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ON FEEDBACK CONTROL OF DELAYED CHAOTIC SYSTEM*

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In this paper two different types of feedback control technique are discussed: the standard feedback control and the time-delay feedback control which have been successfully used in many control systems. In order to understand to what extent the two different types of control technique are useful in delayed chaotic systems, some analytic stabilization conditions for chaos control from the two types of control technique are derived based on Lyapunov stabilization arguments. Similarly, we discuss the tracking problem by applying the time-delay feedback control. Finally, numerical examples are provided.

Keywords: chaos control, stabilization, Lyapunov equation, feedback control
PACC: 0545

I. INTRODUCTION

In recent years, there has been an increasing interest in controlling chaos, and and research of this has grown. Many control methods[1−19] which can be implemented to control chaos have been presented. For example, there are the Ott–Grebogi–Yorke (OGY)[1] method, the modified OGY method,[2,3] the entrainment and migration control technique,[4,5] the periodic impulsive method,[6−12] the continuous parameter adjusting method,[13−15] the learning control method,[16] the digital filter control method,[17,18] and the fuzzy predictive control method.[19] Especially, the feedback control technique has attracted considerable attention.[20−39]

Many systems involve the time delay. Since Mackey and Glass (M–G) first found chaos in the time-delay system, there has been an increasing interest in delayed chaotic systems.[40−48] Chaotic attractors of a first-order differential delayed equation with a M–G delayed function were studied by Farmer.[40] Recently, Lu and He[42,43] observed and analysed chaotic motion in a first-order differential delayed equation for a piece-wise linear delayed function. Tian and Gao[48] have shown that a simple delay system governed by a first-order differential delayed equation may behave chaotically. In this paper we consider a general time-delay chaotic system which is controlled by two different control techniques. For the stabilization problem some typical conditions are derived by using robust stability theory.

II. CONTROL TECHNIQUES

Let us consider the following general chaotic system with an additional feedback force:

$$\begin{aligned}\dot{x}(t) =& Ax(t) + Bx(t-\tau) + f_1(x(t),t) \\ &+ f_2(x(t-\tau),t) + u(t),\end{aligned} \tag{1}$$

where A, B are constant matrices, τ is the time delay, $u(t)$ is the feedback control input, and f is the nonlinear part.

We suppose that the chaotic system (1) has an unstable periodic orbit $\overline{x}(t)$, and is currently in a chaotic state. The purpose of the feedback is to maintain the system on the initially unstable periodic orbit $\overline{x}(t)$ with only an extremely small force $u(t)$.

Firstly, we discuss the case when the target is an unstable fixed point. We discuss two different types of feedback control methods in succession.

The first method is the standard feedback control

$$u(t) = K(\overline{x}(t) - x(t)), \tag{2}$$

where K is an adjustable coefficient, and $\overline{x}(t)$ is the fixed point. Figure 1 shows the block diagram of this technique.

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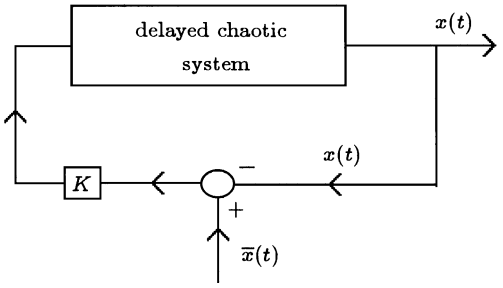


Fig.1. Block diagram of the standard feedback control technique.

Actually, this method has been considered by Huberman and Lumer,[20] and experimentally employed to laminarize chaotic flow in a thermal convection loop by Singer *et al.*[21] Modifications of this technique have been recently demonstrated by the control of unstable steady states in the Lorenz system.[29,30] Namajūnas *et al.*[46] have applied this technique to the M–G system.

The second method is the time-delay feedback control method

$$u(t) = K(x(t-T) - x(t)), \tag{3}$$

where T is the feedback delay time. For $T \ll 1$, the time-delay feedback control technique is equivalent to the derivative control technique.

In recent years, this technique suggested by Pyragas[49] has received much attention.[22−28,31,32,35,36] The block diagram of this technique is shown in Fig.2. From the theoretical standpoint, stabilization by using the control method is an interesting, but difficult, problem. The reason for this is that the dimension of the delay differential equation is infinite. However, recently some theoretical results on the existence of the unstable periodic orbit embedded in the chaotic attractor have been given in Refs.[33,38]. The theoretical results show the limitation of the stabilization on the unstable periodic orbit embedded in the chaotic attractor by using the delayed feedback control method.

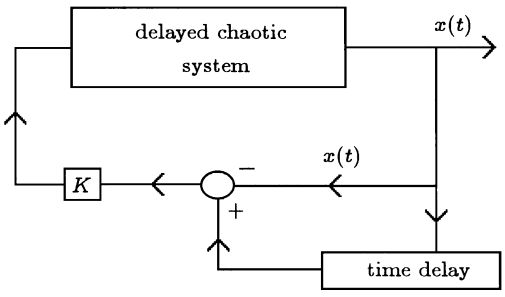


Fig.2. Block diagram of the time-delay feedback control technique.

