the single- and two-mode realizations of $SU(1,1)$ Lie algebra have immediate relevance to the non-classical properties of light.[8] In these works, the single-mode $SU(1,1)$ coherent states (CSs) defined by Perelomov have been discussed in connection with the squeezed states of a single-mode field[8,9] and are a special case of the two-photon CSs of Yuen,[10] namely, the squeezed states.[11] Also, this kind of two-mode bosonic realization of $SU(1,1)$ Lie algebra only includes one photon process in each mode, and the corresponding two-mode $SU(1,1)$ generalized CSs in the two-mode Fock space have been discussed. These two-mode $SU(1,1)$ CSs are called correlated two-mode $SU(1,1)$ CSs[12] which can be obtained from the application of the two-mode squeezed operator onto a two-mode Fock state $|q,0\rangle$ where $q = 0,1,2,\cdots$ Among these, a special case known as the two-mode squeezed vacuum states[13−15] has been much studied and may be produced by a non-degenerate parametric amplifier acting on two-mode vacuum states,[16,17] i.e. $|0,0\rangle$. Similarly, we construct another kind of two-mode bosonic realization of the $SU(1,1)$ Lie algebra in two-photon processes in each mode. At the same time, we derive the corresponding two-mode generalized CSs which are produced from the application of the two-mode displacement operator onto some special two-mode Fock state $|n_1,n_2\rangle$ where n_1, n_2 are 1 or 0. In contrast to the general cases, we call these two-mode $SU(1,1)$ CSs the uncorrelated two-mode $SU(1,1)$ CSs, which have three special cases. Compared with the quantum properties of correlated two-mode $SU(1,1)$ CSs, we find that those of these states are interesting and distinctive.

This paper is organized as follows. In section II the relevant formalism of the $SU(1,1)$ CSs and the representation for the case of a single-mode field and correlated two-mode field are briefly reviewed. In section III, the realization and representation for the case of an uncorrelated two-mode field are introduced. In section IV, the quantum properties of these states—mean photon number, photon bunching and anti-bunching, violations of the Cauchy–Schwarz inequality and two-mode squeezing—are studied. In section V, a brief summary and some remarks are given.

II. REVIEW OF $SU(1,1)$ CSs

The $SU(1,1)$ Lie algebra consists of three generators, K_0, K_+ and K_-, satisfying the commutation relations

$$[K_0, K_\pm] = \pm K_\pm, \quad [K_+, K_-] = -2K_0. \tag{1}$$

The Casimir operator for this algebra is given by $C \equiv K_0^2 - (K_+K_- + K_-K_+)/2$, which has the eigenvalue $k(k-1)$ for a unitary irreducible representation (UIR).

The parameter k is the so-called Bargmann index. For a positive discrete series representation, $k > 0$. The basis is denoted as $|m,k\rangle$, where $m = 0,1,2,\cdots$ and the action of the $SU(1,1)$ generators on these states is

$$K_0|m,k\rangle = (m+k)|m,k\rangle, \tag{2}$$

$$K_+|m,k\rangle = \sqrt{(m+1)(m+2k)}|m+1,k\rangle, \tag{3}$$

$$K_-|m,k\rangle = \sqrt{m(m+2k-1)}|m-1,k\rangle. \tag{4}$$

It is apparent that K_0 is diagonal in these representations.

Following Perelomov,[18] the $SU(1,1)$ generalized CSs $|\xi,k\rangle$ are defined as

$$\begin{aligned}|\xi,k\rangle =&D(\alpha)|0,k\rangle, \quad K_-|0,k\rangle = 0,\\ &K_0|0,k\rangle = k|0,k\rangle,\end{aligned} \tag{5}$$

where

$$\begin{aligned}D(\alpha) =&\exp(\alpha K_+ - \alpha^* K_-)\\ =&\exp(\xi K_+)\exp(\gamma K_0)\exp(-\xi^* K_-),\end{aligned} \tag{6}$$

$\alpha = -(1/2)\theta\exp(-\mathrm{i}\phi), \xi = -\tanh(\theta/2)\exp(-\mathrm{i}\phi), \gamma = \ln(1-|\xi|^2)$, with $\theta \in (-\infty,\infty)$, $\phi \in [0,2\pi]$.

