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MULTI-LAMBDA MATTER IN A CHIRAL HADRONIC MODEL*

Guo Hua(郭 华)[a)c)], Yang Shu(杨 树)[a)d)e)],
Hu Xiang(胡 翔)[a)], and Liu Yu-xin(刘玉鑫)[a)-e)]

[a)] *Department of Technical Physics and Key Laboratory of Heavy Ion Physics, Chinese Ministry of Education, Peking University, Beijing* 100871, *China*

[b)] *Department of Physics, Peking University, Beijing* 100871, *China*

[c)] *Centre of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou* 730000, *China*

[d)] *Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing* 100080, *China*

[e)] *China Centre of Advanced Science and Technology (CCAST), Beijing* 100080, *China*

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Multi-lambda matter is investigated in the framework of a chiral hadronic model. It is shown that multi-lambda matter consisting of $\{N,\ \Lambda\}$ is a metastable state as the strangeness per baryon and the density of hadronic matter are varied. The effective lambda mass decreases as the baryon density increases, and remains larger than that of the nucleon.

Keywords: nuclear matter, a chiral Lagrangian, strange hadronic matter
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I. INTRODUCTION

In recent years, some theoretical investigations have shown that different strange hadronic systems possibly exist in nature[1] and the strangeness per baryon of the nuclear system should be considered as a new degree of freedom, just like the ratio N/Z in asymmetric nuclei. In terms of a derivative scalar coupling model, multi-lambda matter appears to be metastable against particle emission in the density region of 0.5–0.7 times the saturation density.[2] Similar results have also been obtained in a quark–meson coupling model, in which the baryon structure is included.[3] Using an extended relativistic mean-field model, the equation of state of the strangeness-rich matter was studied and compared with Dirac–Brueckner calculations. Then the authors suggested that the quark structure of baryons and the chiral symmetry of strong interaction should be considered.[4] Based on a modified quark–meson coupling model, the influences of baryon structure on strange hadronic matter have been extensively studied, and the main conclusion is that strange hadronic matter is a metastable state.[5] However, it is not clear yet whether such a conclusion still holds when the chiral symmetry of strong interaction is incorporated.

Recently, a chiral hadronic model has been proposed in Ref.[6]. This model respects the underling symmetries of QCD, such as chiral symmetry and broken scale invariance. Intensive investigations show that this model can successfully describe the properties of nuclear matter and finite nuclei.[7] By extending this model to the inclusion of the lambda particle, we will study the properties of multi-lambda matter in this paper.

II. THE MODEL

In the chiral hadronic model in Ref.[6], the degree of freedom of the hyperon has not been included. With Λ and its couplings to mesons being taken into account, the Lagrangian density for a chiral hadronic model can be written as

$$\begin{aligned}\mathcal{L} =&\bar{\psi}_N[\mathrm{i}\gamma_\mu\mathcal{D}^\mu + g_A\gamma^\mu\gamma_5 a_\mu - M_N + g_s\phi]\psi_N \\ &+ \bar{\psi}_\Lambda[\mathrm{i}\gamma_\mu\mathcal{D}^\mu + g_A\gamma^\mu\gamma_5 a_\mu - M_\Lambda + g_s^\Lambda\phi]\psi_\Lambda \\ &+ \frac{1}{2}\left[1+\eta\frac{\phi}{S_0}+\cdots\right] \\ &\cdot\left[\frac{1}{2}f_\pi^2\mathrm{Tr}(\partial_\mu U\partial^\mu U^+) + m_v^2 V_\mu V^\mu\right] \\ &- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4!}\xi(g_v^2 V_\mu V^\mu)^2 + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi \\ &- H_q\left(\frac{S^2}{S_0^2}\right)^{2/d}\left(\frac{1}{2d}\ln\frac{S^2}{S_0^2}-\frac{1}{4}\right)+\cdots, \qquad (1)\end{aligned}$$

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where the self-interaction terms of the scalar and vector fields, and their couplings are included. $\mathcal{D}^\mu = \partial_\mu + \mathrm{i}v_\mu + \mathrm{i}g_v V_\mu$ is a chirally covariant derivative for nucleons, while for lambdas g_v is replaced by g_v^Λ. U, v_μ and a_μ are related to the pion field. $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ is the ω field strength tensor; $g_A = 1.23$ and $f_\pi = 93.0$ MeV are the axial coupling constant and the pion-field decay constant, respectively; η and ξ are real constants. The short notation $S(x)$ represents $S_0 - \phi(x)$ with the scalar field ϕ and $m_s^2 = 4H_q/(d^2 S_0^2)$.