III. ERROR SYSTEM

1) We suppose that the chaotic system (1) has an unstable fixed point $\overline{x}(t)$=constant. By applying the control technique (2) to (1), we have

$$\begin{aligned}\dot{x}(t) =& Ax(t) + Bx(t-\tau) + f_1(x(t),t) \\ &+ f_2(x(t-\tau),t) + K(\overline{x}(t) - x(t)).\end{aligned} \tag{4}$$

Since $\overline{x}(t)$ is constant, we have

$$\begin{aligned}\dot{\overline{x}}(t) =& A\overline{x}(t) + B\overline{x}(t-\tau) \\ &+ f_1(\overline{x}(t),t) + f_2(\overline{x}(t-\tau),t).\end{aligned} \tag{5}$$

Subtracting (5) from (4) yields the error dynamical system

$$\begin{aligned}\dot{e}(t) =& Ae(t) + Be(t-\tau) \\ &+ F_1(e(t),t) + F_2(e(t-\tau),t) - Ke(t) \\ =& (A-K)e(t) + Be(t-\tau) \\ &+ F_1(e(t),t) + F_2(e(t-\tau),t).\end{aligned} \tag{6}$$

2) By applying the time-delay feedback control, we have

$$\begin{aligned}\dot{x}(t) =& Ax(t) + Bx(t-\tau) + f_1(x(t),t) \\ &+ f_2(x(t-\tau),t) + K(x(t-T) - x(t)).\end{aligned} \tag{7}$$

Similarly, we obtain the following error system

$$\begin{aligned}\dot{e}(t) =& Ae(t) + Be(t-\tau) + F_1(e(t),t) \\ &+ F_2(e(t-\tau),t) + K(x(t-T) - x(t)).\end{aligned} \tag{8}$$

For 1) and 2), where $\overline{x}(t)$ =constant is easily derived,

$$e(t) = x(t) - \overline{x}(t),\ e(t-\tau) = x(t-\tau) - \overline{x}(t-\tau),$$

$$\begin{aligned}F_1(e(t),t) =& f_1(x(t),t) - f_1(\overline{x}(t),t) \\ =& f_1((e(t) + \overline{x}(t)),t) - f_1(\overline{x}(t),t),\end{aligned}$$

$$\begin{aligned}F_2(e(t-\tau),t) =& f_2(x(t-\tau),t) - f_2(\overline{x}(t-\tau),t) \\ =& f_2((e(t-\tau) + \overline{x}(t-\tau)),t) \\ &- f_2(\overline{x}(t-\tau),t),\end{aligned}$$

where $F_1(e(t),t) + F_2(e(t-\tau),t)$ has a zero fixed point $F_1(0,t) + F_2(0,t) = 0$, $(t \geq t_0)$. In the practical control problem, $F_1(e(t),t) + F_2(e(t-\tau),t)$ may be uncertain.

The control objective is to force

$$\| e(t) \| \to 0 \quad \text{as} \quad t \to \infty. \tag{9}$$

IV. STABILIZATION PROBLEM

The stability analysis of the time-delay system is a very difficult and important problem. Many approaches have been proposed, but only limited attention has been given to the stabilization of controlled delay chaotic systems. In practical control problems, it is difficult to know the system structure and parameters precisely. Stability analysis capable of dealing with plant uncertainties is an important issue. Robust control theory provides design tools to deal with uncertain and perturbed parts of the system.

In order to understand to what extent the two different types of feedback control technique are useful, some analytic conditions for the chaos control from the two different types of control approach are derived in this paper based on Lyapunov stabilization arguments.

Firstly, we discuss the control technique (2). For the error system (6), since zero is a fixed point of $F_1(e(t),t)+F_2(e(t-\tau),t)$, we have a Taylor expansion

$$\begin{aligned}&F_1(e(t),t)+F_2(e(t-\tau),t)\\=&\beta_0 e(t)+[\text{H.O.T.}]_1+\beta_1 e(t-\tau)+[\text{H.O.T.}]_2,\end{aligned}$$

where $\beta_0=F_1'(e(t),t)$, $\beta_1=F_2'(e(t-\tau),t)$, $[\text{H.O.T.}]_1$ is the higher-order term in $e(t)$, F_i' is the time derivative of F_i, and $[\text{H.O.T.}]_2$ is the higher-order term in $e(t-\tau)$.

Theorem 1 For the control technique (2), if there exists a real matrix K such that the error system (6) satisfies the conditions

1) $(A-K+\beta_0)$ is asymptotically stable,

2) $c_1 \parallel P \parallel \parallel B+\beta_1 \parallel < \lambda_{\min}(Q)$,

where $c_1 \geq 1$, Q is a positive definite matrix, P is the solution of the following Lyapunov equation

$$P(A-K+\beta_0)+(A-K+\beta_0)^{\mathrm{T}}P=-2Q,$$

and if $\|e(t)\|$ is small enough, then the closed-loop control system described by Eqs.(4) and (5) is robustly asymptotically stable with the control goal (9), i.e.

$$\parallel e(t) \parallel \to 0 \quad \text{as} \quad t\to\infty.$$

Now, we discuss the time-delay feedback control technique (3). For $\overline{x}(t-T)=\overline{x}(t)$, we have

$$\begin{aligned}\dot{\overline{x}}(t)=&A\overline{x}(t)+B\overline{x}(t-\tau_1)+f_1(\overline{x}(t),t)\\&+f_2(\overline{x}(t-\tau_1),t)+K(\overline{x}(t-T)-\overline{x}(t)).\end{aligned}$$

Then the error system (8) becomes

$$\begin{aligned}\dot{e}(t)=&Ae(t)+Be(t-\tau_1)+F_1(e(t),t)\\&+F_2(e(t-\tau_1),t)+K(e(t-T)-e(t))\\=&(A-K)e(t)+Be(t-\tau_1)+Ke(t-\tau_2)+F_1(e(t),t)\\&+F_2(e(t-\tau_1),t),\end{aligned}\tag{10}$$

where $\tau_1=\tau$, $\tau_2=T$.