The single-mode bosonic realization of the $SU(1,1)$ Lie algebra is given by[12,19,20]

$$K_+ = \frac{a^{+2}}{2}, \quad K_- = \frac{a^2}{2}, \quad K_0 = \frac{1}{4}(aa^+ + a^+a). \tag{7}$$

In this case the Casimir operator is evaluated to be $C = -3/16$, which implies representation with $k = 1/4$ or $k = 3/4$. The corresponding single-mode $SU(1,1)$ CSs [21] are derived:

$$\begin{aligned}\left|\xi,\frac{1}{4}\right\rangle_s =&D(\alpha)|0\rangle\\ =&(1-|\xi|^2)^{1/4}\sum_{n=0}^{\infty}\frac{\sqrt{(2n)!}}{2^n n!}\xi^n|2n\rangle,\end{aligned} \tag{8}$$

$$\begin{aligned}\left|\xi,\frac{3}{4}\right\rangle_s =&D(\alpha)|1\rangle\\ =&(1-|\xi|^2)^{3/4}\sum_{n=0}^{\infty}\frac{\sqrt{(2n+1)!}}{2^n n!}\xi^n|2n+1\rangle,\end{aligned} \tag{9}$$

where $\left|\xi,\frac{1}{4}\right\rangle_s$ is the squeezed vacuum and $\left|\xi,\frac{3}{4}\right\rangle_s$ is the squeezed 'one-photon' state.

For a type of two-mode bosonic realization, the $SU(1,1)$ Lie algebra is defined as [12,20,22]

$$\begin{aligned}K_+ =&a_1^+a_2^+, \quad K_- = a_1a_2,\\ K_0 =&\frac{1}{2}(a_1^+a_1 + a_2^+a_2 + 1),\end{aligned} \tag{10}$$

with the Casimir operator $C = \frac{1}{4}[(a_1^+a_1 - a_2^+a_2)^2 - 1]$ whose eigenvalue is just the difference between the numbers of photons in modes 1 and 2, i.e. $n_1 - n_2$. Denoting the eigenvalue of $a_1^+a_1 - a_2^+a_2$ as q (without loss of generality, q is assumed to be a positive integer), we obtain the Bargmann index as $k = (1+q)/2$. This type of two-mode $SU(1,1)$ CSs can be derived:

$$\begin{aligned}|\xi,q\rangle =&D(\alpha)|0,k\rangle = D(\alpha)|q,0\rangle\\ =&(1-|\xi|^2)^{(1+q)/2}\sum_{n=0}^{\infty}\sqrt{\frac{(n+q)!}{n!q!}}\xi^n|n+q,n\rangle,\end{aligned} \tag{11}$$

and is called the correlated two-mode $SU(1,1)$ CSs.

III. UNCORRELATED TWO-MODE $SU(1,1)$ CSs

We now turn to another type of two-mode case. If a_1 is the annihilation operator for one mode and a_2 for the other, the $SU(1,1)$ Lie algebra may be realized as[23,24]

$$\begin{aligned}K_+ =&\frac{1}{2}(a_1^{+2} + a_2^{+2}),\\ K_- =&\frac{1}{2}(a_1^2 + a_2^2),\\ K_0 =&\frac{1}{2}(a_1^+a_1 + a_2^+a_2 + 1).\end{aligned} \tag{12}$$

In the usual number representation the basis states are $|n_1\rangle$ and $|n_2\rangle$, so that the full Hilbert space of the two-mode field is simply the direct product

$$|n_1,n_2\rangle = |n_1\rangle \otimes |n_2\rangle. \tag{13}$$

However, it is desirable to map these direct product states, or rather subspaces, of these states onto $\mathcal{D}^+(k)$ UIRs of $SU(1,1)$. To do this, we first need to find the Bargmann index k. From the realization of Eq.(12), the Casimir operator can be shown to be

$$\begin{aligned}C =&\frac{1}{4}[a_1^+a_1 + a_2^+a_2 + 2a_1^+a_1a_2^+a_2 - 1\\ &- (a_1^{+2}a_2^2 + a_1^2a_2^{+2})].\end{aligned} \tag{14}$$

