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SQUEEZING PROPERTIES OF A TRAPPED ION IN THE STANDING-WAVE LASER*

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We investigate the squeezing properties of a trapped ion in a standing-wave laser. Our results show that the squeezing of a trapped ion in the standing-wave laser is dependent on its position in the latter, the detuning parameter and the initial average phonon number.

Keywords: squeezing, trapped ion, standing-wave laser, detuning parameter
PACC: 4250V, 4250, 3290, 0365

I. INTRODUCTION

The squeezing[1] of light and the atom has been a centre of attraction in many theoretical, as well as experimental, studies in the quantum optics field for the past few years. Owing to its potential applications in high-resolution spectroscopy,[2] high-precision atomic fountain clocks,[3] high-precision spin polarization measurements,[4] and generation of squeezed light,[5] squeezing, in particular that of the atom, has been extensively studied. Very recently, attention has been drawn to the squeezing in an ensemble of atoms illuminated with squeezed light,[6] quantum-controlled few-photon states generated by squeezed atoms,[7] and quantum noise and atomic spin polarization measurements.[6]

On the other hand, there has recently been a great deal of interest in trapped ions interacting with laser beams, due to potential applications in the preparation of non-classical states of the vibrational motion of ions,[8−11] precision spectroscopy,[11] quantum computation,[12] and quantum state reconstruction.[13,14] Some authors have extensively investigated many dynamical properties of the trapped ion system, but less attention has been paid to the squeezing properties of these ions. In this paper, we investigate the squeezing properties of the trapped ions in a standing-wave laser by transforming the trapped ion system to the Jaynes–Cummings type system under the Lamb–Dicke limit.

II. THE MODEL

Here, we consider a single two-level ion trapped in a harmonic potential of frequency ν and interacting with a standing-wave laser. When the trap frequency ν is much greater than the ionic decay rate, the Hamiltonian is[15]

$$H = \nu a^+ a + \frac{\Delta}{2}\sigma_z + \frac{\Omega_0}{2}\sigma_x \cos[\eta(a + a^+) + \phi], \quad (1)$$

where a, a^+ are the annihilation and creation operators for the phonons or the vibrational states of the trap, $\Delta = \omega_0 - \omega_L$ is the detuning between the transition and laser frequencies, $\sigma_x = \sigma_{21} + \sigma_{12}$ and $\sigma_z = \sigma_{22} - \sigma_{11}$ are the two-level polarization and inversion operators, respectively, Ω_0 is the Rabi frequency proportional to the amplitude of the standing-wave laser, η is the Lamb–Dicke parameter, and ϕ accounts for the relative position of the ion in the standing-wave laser. In particular, $\phi = \pi/2$ corresponds to an ion centred at a node of the standing-wave laser. We expand Eq.(1) in powers of the Lamb–Dicke parameter η to obtain

$$\begin{aligned} H =& \nu a^+ a + \frac{\Delta}{2}\sigma_z + \frac{\Omega_0}{2}\cos(\phi)\sigma_x \\ & - \eta\frac{\Omega_0}{2}\sin(\phi)\sigma_x(a + a^+) + O(\eta^2), \end{aligned} \quad (2)$$

where higher-order terms of η are negligible in the Lamb–Dicke limit. We have recently shown that by introducing a unitary transform operator,[16]

$$T = x\sigma_x + y\sigma_z, \quad (3)$$

with

$$\begin{aligned} x = \sqrt{\frac{\Delta + \Omega}{2\Omega}}, \quad y = \frac{\Omega_0 \cos(\phi)}{\sqrt{2\Omega(\Delta + \Omega)}}, \\ \Omega = \sqrt{\Delta^2 + \Omega_0^2 \cos^2(\phi)}, \end{aligned} \quad (4)$$