Since zero is a fixed point of $F_1(e(t),t)+F_2(e(t-\tau),t)$, we have a Taylor expansion

$$\begin{aligned}&F_1(e(t),t)+F_2(e(t-\tau),t)\\=&\beta_0 e(t)+[\text{H.O.T.}]_1+\beta_1 e(t-\tau)+[\text{H.O.T.}]_2,\end{aligned}$$

where $\beta_0=F_1'(e(t),t)$, $\beta_1=F_2'(e(t-\tau),t)$, $[\text{H.O.T.}]_1$ and $[\text{H.O.T.}]_2$ are the higher-order terms.

Theorem 2 For the control technique (3), if there exists a real matrix K such that error system (10) satisfies the conditions

1) $(A-K+\beta_0)$ is asymptotically stable,

2) $c_1 \parallel P \parallel \parallel B+\beta_1 \parallel +c_1 \parallel P \parallel \parallel K \parallel < \lambda_{\min}(Q)$,

where $c_1 \geq 1$, Q is a positive definite matrix, P is the solution of the following Lyapunov equation

$$P(A-K+\beta_0)+(A-K+\beta_0)^{\mathrm{T}}P=-2Q,$$

and if $\|e(t)\|$ is small enough, then the closed loop control system described by Eqs.(5) and (7) is robustly asymptotically stable with the control goal (9), i.e.

$$\parallel e(t) \parallel \to 0 \quad \text{as} \quad t\to\infty.$$

Remark 1 If $\tau_2 \approx \tau_1$, we assume $\tau_2=\tau_1$, then the condition 2) in Theorem 2 can be replaced by $c_1 \parallel P \parallel \parallel B+\beta_1+K \parallel < \lambda_{\min}(Q)$.

In order to prove the theorem, we give a lemma.

Lemma 1[50] For any $x,y\in R^n, \epsilon>0$, we have

$$2x^{\mathrm{T}}y \leq \epsilon x^{\mathrm{T}}x+\frac{1}{\epsilon}y^{\mathrm{T}}y.$$

The proof of Theorem 2 is given below:

[Proof] Construct a Lyapunov equation of the form

$$\begin{aligned}V(x(t))=&x^{\mathrm{T}}(t)Px(t)+\epsilon \parallel P \parallel \parallel B\\&+\beta_1 \parallel \int_{t-\tau_1}^{t} x^{\mathrm{T}}(s)x(s)\mathrm{d}s\\&+\epsilon \parallel P \parallel \parallel K \parallel \int_{t-\tau_2}^{t} x^{\mathrm{T}}(s)x(s)\mathrm{d}s,\end{aligned}$$

then

$$\begin{aligned}
&\dot{V}(x(t))\\
=&2x^{\mathrm{T}}(t)P(A-K)x(t)+2x^{\mathrm{T}}(t)PBx(t-\tau_1)\\
&+2x^{\mathrm{T}}(t)PKx(t-\tau_2)+2x^{\mathrm{T}}(t)P(F_1(x,t)\\
&+F_2(x(t-\tau_1),t))\\
&+\epsilon\parallel P\parallel\parallel B+\beta_1\parallel[x^{\mathrm{T}}(t)x(t)\\
&-x^{\mathrm{T}}(t-\tau_1)x(t-\tau_1)]\\
&+\epsilon\parallel P\parallel\parallel K\parallel[x^{\mathrm{T}}(t)x(t)-x^{\mathrm{T}}(t-\tau_2)x(t-\tau_2)]\\
=&2x^{\mathrm{T}}(t)P(A-K)x(t)+2x^{\mathrm{T}}(t)PBx(t-\tau_1)\\
&+2x^{\mathrm{T}}(t)PKx(t-\tau_2)\\
&+2x^{\mathrm{T}}(t)P\beta_0x(t)+2x^{\mathrm{T}}(t)P\beta_1x(t-\tau_1)\\
&+2x^{\mathrm{T}}(t)P([\mathrm{H.O.T.}]_1+[\mathrm{H.O.T.}]_2)\\
&+\epsilon\parallel P\parallel\parallel B+\beta_1\parallel\\
&\cdot[x^{\mathrm{T}}(t)x(t)-x^{\mathrm{T}}(t-\tau_1)x(t-\tau_1)]\\
&+\epsilon\parallel P\parallel\parallel K\parallel[x^{\mathrm{T}}(t)x(t)-x^{\mathrm{T}}(t-\tau_2)x(t-\tau_2)].
\end{aligned}$$

Using Lemma 1, we have

$$\begin{aligned}
\dot{V}(x(t))\leq&-2\lambda_{\min}(Q)\parallel x(t)\parallel^2\\
&+\frac{1}{\epsilon}\parallel P\parallel\parallel B+\beta_1\parallel\parallel x(t)\parallel^2\\
&+\epsilon\parallel P\parallel\parallel K\parallel\parallel x(t)\parallel^2\\
&+\epsilon\parallel P\parallel\parallel B+\beta_1\parallel\parallel x(t)\parallel^2\\
&+\frac{1}{\epsilon}\parallel P\parallel\parallel K\parallel\parallel x(t)\parallel^2\\
&+2x^{\mathrm{T}}(t)P([\mathrm{H.O.T.}]_1+P[\mathrm{H.O.T.}]_2). \quad (11)
\end{aligned}$$

Let

$$2c_1=\epsilon+\frac{1}{\epsilon}\geq 2. \quad (12)$$

Then, for any small $\|x(t)\|$, we have

$$\begin{aligned}
\dot{V}(x(t))\leq&-2\{\lambda_{\min}(Q)-c_1\parallel P\parallel\parallel B\\
&+\beta_1\parallel-c_1\parallel P\parallel\parallel K\parallel\}\parallel x(t)\parallel.
\end{aligned}$$

From the Lyapunov stability theorem, the error system is robustly asymptotically stable, and the proof is completed.