We operate the Casimir operator onto the basis states $|n_1,n_2\rangle$ and obtain:

$$\begin{aligned}&C|n_1,n_2\rangle\\ =&\frac{1}{4}[(n_1+n_2+2n_1n_2-1)|n_1,n_2\rangle\\ &-\sqrt{(n_1+1)(n_1+2)n_2(n_2-1)}|n_1+2,n_2-2\rangle\\ &-\sqrt{n_1(n_1-1)(n_2+1)(n_2+2)}|n_1-2,n_2+2\rangle].\end{aligned} \tag{15}$$

From this equation we know that $|n_1, n_2\rangle$ are not the eigenvectors of C for arbitrary n_1 and n_2. However, given n_1 and n_2 are limited to 1 or 0, the Eq.(15) becomes the eigenvalue equation of C. Furthermore, C satisfies

$$C|m,k\rangle = k(k-1)|m,k\rangle. \tag{16}$$

Comparing Eq.(16) with Eq.(15) where n_1 and n_2 are 1 or 0, we will obtain the following equations which the Bargmann index k satisfies:

$$k(k-1) = -1/4, 0, 3/4. \tag{17}$$

Solving the equations and making k positive, the Bargmann index k is equal to $1/2$, 1 or $3/2$.

Now from Eq.(12) we have

$$K_0|n_1,n_2\rangle = \frac{1}{2}(n_1+n_2+1)|n_1,n_2\rangle. \tag{18}$$

Comparing Eq.(18) with $K_0|0,k\rangle = k|0,k\rangle$, one may derive the following relations:

$$|0,k\rangle = \begin{cases} |0,0\rangle & k=1/2, \\ |0,1\rangle \text{ or } |1,0\rangle & k=1, \\ |1,1\rangle & k=3/2. \end{cases} \tag{19}$$

Using Eqs.(5), (6) and (19), we obtained

$$\begin{aligned} \left|\xi,\frac{1}{2}\right\rangle =& D(\alpha)|0,0\rangle = (1-|\xi|^2)^{1/2} \\ & \cdot \sum_{n_1,n_2=0}^{\infty} \frac{\sqrt{(2n_1)!(2n_2)!}}{2^{n_1+n_2}n_1!n_2!} \\ & \cdot \xi^{n_1+n_2}|2n_1,2n_2\rangle, \end{aligned} \tag{20}$$

$$\begin{aligned} |\xi,1\rangle =& D(\alpha)|0,1\rangle = (1-|\xi|^2) \\ & \cdot \sum_{n_1,n_2=0}^{\infty} \frac{\sqrt{(2n_1+1)!(2n_2)!}}{2^{n_1+n_2}n_1!n_2!} \\ & \cdot \xi^{n_1+n_2}|2n_1+1,2n_2\rangle, \end{aligned} \tag{21}$$

$$\begin{aligned} \left|\xi,\frac{3}{2}\right\rangle =& D(\alpha)|1,1\rangle = (1-|\xi|^2)^{3/2} \\ & \cdot \sum_{n_1,n_2=0}^{\infty} \frac{\sqrt{(2n_1+1)!(2n_2+1)!}}{2^{n_1+n_2}n_1!n_2!} \\ & \cdot \xi^{n_1+n_2}|2n_1+1,2n_2+1\rangle, \end{aligned} \tag{22}$$

where we have already considered $|0,1\rangle$ and $|1,0\rangle$ as the same state. These generalized $SU(1,1)$ CSs are called uncorrelated two-mode $SU(1,1)$ CSs.

Moreover, comparing Eqs.(8) and (9) with Eqs.(20) and (21), we find that $\left|\xi,\frac{1}{2}\right\rangle = \left|\xi,\frac{1}{4}\right\rangle_s \otimes \left|\xi,\frac{1}{4}\right\rangle_s$, $|\xi,1\rangle = \left|\xi,\frac{1}{4}\right\rangle_s \otimes \left|\xi,\frac{3}{4}\right\rangle_s$ and $\left|\xi,\frac{3}{2}\right\rangle = \left|\xi,\frac{3}{4}\right\rangle_s \otimes \left|\xi,\frac{3}{4}\right\rangle_s$. From the bosonic realization of the single-mode and this type of two-mode $SU(1,1)$ Lie algebra, i.e. Eqs.(7) and (12), it is obvious that these results are self-consistent.

