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INVERSE SYNCHRONIZATION OF CHAOTIC SYSTEMS IN AN ERBIUM-DOPED FIBRE DUAL-RING LASER USING THE MUTUAL COUPLING METHOD

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Inverse synchronization of chaos is a type of synchronization in which the dynamical variables of two chaotic systems are inversely equal. In this paper, we present a scheme for inverse synchronization of two chaotic systems in an erbium-doped fibre dual-ring laser using the mutual coupling method. For realistic values of the systems, we demonstrate two kinds of results, as follows. (1) Two independent identical chaotic systems can go into inversely synchronized chaotic oscillation for coupling greater than 0.03. (2) When some parameter of one system varies, the state of the coupled systems could go into some periodic states directly or by inverse bifurcation. Simultaneously, they will lose the synchronization as the parameter changes.

Keywords: chaos synchronization; strange attractor; erbium-doped fibre dual-ring laser
PACC: 0545, 4255N, 4260

I. INTRODUCTION

Erbium-doped fibre lasers have attracted the attention of numerous workers for their potential applications in various fields such as communication, sensing and medicine.[1] Recently, the dynamical behaviours of erbium-doped fibre lasers have been investigated.[2−10] In this paper, we mainly investigate the chaotic synchronization in an erbium-doped fibre dual-ring laser.

Chaotic synchronization of nonlinear dynamical systems is one of the most important topics in the field of nonlinear science. The concept of chaos synchronization was first introduced by Pecora and Carroll,[11−12] and is known as the P–C synchronized method. Following this, some other methods[13−19] were introduced, and chaotic synchronization in lasers[20−23] and other systems were realized.[24] But this kind of synchronization was understood to be the perfect coincidence of the chaotic dynamics of the two systems (called identical synchronization). Now, it is recognized that this form of synchronization is only a particular case, and in fact more complicated forms of synchronization exist, called generalized synchronization,[25] in which a functional relationship between the dynamical variables exists. So, like the identical synchronization, the inverse synchronization of chaos is one particular form of generalized synchronization, in which the chaotic variables of two systems are inversely equal.

So far, reports on chaos synchronization in erbium-doped fibre lasers have been confined to chaotic lasers with long cavity[26−27] and short cavity by modulating the loss parameter.[28] They all use the unidirectional coupling method. In this paper, using the mutual coupling method, we propose a scheme of inversely chaotic synchronization between two independent chaotic systems in an erbium-doped fibre dual-ring laser and realize the inverse synchronization of two chaotic systems.

II. THE CHAOTIC SYSTEM IN AN ERBIUM-DOPED FIBRE DUAL-RING LASER

An erbium-doped fibre dual-ring laser consists of two erbium-doped fibre-ring lasers with one polarized mode. The scheme is shown in Fig.1.

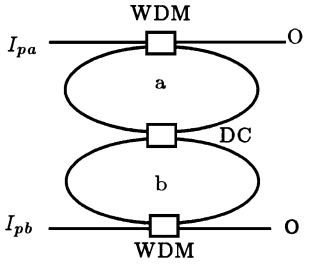


Fig.1. Erbium-doped fibre dual-ring laser system: I_{pa}, I_{pb}, pump light; DC, directional coupler; WDM, wavelength division multiplexing; O, output.

According to the report,[10] two erbium-doped fibre ring lasers with one polarized mode are in the steady state. When they are coupled to each other

through a directional coupler, they could enter periodic states. As the gain coefficient of ring b changes, the system could be routed to chaos and developed chaos. The dynamical equations of the coupled system are as follows:

$$\dot{E}_a = -\kappa_a(E_a + \eta_0 E_b) + g_a E_a D_a, \quad (1)$$

$$\dot{E}_b = -\kappa_b(E_b - \eta_0 E_a) + g_b E_b D_b, \quad (2)$$

$$\dot{D}_a = -(1 + I_{pa} + |E_a|^2)D_a + I_{pa} - 1, \quad (3)$$

$$\dot{D}_b = -(1 + I_{pb} + |E_b|^2)D_b + I_{pb} - 1, \quad (4)$$

where E_a and E_b are the lasing fields of rings a and b, respectively; D_a and D_b are the population inversions of rings a and b, respectively; κ_a and κ_b are the decay rates of rings a and b, respectively; and g_a and g_b are the gain coefficients of rings a and ring b, respectively. I_{pa} and I_{pb} represent the pump parameter in the respective fibre ring lasers. $|E_a|^2$ and $|E_b|^2$ represent the lasing intensities of rings a and b, respectively. η_0 is the coupling coefficient of the directional coupler (DC) at wavelength $\lambda = 1.5\mu$m. Moreover, the time in the equations is normalized to τ_2; τ_2 is the lifetime of the population of the lasing upper level in erbium.