The proof of Theorem 1 follows from the proof of Theorem 2, and is omitted.

V. TRACKING PROBLEM

Chen and Yu[38] have discussed the stabilization and tracking problem controlled by time-delay feedback, but they only discussed the delay-free system. In this section we extend the method of Chen and Yu to the time-delay chaotic system. We suppose that system (1) has an unstable periodic solution $\overline{x}(t)$ with period $t_{\mathrm{p}}>0$. The main purpose of this section is to apply time-delay feedback control so that the controlled system orbit can track the target:

$$\lim_{t\to\infty}\parallel x(t)-\overline{x}(t)\parallel=0.$$

Since $\overline{x}(t)$ is an unstable periodic solution, we have

$$\begin{aligned}
\dot{\overline{x}}(t)=&A\overline{x}(t)+B\overline{x}(t-\tau)\\
&+f_1(\overline{x}(t),t)+f_2(\overline{x}(t-\tau),t), \quad (13)
\end{aligned}$$

and

$$\begin{aligned}
\dot{\overline{x}}(t-t_{\mathrm{p}})=&A\overline{x}(t-t_{\mathrm{p}})+B\overline{x}(t-t_{\mathrm{p}}-\tau)\\
&+f_1(\overline{x}(t-t_{\mathrm{p}}),t)+f_2(\overline{x}(t-\tau-t_{\mathrm{p}}),t).
\end{aligned}$$

In this case, the error system becomes:

$$\begin{aligned}
\dot{e}'(t)=&Ae'(t)+Be'(t-\tau_1)+F_1(e'(t),t)\\
&+F_2(e'(t-\tau_1),t)-K(x(t)-x(t-\tau_2)), \quad (14)
\end{aligned}$$

where

$$\begin{aligned}
e'(t)=&x(t)-\overline{x}(t-t_{\mathrm{p}}),\\
e'(t-\tau_1)=&x(t-\tau_1)-\overline{x}(t-t_{\mathrm{p}}-\tau_1),\\
F_1(e'(t),t)=&f_1(x(t),t)-f_1(\overline{x}(t-t_{\mathrm{p}}),t),\quad \tau_1=\tau,\\
&F_2(e'(t-\tau_1),t)\\
=&f_2(x(t-\tau_1),t)-f_2(\overline{x}(t-\tau_1-t_{\mathrm{p}}),t),
\end{aligned}$$

and the controlled objective is to force

$$x(t)\to\overline{x}(t)\quad\text{as}\quad t\to\infty. \quad (15)$$

Lemma 2[38] If the controlled orbit $x(t)$ tracks the inherently unstable orbit $\overline{x}(t)$ as $t\to\infty$, and if the matrix K is non-singular, then the delay time τ_2 is (an integer multiple of) the period t_{p} of $\overline{x}(t)$.

We extend the result to the time-delay system, and then have:

Corollary 1 For the time-delay system, Lemma 2 also holds and the delayed time τ_2 is equivalent to the period time t_{p} of $\overline{x}(t)$, then

$$\overline{x}(t)=\overline{x}(t-t_{\mathrm{p}})=x(t-\tau_2).$$

For the error system (14), we have

$$\begin{aligned}
\dot{e}'(t)=&Ae'(t)+Be'(t-\tau_1)+F_1(e'(t),t)\\
&+F_2(e'(t-\tau_1),t)-K(x(t)-x(t-\tau_2))\\
=&Ae'(t)+Be'(t-\tau_1)+F_1(e'(t),t)\\
&+F_2(e'(t-\tau_1),t)-Ke'(t)\\
=&(A-K)e'(t)+Be'(t-\tau_1)\\
&+F_1(e'(t),t)+F_2(e'(t-\tau_1),t). \quad (16)
\end{aligned}$$

Similarly, assuming zero is a fixed point of $F_1(e'(t),t)+F_2(e'(t-\tau),t)$, we have a Taylor expansion

$$\begin{aligned}&F_1(e'(t),t)+F_2(e'(t-\tau),t)\\=&\beta_0 e'(t)+[\text{H.O.T.}]_1'+\beta_1 e'(t-\tau)+[\text{H.O.T.}]_2',\end{aligned}$$

where $\beta_0=F_2'(e'(t),t)$, $\beta_1=F_2'(e'(t-\tau),t)$, $[\text{H.O.T.}]_1'$ and $[\text{H.O.T.}]_2'$ are the higher-order terms.