In the mean field approximation (MFA) for symmetric nuclear matter, there are no contributions from the π-meson field, and the other meson field operators are replaced by their expection values denoted as $\phi \to \phi_0$, and $V_\mu \to \delta_{\mu 0} V_0$. The Lagrangian density in the MFA can be rewritten as

$$\begin{aligned}\mathcal{L}_{\mathrm{MFA}} =& \bar{\psi}_N[\mathrm{i}\gamma_\mu\partial^\mu - (M_N - g_s\phi_0) - g_v\gamma^0 V_0]\psi_N \\ &+ \bar{\psi}_\Lambda[\mathrm{i}\gamma_\mu\partial^\mu - (M_\Lambda - g_s^\Lambda\phi_0) - g_v^\Lambda\gamma^0 V_0]\psi_\Lambda \\ &+ \frac{1}{2}m_v^2 V_0^2\left(1+\eta\frac{\phi_0}{S_0}\right) + \frac{1}{4!}\xi(g_v V_0)^4 \\ &- H_q\left(1-\frac{\phi_0}{S_0}\right)^{4/d}\left[\frac{1}{d}\ln\left(1-\frac{\phi_0}{S_0}\right) - \frac{1}{4}\right]. \quad (2)\end{aligned}$$

The equations of motion are derived as

$$[\mathrm{i}\gamma_\mu\partial^\mu - (M_N - g_s\phi_0) - g_v\gamma^0 V_0]\psi_N = 0\,, \quad (3)$$

$$[\mathrm{i}\gamma_\mu\partial^\mu - (M_\Lambda - g_s^\Lambda\phi_0) - g_v^\Lambda\gamma^0 V_0]\psi_\Lambda = 0\,, \quad (4)$$

for nucleons and lambdas, and

$$\begin{aligned}&\frac{\eta}{2S_0}m_v^2 V_0^2 + m_s^2 S_0\left(1-\frac{\phi_0}{S_0}\right)^{(\frac{4}{d}-1)}\ln\left(1-\frac{\phi_0}{S_0}\right) \\ &= -\left[g_s\rho_S(N) + g_s^\Lambda\rho_S(\Lambda)\right], \quad (5) \\ &\frac{1}{6}\xi g_v^4 V_0^3 + \left(1+\eta\frac{\phi_0}{S_0}\right)m_v^2 V_0 \\ &= [g_v\rho_B(N) + g_v^\Lambda\rho_B(\Lambda)], \quad (6)\end{aligned}$$

for the σ- and ω-meson fields. The scalar and vector densities are expressed as

$$\begin{aligned}&\rho_S(i) \equiv <\bar{\psi}_i\psi_i>, \\ &\rho_B(i) \equiv <\bar{\psi}_i\gamma^0\psi_i>, \quad i = (N, \Lambda). \quad (7)\end{aligned}$$

The energy density for multi-lambda matter is

$$\begin{aligned}\epsilon =& \frac{4}{(2\pi)^3}\int_0^{K_{\mathrm{F}_N}} \mathrm{d}^3k(k^2 + M_N^{\star 2})^{1/2} \\ &+ \frac{2}{(2\pi)^3}\int_0^{K_{\mathrm{F}\Lambda}} \mathrm{d}^3k(k^2 + M_\Lambda^{\star 2})^{1/2} \\ &+ g_v V_0\rho_B(N) + g_v^\Lambda V_0\rho_B(\Lambda) \\ &- \frac{1}{2}\left(1+\eta\frac{\phi_0}{S_0}\right)m_v^2 V_0^2 - \frac{1}{4!}\xi(g_v V_0)^4 \\ &+ H_q\left\{\left(1-\frac{\phi_0}{S_0}\right)^{4/d}\left[\frac{1}{d}\ln\left(1-\frac{\phi_0}{S_0}\right) - \frac{1}{4}\right] + \frac{1}{4}\right\}, \quad (8)\end{aligned}$$

with the effective nucleon and lambda masses defined as

$$M_N^\star = M_N - g_s\phi_0\,, \qquad M_\Lambda^\star = M_\Lambda - g_s^\Lambda\phi_0\,. \quad (9)$$

The baryon density is given by

$$\rho = \rho_B(N) + \rho_B(\Lambda) = \frac{2}{3\pi^2}K_{\mathrm{F}_N}^3 + \frac{1}{3\pi^2}K_{\mathrm{F}\Lambda}^3, \quad (10)$$

where K_{F_N} and $K_{\mathrm{F}\Lambda}$ are the Fermi momenta for nucleons and lambdas, respectively. The value of strangeness per baryon is

$$Y_\Lambda \equiv \frac{\rho_B(\Lambda)}{\rho_B(N) + \rho_B(\Lambda)}\,, \quad (11)$$

with $Y_\Lambda = 0$ for symmetric nuclear matter, and $Y_\Lambda = 1$ for pure lambda matter. The energy per baryon for multi-lambda matter is expressed as

$$\frac{E}{B} = \frac{\epsilon}{\rho} - (1 - Y_\Lambda)M_N - Y_\Lambda M_\Lambda\,. \quad (12)$$

With the parameter sets for a chiral hadronic model listed in Table 1, Eqs.(3)–(6) can be solved self-consistently, then the properties of multi-lambda matter in a chiral hadronic model can be discussed.