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the Hamiltonian (2) can be transformed into the standard Jaynes–Cummings model (JCM) form:

$$\begin{aligned} H' =& THT^{+} \\ =& \nu a^{+}a + \frac{\Omega}{2}\sigma_z + g(\sigma_{21}a + \sigma_{12}a^{+}) \\ =& H_{JCM}, \end{aligned} \tag{5}$$

where $g = \eta\dfrac{\Omega_0\Delta}{2\Omega}\sin(\phi)$. Therefore, we can directly use the results of the normal JCM in studying the dynamics of the trapped ion in any position of the standing-wave laser. For example, the time evolution operator of the trapped ion system can be obtained by applying the unitary transformation T to the JCM evolution operator $U_{\rm JCM}$ as

$$U_{\rm ion} = T^{+}U_{\rm JCM}(t)T, \tag{6}$$

with

$$U_{JCM} = \begin{bmatrix} \mathrm{e}^{-i\nu(aa^{+}+1/2)t}\left[\cos(At) - i\dfrac{\delta}{2}\dfrac{\sin(At)}{2A}\right] & -ig\mathrm{e}^{-i\nu(aa^{+}+1/2)t}a\dfrac{\sin(Bt)}{B} \\ -ig\mathrm{e}^{-i\nu(a^{+}a-1/2)t}a^{+}\dfrac{\sin(At)}{A} & \mathrm{e}^{-i\nu(a^{+}a-1/2)t}\left[\cos(Bt) + i\dfrac{\delta}{2}\dfrac{\sin(Bt)}{B}\right] \end{bmatrix}, \tag{7}$$

where $A = \sqrt{\delta^2/4 + g^2aa^{+}}$, $B = \sqrt{\delta^2/4 + g^2a^{+}a}$, and $\delta = \Omega - \nu$. Correspondingly, the total density operator is given by

$$\rho(t) = U_{\rm ion}(t)\rho(0)U^{+}_{\rm ion}(t). \tag{8}$$

For the ion initially in the ground state and the phonons in a coherent state, $\rho(0)$ is of the form

$$\rho(0) = \begin{bmatrix} 0 & 0 \\ 0 & \rho_p(0) \end{bmatrix}, \tag{9}$$

with

$$\rho_p(0) = \sum_{n,m}^{\infty} P_n P^{*}_m |n\rangle\langle m|, \tag{10}$$

and

$$P_n = \exp\left(-\frac{1}{2}|\alpha|^2\right)\frac{\alpha^n}{\sqrt{n!}}, \tag{11}$$

where $\alpha = \sqrt{\bar{n}}\exp(i\beta), \bar{n}$ is the initial average phonon number, and β is the phase of the phonon coherent state. The total density operator after a straightforward calculation can be expressed in the form:

$$\rho(t) = \begin{bmatrix} |D\rangle\langle D| & |D\rangle\langle E| \\ |E\rangle\langle D| & |E\rangle\langle E| \end{bmatrix}, \tag{12}$$

where

$$\begin{aligned} |D(t)\rangle =& \sum_{n=0}^{\infty} P_n \exp(-in\nu t)\Big\{\exp\left(-i\frac{\nu t}{2}\right) \\ & \cdot [xy\zeta_n|n\rangle - iy^2\eta_n|n+1\rangle] \\ & - \exp\left(i\frac{\nu t}{2}\right)[xy\zeta_{n-1}|n\rangle - ix^2\eta_{n-1}|n-1\rangle]\Big\}, \end{aligned} \tag{13}$$

$$\begin{aligned} |E(t)\rangle =& \sum_{n=0}^{\infty} P_n \exp(-in\nu t)\Big\{\exp\left(-i\frac{\nu t}{2}\right) \\ & \cdot [y^2\zeta_n|n\rangle + ixy\eta_n|n+1\rangle] \\ & + \exp\left(i\frac{\nu t}{2}\right)[x^2\zeta_{n-1}|n\rangle + ixy\eta_{n-1}|n-1\rangle]\Big\}, \end{aligned} \tag{14}$$

with

$$\begin{aligned} \zeta_n &= \cos A_n t - i\frac{\delta}{2}\frac{\sin A_n t}{A_n}, \\ \eta_n &= \frac{g\sqrt{n+1}\sin A_n t}{A_n}, \\ A_n &= \sqrt{\frac{\delta^2}{4} + g^2(n+1)}. \end{aligned} \tag{15}$$

Tracing $\rho(t)$ over the phonon variables gives rise to the reduced ionic density operator

$$\rho_{\rm ion} = \begin{bmatrix} \langle D(t)|D(t)\rangle & \langle E(t)|D(t)\rangle \\ \langle D(t)|E(t)\rangle & \langle E(t)|E(t)\rangle \end{bmatrix}. \tag{16}$$

In what follows, we shall explore the squeezing properties of the trapped ion in a standing-wave laser.