Theorem 3 For the error dynamical system (16), if the following conditions are satisfied:

1) $(A-K+\beta_0)$ is asymptotically stable,

2) $c_1 \parallel P \parallel \parallel B+\beta_1 \parallel < \lambda_{\min}(Q)$,

where $c_1 \geq 1$, Q is a positive definite matrix, P is the solution of the following Lyapunov equation

$$P(A-K+\beta_0)+(A-K+\beta_0)^{\mathrm{T}}P=-2Q,$$

then the closed-loop control system described by Eqs.(7) and (13) is robustly asymptotically stable with the control goal (15), i.e.

$$x(t)\to\overline{x}(t) \quad \text{as} \quad t\to\infty.$$

This follows directly from the proof of Theorem 2. In order to achieve a successful tracking, the period $t_{\mathrm{p}}=\tau$ has to be known before hand. Chen and Yu[38] suggested a method for estimating an unknown period constant.

VI. NUMERICAL ILLUSTRATIONS

We give two numerical control examples of the time-delay systems in order to test the proposed control systems.

Example 1 The time-delay Logistic system is given as[48]

$$\dot{x}(t)=-\lambda x(t)+rx(t-\tau)(1-x(t-\tau)). \qquad (17)$$

This system has two unstable fixed points

$$\overline{x}_0=0, \quad \overline{x}_1=1-\frac{\lambda}{r}.$$

In this example, we shall stabilize the unstable fixed point $\overline{x}_1$. From Ref.[48], when $\lambda=26$, $r=104$ and $\tau=0.5$, the system is chaotic, and the system has an unstable fixed point $\overline{x}_1=0.75$. The chaotic behaviour of this system is shown in Figs.3 and 4.

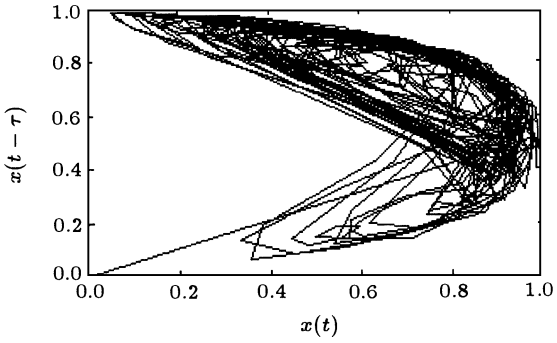


Fig.3. The phase plane simulations of chaotic behaviour for $\lambda=26$ and $r=104$.

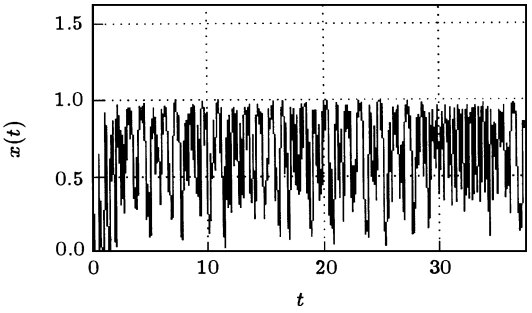


Fig.4. The time simulations of chaotic behaviour for $\lambda=26$ and $r=104$.

We have previously discussed the two control methods. For the first control method, we easily obtain

$$\begin{aligned}\dot{e}(t)=&(-26-K)e(t)+104e(t-\tau)\\&-104e(t-\tau)(e(t-\tau)+1.5),\end{aligned}$$

for

$$\begin{aligned}&F_1(e(t),t)+F_2(e(t-\tau),t)\\=&\beta_0 e(t)+[\text{H.O.T.}]_1+\beta_1 e(t-\tau)+[\text{H.O.T.}]_2,\end{aligned}$$

then $\beta_0=0$, $\beta_1=-156$.

According to Theorem 1, we let $c_1=1.001$, then we have $K>26.052$. It is clear that the choice of $K=100$ satisfies the required condition. Figure 5 shows the result of this control method.

For the time-delay feedback control technique (3), we let $c_1=1.001$, and if $\tau\neq\tau_2$ it follows from Theorem 2 that the control parameter K does not exist. In our simulation process, when $\tau\neq\tau_2$, we can hardly find K such that the system is stable. This exactly proves the effectiveness of our main results. Also, if $\tau=\tau_2=0.5$, we let $c_1=1.001$ according to Remark 1 and we have $13.01<K<78052$. The choice of $K=100$ satisfies the required condition. The result of this control method is shown in Fig.6.

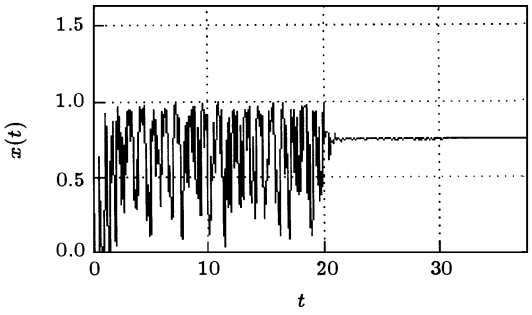


Fig.5. Stabilization of the unstable equilibrium point by using the standard feedback control method.

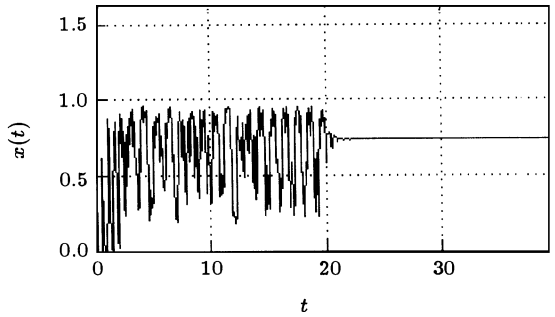


Fig.6. Stabilization of the unstable equilibrium point by using the time-delay feedback control method.