IV. PROPERTIES OF UNCORRELATED TWO-MODE $SU(1,1)$ CSs

A. The mean photon number

Let $N_1 = \langle a_1^+ a_1\rangle$ and $N_2 = \langle a_2^+ a_2\rangle$ be the mean photon number in modes 1 and 2, respectively. For $\left|\xi,\frac{1}{2}\right\rangle$ one gets

$$N_1 = N_2 = \frac{|\xi|^2}{1-|\xi|^2}. \tag{23}$$

For $|\xi,1\rangle$ one gets

$$N_1 = \frac{1+2|\xi|^2}{1-|\xi|^2}, \tag{24}$$

$$N_2 = \frac{|\xi|^2}{1-|\xi|^2}. \tag{25}$$

For $\left|\xi,\frac{3}{2}\right\rangle$ one gets

$$N_1 = N_2 = \frac{1+2|\xi|^2}{1-|\xi|^2}. \tag{26}$$

From these relations, we know that, for $\left|\xi,\frac{1}{2}\right\rangle$ and $\left|\xi,\frac{3}{2}\right\rangle$, the mean photon number in modes 1 and 2 are equal, while for $|\xi,1\rangle$, the mean photon number in modes 1 and 2 are different. When $|\xi|$ increases, the mean photon in each case increases monotonically. Furthermore, the mean photon number $N_i \to \infty (i=1,2)$ as $|\xi| \to 1$.

The correlation coefficient of modes 1 and 2 is

$$C_{12} = \langle a_1^+ a_1 a_2^+ a_2\rangle - \langle a_2^+ a_2\rangle\langle a_1^+ a_1\rangle. \tag{27}$$

We can calculate $C_{12} = 0$ for each of the above three states, which proves that modes 1 and 2 are uncorrelated. This result is distinguished apparently from the correlated two-mode $SU(1,1)$ CSs.[12]

B. Bunching and anti-bunching

To test for photon bunching and anti-bunching, we consider the second-order zero-time coherence functions for modes 1 and 2, which are defined as:

$$g_1^{(2)}(0) = \frac{\langle a_1^{+2} a_1^2\rangle}{\langle a_1^+ a_1\rangle^2} = \frac{\langle (a_1^+ a_1)^2\rangle - \langle a_1^+ a_1\rangle}{\langle a_1^+ a_1\rangle^2}, \tag{28}$$

$$g_2^{(2)}(0)=\frac{\langle a_2^{+2}a_2^2\rangle}{\langle a_2^+a_2\rangle^2}=\frac{\langle(a_2^+a_2)^2\rangle-\langle a_2^+a_2\rangle}{\langle a_2^+a_2\rangle^2}. \quad (29)$$

Bunching is present whenever $g^{(2)}(0)>1$, and $g^{(2)}(0)<1$ is called photon anti-bunching.
For $\left|\xi,\frac{1}{2}\right\rangle$

$$g_1^{(2)}(0)=g_2^{(2)}(0)=\frac{1+2|\xi|^2}{|\xi|^2}. \quad (30)$$

For $|\xi,1\rangle$

$$g_1^{(2)}(0)=\frac{3|\xi|^2(3+2|\xi|^2)^2}{(1+2|\xi|^2)^2}, \quad (31)$$

$$g_2^{(2)}(0)=\frac{1+2|\xi|^2}{|\xi|^2}. \quad (32)$$

For $\left|\xi,\frac{3}{2}\right\rangle$

$$g_1^{(2)}(0)=g_2^{(2)}(0)=\frac{3|\xi|^2(3+2|\xi|^2)^2}{(1+2|\xi|^2)^2}. \quad (33)$$