Ⅲ. SQUEEZING PROPERTIES OF A TRAPPED ION

To analyse the squeezing properties of a trapped ion, we define the two Hermitian quadrature operators

$$\begin{aligned} S_1 &= \frac{1}{2}(S_{+} + S_{-}), \\ S_2 &= \frac{1}{2i}(S_{+} - S_{-}). \end{aligned} \tag{17}$$

In fact, S_1 and S_2 correspond to the in-phase and out-of-phase components of the amplitude of the ionic polarization, respectively. They obey the commutation relation

$$[S_1, S_2] = iS_3. \tag{18}$$

Correspondingly, the Heisenberg uncertainty relation is

$$(\Delta S_1)^2(\Delta S_2)^2 \geq \frac{1}{4}|\langle S_3\rangle|^2, \tag{19}$$

where $(\Delta S_l)^2 = \langle S_l^2\rangle - \langle S_l\rangle^2$ is the variance in the component $S_l (l = 1 \text{ or } 2)$ of the ionic dipole.

The fluctuations in the component S_l ($l = 1$ or 2) of the ionic dipole are said to be squeezed if S_l satisfies the condition

$$V(S_l) = (\Delta S_l)^2 - \frac{1}{2}|S_3| < 0, \quad (l = 1 \text{ or } 2). \tag{20}$$

By using the reduced ionic density operator given by Eq.(16), the expectation values of the ionic operators S_l ($l = 1$, 2 and 3) are given by

$$\langle S_1\rangle = \mathrm{Re}\langle D|E\rangle, \qquad \langle S_2\rangle = -\mathrm{Im}\langle D|E\rangle,$$
$$\langle S_3\rangle = \frac{1}{2}(\langle D|D\rangle - \langle E|E\rangle). \tag{21}$$

Substituting Eq.(21) into Eq.(20) gives

$$V(S_1) = \frac{1}{4} - (\mathrm{Re}\langle D|E\rangle)^2 - \frac{1}{4}|\langle D|D\rangle - \langle E|E\rangle|,$$
$$V(S_2) = \frac{1}{4} - (\mathrm{Im}\langle D|E\rangle)^2 - \frac{1}{4}|\langle D|D\rangle - \langle E|E\rangle|. \tag{22}$$

We will discuss the squeezing properties of the trapped ion in the two cases of $\delta = \Omega - \nu = -\Omega$ and $\delta = 0$ for the ion located at different positions in the standing-wave laser.

Figure 1 presents the time dependence of $V(S_2)$, indicating the squeezing in S_2 for $\Omega_0 = 6\pi\times10^5\mathrm{s}^{-1}$,[17] $\Delta = \Omega_0/20, \eta = 0.01$. Here, the case when the trapped ion is located at the node of the standing-wave laser ($\phi = \pi/2$) is considered. Figures 1(a) and 1(b) show the time evolution of $V(S_2)$ under the "resonant" condition $\delta = 0$, for the initial average phonon number $\bar{n} = 1$ and $\bar{n} = 25$, respectively. It can be seen that at the early stage of the time evolution, the quantum fluctuation in S_2 of the trapped ion can be squeezed, but the time of squeezing is decreased with an increase in the initial average phonon number $\bar{n}$. In Figs.1(c) and 1(d), the off-resonant cases $\delta = -\Omega$ are considered for the initial average phonon number $\bar{n} = 1$ and 25, respectively. It is observed that, for the off-resonant case, the squeezing time of the trapped ion is long, compared to that under the resonant condition.