Assuming that the nonlinear part has a parameter error, then $x^2(t-\tau)$ becomes $\delta x^2(t-\tau)$. If $\|1-\delta\|$ is not big enough, we have

$$\begin{aligned}&F_1(e(t),t)+F_2(e(t-\tau),t)\\=&\beta_{01}e(t)+[\text{H.O.T.}]_{11}+\beta_{12}e(t-\tau)+[\text{H.O.T.}]_{22}\\=&\beta_0 e(t)+\beta_1 e(t-\tau)+[\text{H.O.T.}]_{11}+[\text{H.O.T.}]_{22}+\nabla,\end{aligned}$$

where $[\text{H.O.T.}]_{11}+[\text{H.O.T.}]_{22}+\nabla$ is small enough. According to the theorems we do not need to change the control parameter to obtain a satisfactory control result. Figures 7–10 show the controlled results when $\delta=0.9$ and $\delta=1.2$ by using two different types of feedback control method.

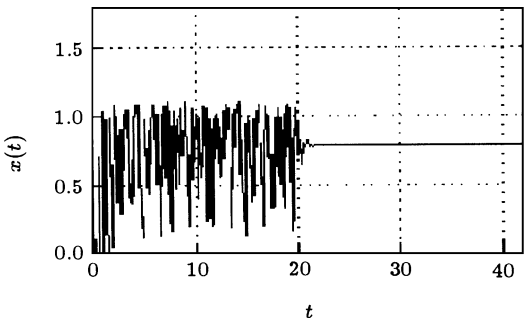


Fig.7. Stabilization of the unstable equilibrium point when uncertain parameter $\delta=0.9$, using the standard feedback control with $K=100$.

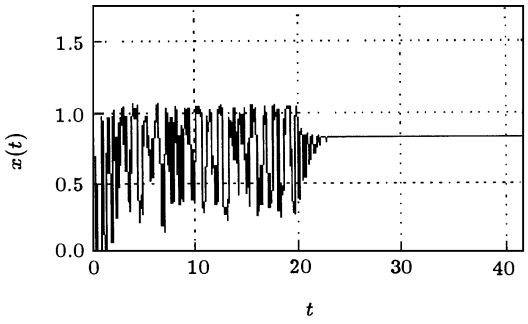


Fig.8. Stabilization of the unstable equilibrium point when uncertain parameter $\delta=0.9$, using the time-delay feedback control with $K=100$ and $\tau_2=0.5$.

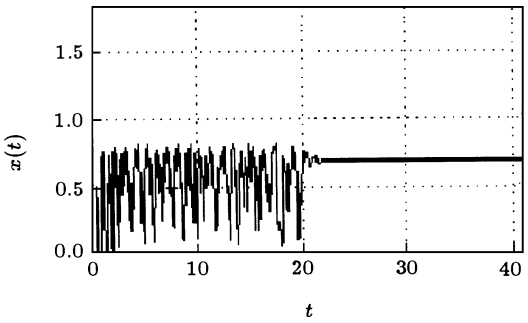


Fig.9. Stabilization of the unstable equilibrium point when uncertain parameter $\delta=1.2$, using the standard feedback control with $K=100$.

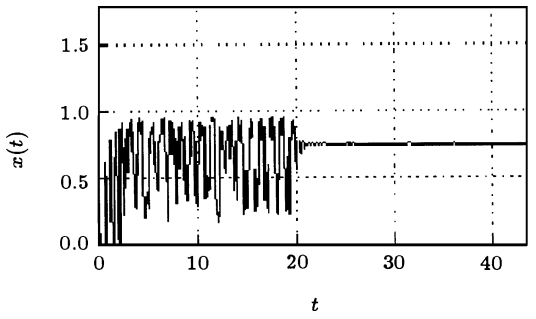


Fig.10. Stabilization of the unstable equilibrium point when uncertain parameter $\delta=1.2$, using the time-delay feedback control with $K=100$ and $\tau_2=0.5$.

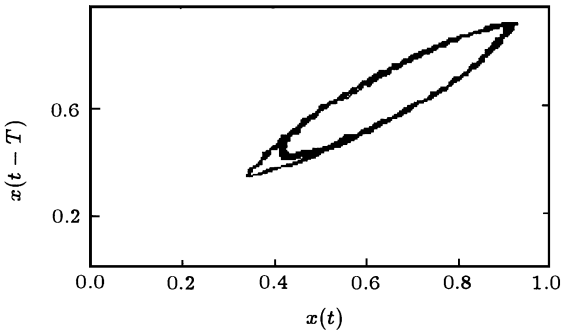


Fig.11. The phase plane simulations of controlled period-three orbit with $t_{\text{p}}=0.9999$, using the time-delay feedback control with $t_{\text{p}}=\tau_2=0.9999$ and $K=100$.

Now, we discuss the tracking problem. By applying the estimating method,[38] and according to Theorem 3, we let $c_1 = 1.001$, and we have $K > 26.052$. We know that the system exhibits a period-three oscillation with period $t_p = 0.9999$. We choose $t_p = \tau_2$ and $K = 100$; the corresponding results are shown in Figs.11 and 12.