For $\left|\xi,\frac{1}{2}\right\rangle$, $g_1^{(2)}(0)=g_2^{(2)}(0)>1$ for all $|\xi|$ in the range $0\le|\xi|<1$, which indicates that $SU(1,1)$ CSs $\left|\xi,\frac{1}{2}\right\rangle$ are bunching in each mode. However, for $|\xi,1\rangle$, $g_1^{(2)}(0)$ may be smaller than unity, which shows that $SU(1,1)$ CSs $|\xi,1\rangle$ in mode 1 will be anti-bunching for some values of $|\xi|$. On the other hand, $g_2^{(2)}(0)>1$ for all $|\xi|$, which shows that $SU(1,1)$ CSs $|\xi,1\rangle$ in mode 2 will be bunching. For $\left|\xi,\frac{3}{2}\right\rangle$, $SU(1,1)$ CSs $\left|\xi,\frac{3}{2}\right\rangle$ in modes 1 and 2 may be anti-bunching. In Fig.1, we plot $g_i^{(2)}(0)$ versus $|\xi|$.

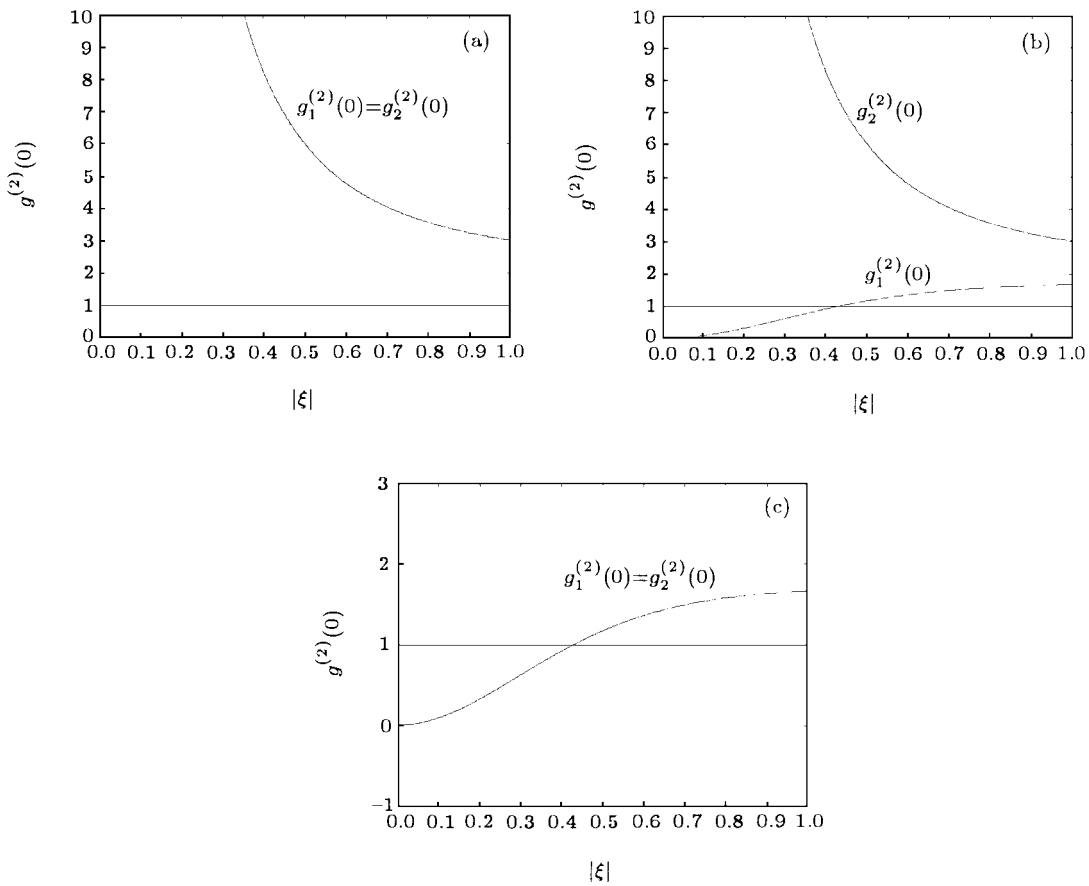


Fig.1. The parameter $g_i^{(2)}(0)$ $(i=1,2)$ which measures bunching and anti-bunching versus $|\xi|$ for different uncorrelated two-mode $SU(1,1)$ CSs: (a) $\left|\xi,\frac{1}{2}\right\rangle$; (b) $|\xi,1\rangle$; (c) $\left|\xi,\frac{3}{2}\right\rangle$.