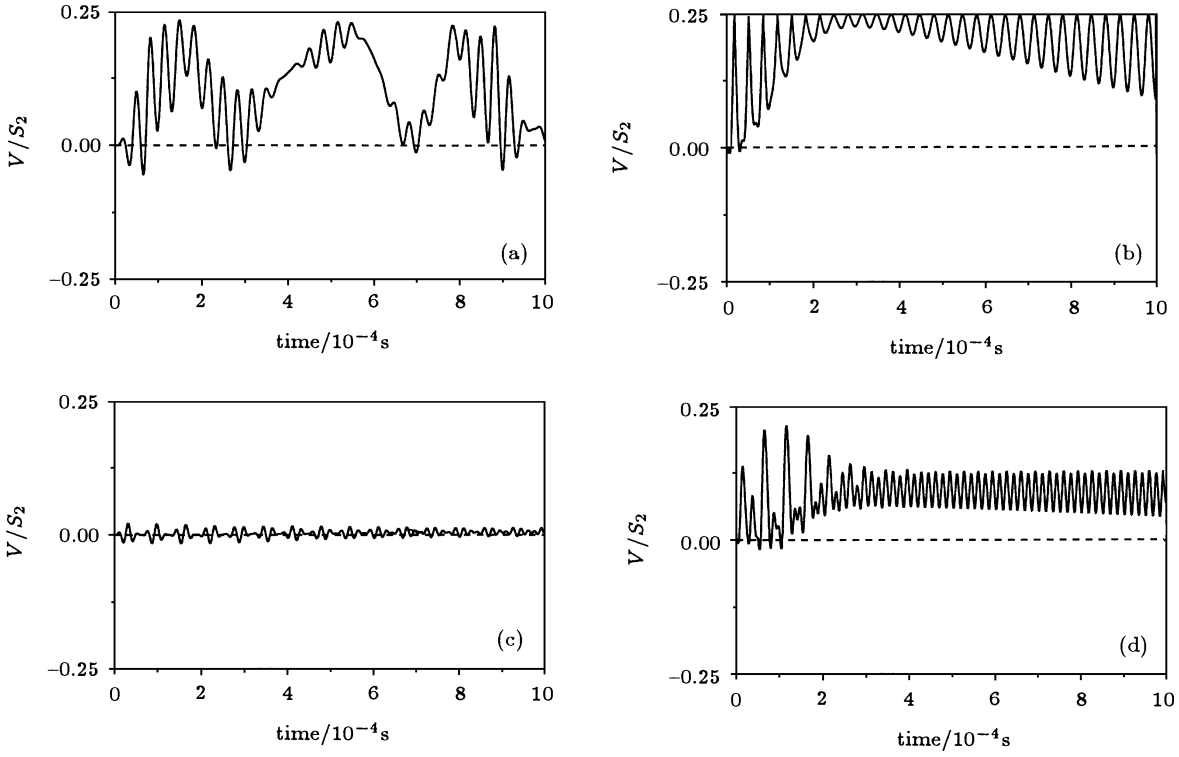


Fig.1. Time evolution of the squeezing factor $V(S_2)$ of a trapped ion located at the node of the standing-wave laser($\phi = \pi/2$). The ion is initially in the ground state $|-\rangle$ and the phonons in the coherent state. $\Omega_0 = 6\pi \times 10^5 s^{-1}$, $\Delta = \Omega_0/20$, and $\eta = 0.05$. (a) $\delta = 0, \bar{n} = 1$; (b) $\delta = 0, \bar{n} = 25$; (c) $\delta = -\Omega, \bar{n} = 1$; (d) $\delta = -\Omega, \bar{n} = 25$.

Figure 2 shows the time evolution of $V(S_2)$ corresponding to the squeezing of S_2 for the case that the trapped ion deviates from the node of the standing-wave laser ($\phi = \pi/4$) and the other parameters are same as in Fig.1. Under the resonant condition, for the small average phonon numbers ($\bar{n} = 1$), the time dependence of $V(S_2)$ shows a chaotic behaviour, and the trapped ion exhibits irregular squeezing in S_2, as in Fig.2(a); while for large average phonon numbers ($\bar{n} = 25$), the squeezing in S_2 cannot occur, except for the initial moment, as in Fig.2(b). In Figs.2(c) and 2(d), the off-resonant case $\delta = -\Omega$ is considered. It is obvious that the trapped ion shows the periodical squeezing effects in S_2, and the squeezing time is also decreased with an increase in the average phonon number $\bar{n}$.