Table 1. Parameter sets of Ref.[6] for a chiral hadronic model. M_N=939 MeV and M_Λ= 1116 MeV are used. The scalar meson mass m_σ and ${H_q}^{1/4}$ are given in MeV.

Sets	g_s^2	m_σ	g_v^2	S_0	ξ	η	d	${H_q}^{1/4}$
T1	99.3	509	154.5	90.6	0.0402	-0.496	2.70	250
T2	96.3	529	138.0	95.6	0.0342	-0.701	2.20	236
T3	109.5	508	178.6	89.8	0.0346	-0.160	3.50	283

Ⅲ. RESULTS AND DISCUSSION

Besides the parameter sets presented in Table 1, the additional coupling constants g_v^Λ and g_s^Λ should be determined. As in Ref.[8] the correct binding energy for the lowest Λ level in nuclear matter at the saturation density satisfies

$$E_\Lambda = g_v^\Lambda V_0 - g_s^\Lambda \phi_0 \approx -28\text{MeV},$$

from which the correlation between g_v^Λ and g_s^Λ is calculated, as plotted in Fig.1. It is seen that a linear relationship between the ratios of g_v^Λ/g_v and g_s^Λ/g_s is given and the differences of these ratios among the parameter sets T1, T2 and T3 are quite small. According to $SU(6)$ symmetry, one chooses $g_v^\Lambda/g_v = 2/3$,[9] and uses the above formulae for $E_\Lambda \approx -28$ MeV. Then the ratios of the additional coupling constants in the following calculations can be fixed to be $g_s^\Lambda/g_s = 0.6112$, 0.6157 and 0.6151 for the parameter sets T1, T2 and T3, respectively.

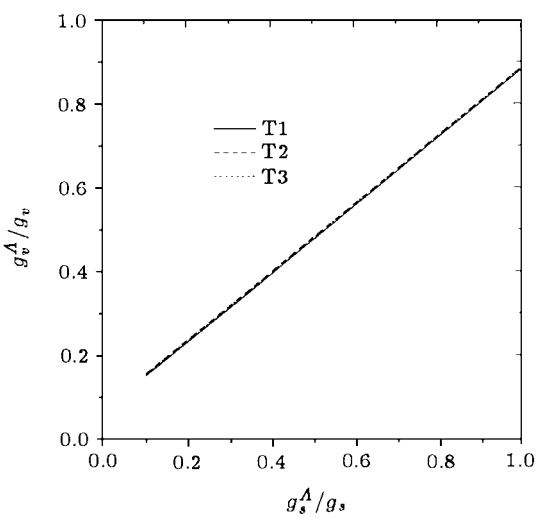


Fig.1. Relation between the two lambda–meson coupling constants.

We plot the energy per nucleon with different lambda fractions for the parameter sets T1, T2 and T3 in Figs.2(a), 2(b) and 2(c), respectively. It is shown that variations of the energy per baryon with different parameter sets are quite similar to each other. These figures display that the absolute minimum of the binding energy per baryon increases until the lambda fraction reaches about 0.1. The system becomes less bound when more lambdas ($Y_\Lambda \geq 0.1$) are added, and then unbound with the lambda fraction $Y_\Lambda \geq 0.68$. These results are generally consistent with those predicted in Refs.[2,3].