C. Violations of the Cauchy–Schwarz inequality

The Cauchy–Schwarz inequality is given as [25]

$$\langle a_1^{+2}a_1^2\rangle\langle a_2^{+2}a_2^2\rangle\ge\langle a_1^+a_1a_2^+a_2\rangle^2. \quad (34)$$

Violations of this inequality indicate strong quantum properties. As a measure of this violation we consider the quantity I_0.[26]

$$I_0\equiv\frac{(\langle a_1^{+2}a_1^2\rangle\langle a_2^{+2}a_2^2\rangle)^{1/2}}{|\langle a_1^+a_1a_2^+a_2\rangle|}-1, \quad (35)$$

which becomes negative if inequality (34) is violated. In Fig.2, it is shown that only $SU(1,1)$ CSs $\left|\xi, \frac{3}{2}\right\rangle$ seem to violate the inequality in some range of $|\xi|$. For these states, when $|\xi| \to 0$ there is the greatest violation and the violation decreases as $|\xi|$ increases.

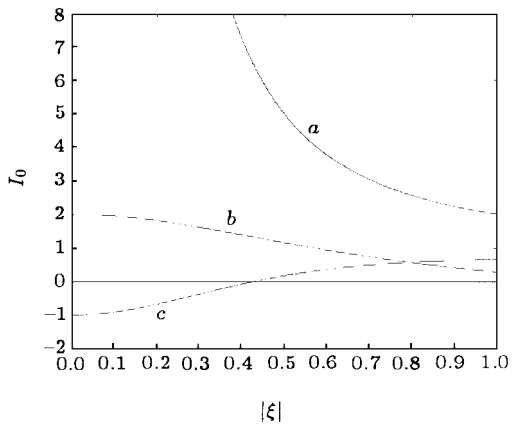


Fig.2. The violation of the Cauchy–Schwarz inequality measured by parameter I_0 for three uncorrelated two-mode $SU(1,1)$ CSs as a function of $|\xi|$: curve a, $\left|\xi, \frac{1}{2}\right\rangle$; curve b, $|\xi, 1\rangle$; curve c, $\left|\xi, \frac{3}{2}\right\rangle$.

D. Squeezing

Finally, we examine the squeezing properties of these states. The two-mode quadrature operators are defined as

$$X_1 \equiv \frac{1}{2^{3/2}}(a_1 + a_1^+ + a_2 + a_2^+), \tag{36}$$

$$X_2 \equiv \frac{i}{2^{3/2}}(a_1 - a_1^+ + a_2 - a_2^+), \tag{37}$$

which satisfy the commutation relation $[X_1, X_2] = i/2$, leading to the uncertainty relation

$$\langle(\Delta X_1)^2\rangle\langle(\Delta X_2)^2\rangle \geq 1/16. \tag{38}$$

Squeezing exists whenever the variance of either quadrature component is less than $1/4$. We therefore characterize the degree of squeezing by the parameters