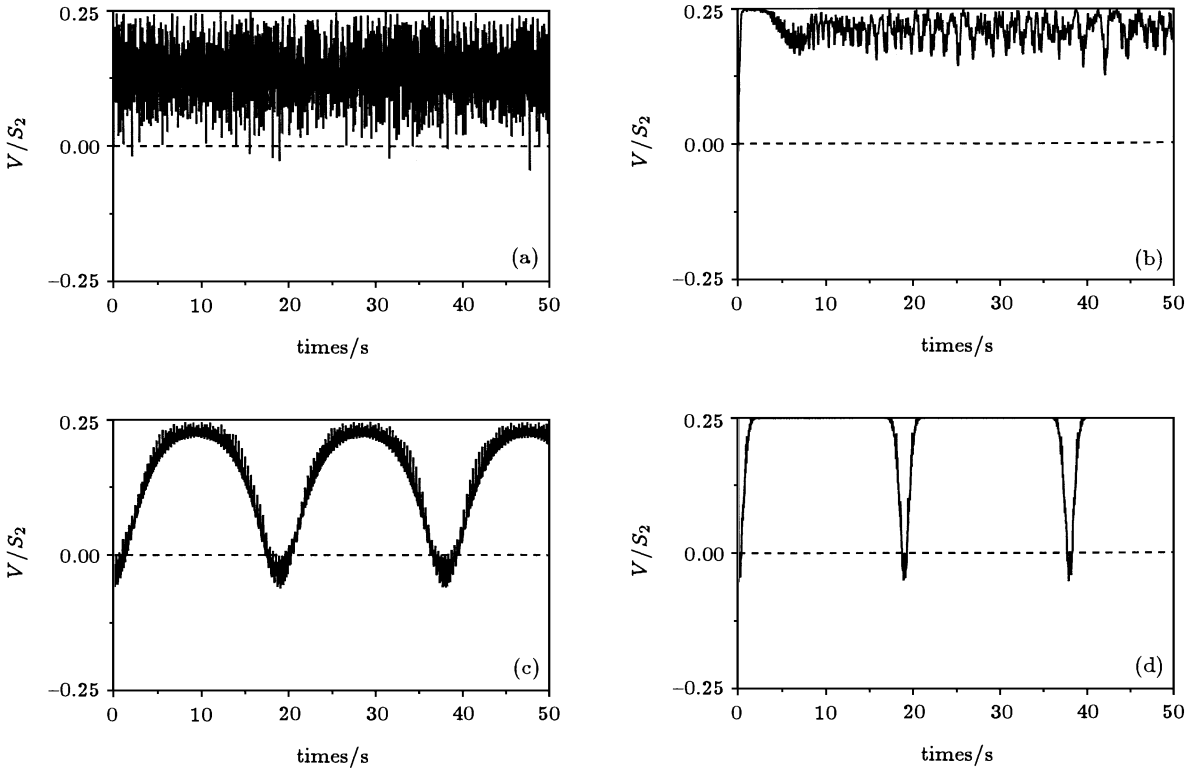


Fig.2. Time evolution of the squeezing factor $V(S_2)$ of a trapped ion deviating from the node of the standing-wave laser ($\phi = \pi/4$). The ion is initially in the ground state $|-\rangle$ and the phonons in the coherent state. $\Omega_0 = 6\pi \times 10^5 s^{-1}$, $\Delta = \Omega_0/20$, and $\eta = 0.05$. (a) $\delta = 0, \bar{n} = 1$; (b) $\delta = 0, \bar{n} = 25$; (c) $\delta = -\Omega, \bar{n} = 1$; (d) $\delta = -\Omega, \bar{n} = 25$.

In conclusion, we have investigated the squeezing properties of a trapped ion in a standing-wave laser. Our results show that the squeezing of a trapped ion in the standing-wave laser depends on its position in the laser, the detuning parameter δ and the initial average phonon number $\bar{n}$. We can come to the following conclusions. Firstly, when the trapped ion is located at the node of the standing-wave laser, the trapped ion shows the squeezing effects only in the short-time range(10^{-4}s). Secondly, as the trapped ion deviates from the node of the laser, the squeezing effects can occur in the long-time range, and the trapped ion exhibits periodical squeezing effects under the off-resonant condition. Finally, the squeezing time is decreased with an increase in the initial average phonon numbers.

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