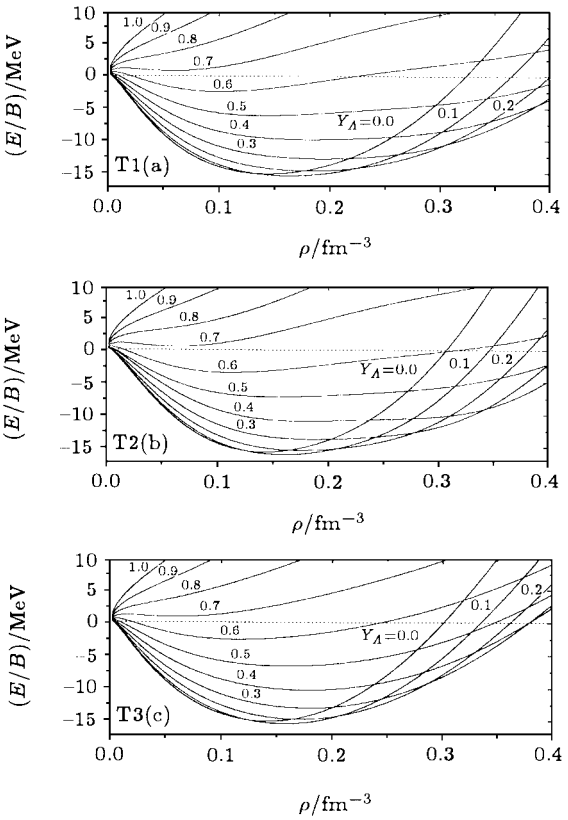


Fig.2. Saturation curves against the baryon density with various Y_Λ.

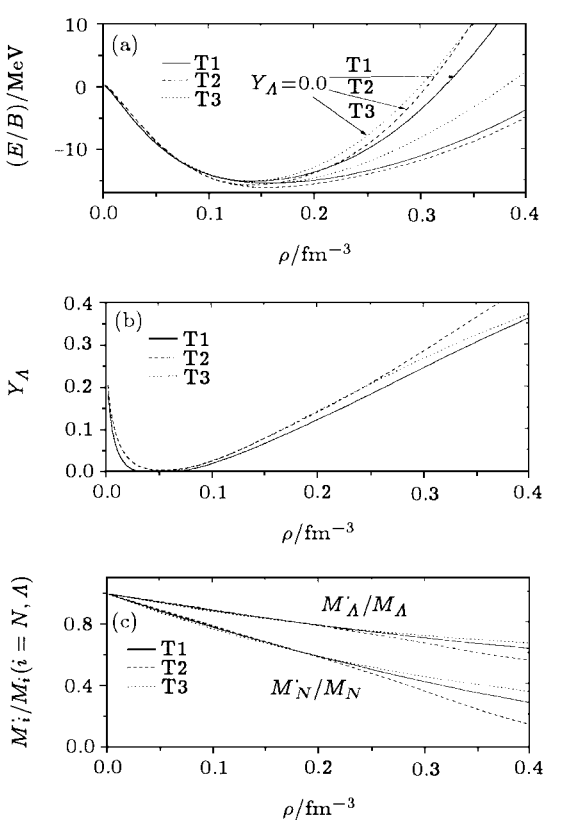


Fig.3. (a) Minimum of the energy per baryon, (b) lambda fractions Y_Λ, and (c) the effective baryonic masses versus the baryon density.

Figure 3 shows the minimum of the energy per baryon, the fractions of lambdas, and the effective baryonic masses as functions of baryon density. It is clear that the absolute minimum of the energy per baryon for the parameter sets T1, T2 and T3 is about -16.5 MeV with $\rho \approx 0.17\,\mathrm{fm}^{-3}$ and $Y_\Lambda \approx 0.1$ (see Fig.3(a)). In the low-density region ($0 \leq \rho \leq 0.075\,\mathrm{fm}^{-3}$), the theoretical values of Y_Λ do not vanish, indicating that lambda particles are present. These theoretical predictions are unrealistic since lambda particles cannot occur in such a low-density region ($0 \leq \rho \leq 0.075\,\mathrm{fm}^{-3}$). Similar theoretical results also appear in other models (see Refs.[2] and [3]). Therefore, it seems reasonable that lambda particles will appear at the baryon density $\rho \geq 0.075\,\mathrm{fm}^{-3}$ in a chiral hadronic model. It is also seen in Figs.3(a) and 3(b) that the differences of the results for E/B or Y_Λ among the curves given by parameter sets T1, T2 and T3 are small in the low-density region ($\rho \leq 0.1\,\mathrm{fm}^{-3}$), and turn out to be large when the density of the hadronic matter is high. Whereas the effective baryonic masses are not sensitive to the variation of the baryon density for the different parameter sets (see Fig.3(c)). The curves of E/B for parameter sets T1, T2 and T3 without lambdas ($Y_\Lambda = 0$) are also plotted in Fig.3(a). In comparison with the curves for $Y_\Lambda \neq 0$, it is obvious that the equations of state for nuclear matter with some lambdas are softer. Figure 3(c) shows that the effective baryonic masses decrease as the baryon density increases, and the effective lambda mass is always larger than that of nucleons.

In short, we conclude that multi-lambda matter could be loosely bound in the framework of a chiral hadronic model. It seems impossible that there is a large fraction of lambdas, since the $\{N, \Lambda\}$ system is unstable for a too large fraction of lambdas. Furthermore, the effective lambda mass is larger than that of nucleons for the $\{\ N,\ \Lambda\ \}$ system.

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