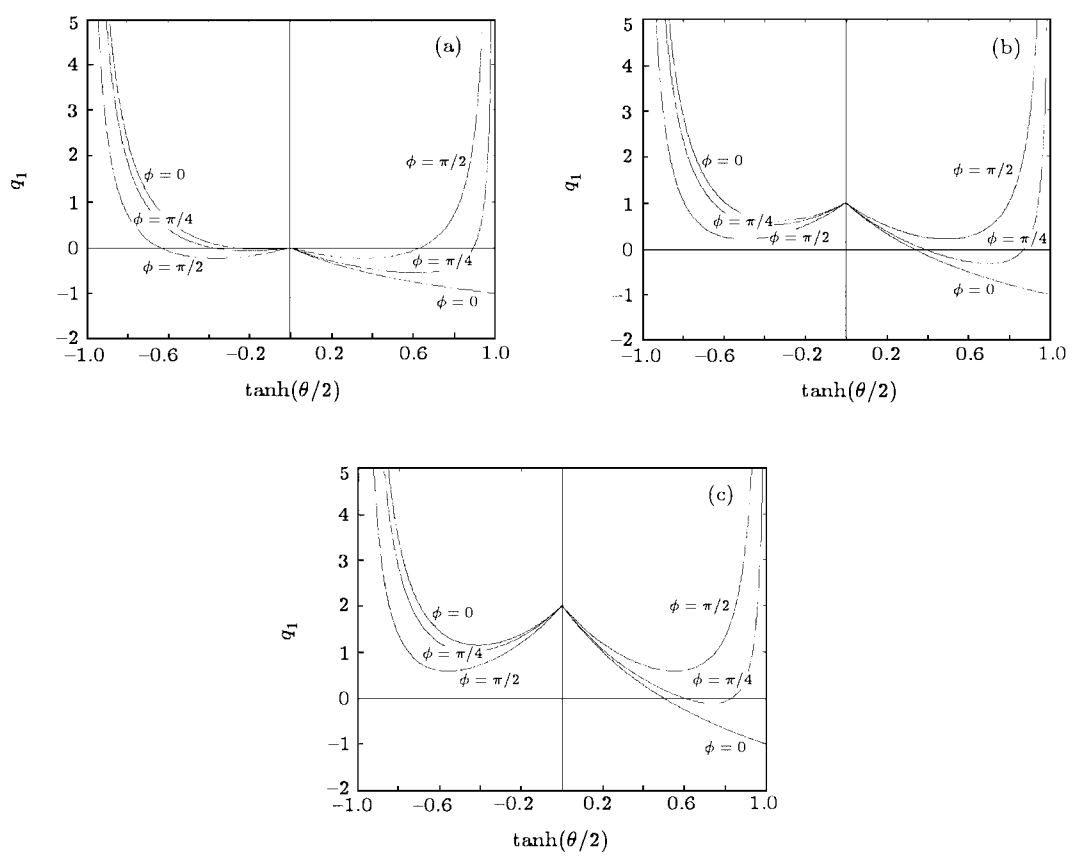


Fig.3. Two-mode squeezing q_1 for different uncorrelated two-mode $SU(1,1)$ CSs and three values of the angle ϕ: (a) $\left|\xi, \frac{1}{2}\right\rangle$; (b) $|\xi, 1\rangle$; (c) $\left|\xi, \frac{3}{2}\right\rangle$.

$$q_i \equiv 4\langle(\Delta X_i)^2\rangle - 1, \quad i = 1, 2, \tag{39}$$

where squeezing exists for $-1 \leq q_i < 0$. A maximum (100%) $SU(1,1)$ squeezing is obtained for $q_i = -1$.

In Fig.3 we plot q_1 for three $SU(1,1)$ CSs, respectively. We find that in each case, the degree of squeezing in the X_1 quadrature depends on the phase ϕ of the complex parameter ξ defined under Eq.(6) as $\xi = -\tanh(\theta/2)\exp(-\mathrm{i}\phi)$. In Fig.3 the phase ϕ was set to $0, \pi/4, \pi/2$. When $\phi = 0$ there is maximum $SU(1,1)$ squeezing in X_1 for three cases. In the range of $\pi/2 \leq \phi \leq 0$ and $\tanh(\theta/2) > 0$, the greater ϕ is, the smaller is the squeezing in X_1. At the same time, for a fixed ξ, the degree of squeezing in X_1 decreases with the increases of the Bargmann index k.

V. CONCLUSIONS

In this paper, we have constructed the uncorrelated two-mode bosonic realization of the $SU(1,1)$ Lie algebra, on the basis of which three $SU(1,1)$ generalized coherent states in two-mode Fock space are derived. For these states the mean photon number and the non-classical properties of photon anti-bunching, violations of the Cauchy–Schwarz inequality and two-mode squeezing are studied. It is shown that the uncorrelated two-mode $SU(1,1)$ CSs may exhibit the non-classical properties which only depend on the value of ξ. These properties are different from those of correlated two-mode $SU(1,1)$ CSs. For example, the non-classical properties of correlated two-mode $SU(1,1)$ CSs not only rely on the value of ξ, but also depend on the value of q (the eigenvalue of $|n_1 - n_2|$). For different values of q and a fixed ξ, they perhaps may exhibit different quantum properties.[12] As for the practical applications of uncorrelated two-mode $SU(1,1)$ CSs, further detailed study is needed.

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