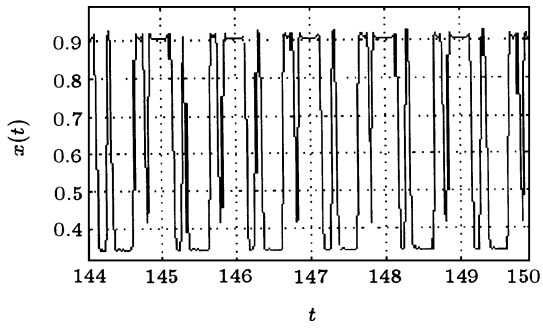


Fig.12. The time simulations of controlled period-three orbit with $t_p = 0.9999$, using the time-delay feedback control with $t_p = \tau_2 = 0.9999$ and $K = 100$.

Example 2 The M–G system[51]

$$\dot{x}(t) = -cx(t) + ax(t-\tau)/(1 + x(t-\tau)^b).$$

This system has two unstable fixed points

$$\overline{x}_0 = 0, \quad \overline{x}_1 = \left(1 - \frac{a}{c}\right)^{1/b}.$$

In this example, we shall stabilize the unstable fixed point $\overline{x}_0 = 0$. When $c = 1$, $a = -2$, $b = 10$ and $\tau = 10$, the system is chaotic. The chaotic behaviour of this system is shown in Fig.13.

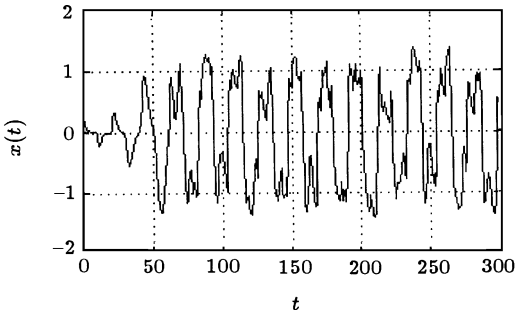


Fig.13. The time simulation of chaotic behaviour for $a = -2$, $b = 10$ and $c = 1$.

For the first control method, we easily obtain

$$\dot{e}(t) = (-1-k)e(t) - 2e(t-\tau)/[1 + (e(t-\tau))^{10}],$$

for

$$\begin{aligned} &F_1(e(t),t) + F_2(e(t-\tau),t) \\ =&\beta_0 e(t) + [\text{H.O.T.}]_1 + \beta_1 e(t-\tau) + [\text{H.O.T.}]_2, \end{aligned}$$

then $\beta_0 = 0$, $\beta_1 = -2$.

According to Theorem 1, we let $c_1 = 1.01$, then we have $K > 1.01$. It is clear that the choice of $K = 30$ satisfies the required condition. Figure 14 shows the result of this control method.

For the time-delay feedback control technique (3), we let $c_1 = 1.01$. If $\tau \neq \tau_2$, it follows from Theorem 2 that the control parameter K does not exist. In our simulation process, when $\tau \neq \tau_2$, we can hardly find K such that the system is stable. This exactly proves the effectiveness of our main results. If the $\tau = \tau_2 = 10$, we let $c_1 = 1.01$, according to note 1, and we have $0.49 < K < 302$. The choice of $K = 2$ satisfies the required condition. The result of this control method is shown in Fig.15.

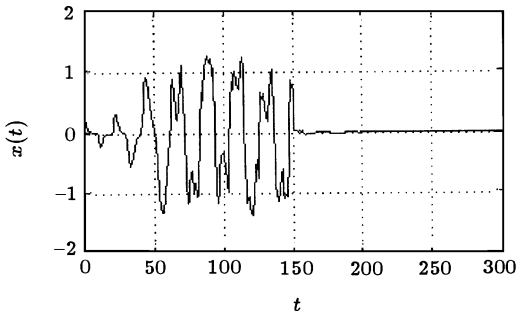


Fig.14. Stabilization of the unstable equilibrium point by using the standard feedback control with $K = 30$.

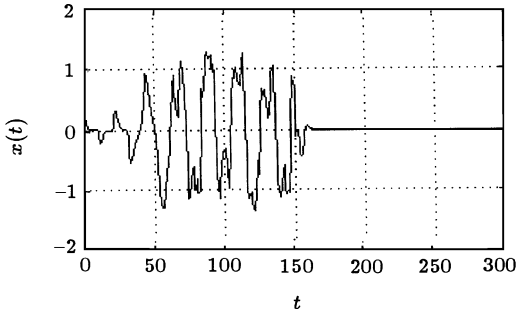


Fig.15. Stabilization of the unstable equilibrium point by using the time-delay feedback control with $K = 2$.

Assuming that the nonlinear part has a parameter error, then $x(t-\tau)/[1 + x(t-\tau)^{10}]$ becomes $\delta x(t-\tau)/[1 + x(t-\tau)^{10}]$. If $\|1 - \delta\|$ is not big enough, we have

$$\begin{aligned} &F_1(e(t),t) + F_2(e(t-\tau),t) \\ =&\beta_{01} e(t) + [\text{H.O.T.}]_{11} + \beta_{12} e(t-\tau) + [\text{H.O.T.}]_{22} \\ =&\beta_0 e(t) + \beta_1 e(t-\tau) + [\text{H.O.T.}]_{11} + [\text{H.O.T.}]_{22} + \nabla, \end{aligned}$$

where $[\text{H.O.T.}]_{11} + [\text{H.O.T.}]_{22} + \nabla$ is small enough. According to the theorems we do not need to change the control parameter to obtain the satisfactory control result. Figures 16–19 show the controlled results when $\delta = 0.9$ and $\delta = 1.2$ by using two different types of feedback control method.

Now, we discuss the tracking problem. By applying the estimating method,[38] and according to Theorem 3, we let $c_1 = 1.01$ and we have $K > 1.01$. We know that the system exhibits a period-two oscillation with period $t_\mathrm{p} = 3.1$ and we choose $t_\mathrm{p} = \tau_2$, $K = 30$; the corresponding results are shown in Figs.20 and 21.

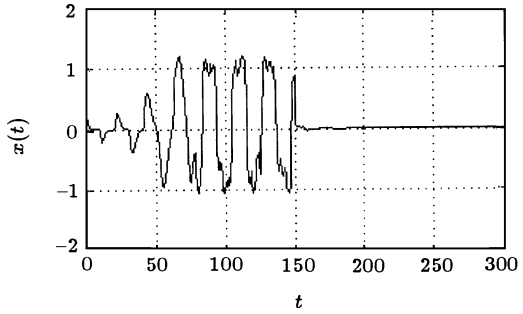


Fig.16. Stabilization of the unstable equilibrium point when uncertain parameter $\delta = 0.9$, using the standard feedback control with $K = 30$.

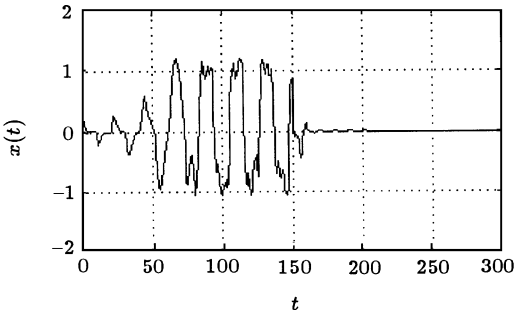


Fig.17. Stabilization of the unstable equilibrium point when uncertain parameter $\delta = 0.9$, using the time-delay feedback control with $K = 2$ and $\tau_2 = 10$.

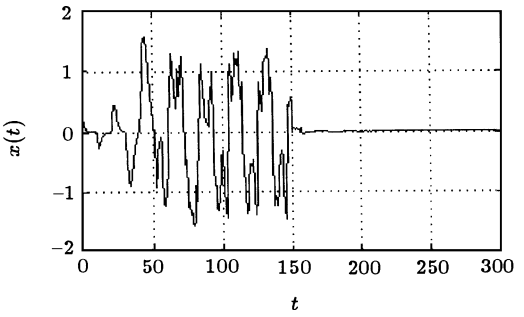


Fig.18. Stabilization of the unstable equilibrium point when uncertain parameter $\delta = 1.2$, using the standard feedback control with $K = 30$.

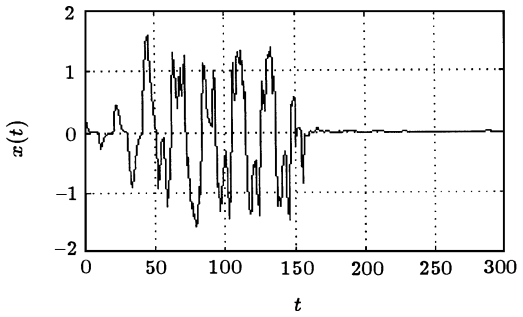


Fig.19. Stabilization of the unstable equilibrium point when uncertain parameter $\delta = 1.2$, using the time-delay feedback control with $K = 2$ and $\tau_2 = 10$.

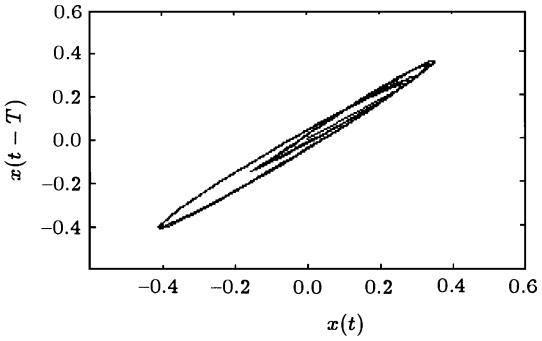


Fig.20. The phase plane simulations of controlled period-two orbit with $t_\mathrm{p} = 3.1$, using the time-delay feedback control with $t_\mathrm{p} = \tau_2 = 3.1$ and $K = 30$.

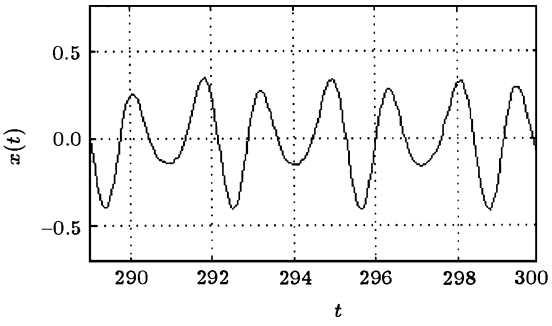


Fig.21. The time simulations of controlled period-two orbit with $t_\mathrm{p} = 3.1$, using the time-delay feedback control with $t_\mathrm{p} = \tau_2 = 3.1$ and $K = 30$.

VII. CONCLUSION

Many control methods have been used in the delayed chaotic system, but only limited attention has been given to the stability analysis of the controlled delay chaotic system. In this paper, two different types of feedback control methods, which are used to stabilize the unstable steady state in the delayed chaotic system, are considered, and we also provide the theoretical analysis of the close-loop controlled delay system, based on the Lyapunov stability theorem.

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