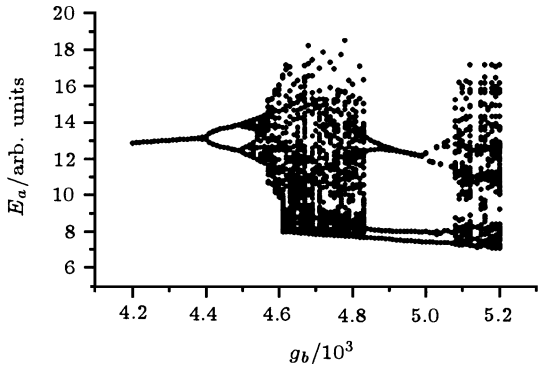


Fig.2. The bifurcation diagrams of the system via g_b.

To elucidate the method and the process of chaos generation, we take the parameters as:[10] κ_a, κ_b=1000; g_a=10 500; g_b=4200; η_0=0.2; I_{pa}, I_{pb}=4; τ_2=10ms. Firstly, we take the gain coefficient g_b as the independently changing parameter and investigate the changing process of the systems. As reported,[10] when g_b varies from 4200 to 4800, the two systems would go into chaos and developed chaos follows the period-doubling bifurcation. This is obviously found in the Fig.2. Moreover, we find that when g_b increases to 4840, the system can go directly into a more complicated periodic state from the developed chaos. At this time their attractor and sequence are shown in Fig.3. It is not until g_b increases to more than 5060 that the system would go directly into developed chaos again. This effect has not been reported in Ref.[10]

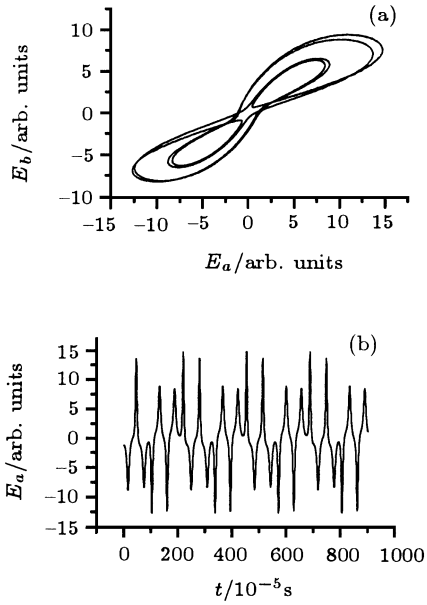


Fig.3. The periodic state (a) the attractor on the phase plane E_a-E_b (b) the sequence of E_a.

Secondly, we take the decay rates of the two lasers (κ_a=κ_b) as the changing parameter, and keep g_b as 4800. The numerical result shows that the coupled system could go into chaos and developed chaos if the decay rates (κ_a, κ_b) change from 1250 to 850, just as in the case of changing g_b. At this time, the system is in the complicated periodic state when κ_a, κ_b vary from 991 to 929, and the attractor and the bifurcation process of the system are similar to that of the above case.

Ⅲ. SYNCHRONIZATION SCHEME AND NUMERICAL ANALYSIS

A. Scheme for Synchronization

To synchronize two chaotic systems in an erbium-doped fibre dual-ring laser, we use the method of mutual coupling and propose the block diagram shown in Fig.4.

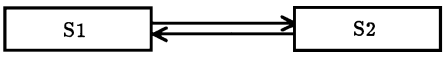


Fig.4. Synchronization scheme: S1 and S2, chaotic systems in erbium-doped fibre dual-ring lasers.

The outputs of two chaotic systems in ring a are coupled to each other through a directional cou-

pler. According to the coupling theory of the optical wave,[29] when the light in one fibre is coupled to the other fibre through a directional coupler, the coupled lasing field to the other has a phase change $\pi/2$, such as the wavelength division multiplexing (WDM). In the scheme, the lasing field of one system is coupled to the other through one directional coupler and one WDM, in which the WDM is used for the output and the directional coupler for the output to be coupled to the other. So when the lasing field of one of these is coupled to the other, the coupled field has a phase change of π from the lasing field. According to the dynamical equations[10] and considering the effect of the directional coupler and WDM, we express the dynamical equations of the two systems in the scheme of synchronization as follows:

$$\dot{E}_{ma} = -\kappa_{ma}(E_{ma}+\eta_0 E_{mb}+\rho_{mn}E_{na})+g_{ma}E_{ma}D_{ma}, \quad (5)$$

$$\dot{E}_{mb} = -\kappa_{mb}(E_{mb}-\eta_0 E_{ma})+g_{mb}E_{ma}D_{mb}, \quad (6)$$

$$\dot{D}_{ma} = -(1+I_{pa}+|E_{ma}|^2)D_{ma}+I_{pa}-1, \quad (7)$$

$$\dot{D}_{mb} = -(1+I_{pb}+|E_{mb}|^2)D_{mb}+I_{pb}-1, \quad (8)$$

where m, n=1,2, refer to the two chaotic systems, respectively. ρ_{mn} represents the coupling coefficient (ρ_{mn}=$\eta_w\eta_1$), η_w, η_1 represent the coupling coefficient of the WDM and the directional coupler at the wavelength of 1.5μm, respectively. The meaning of the other symbols is the same as above.

B. Numerical Simulation and Analysis

To elucidate the synchronization of chaos in the scheme, we consider at first the case when the two systems are identical and both are in developed chaos. In the numerical simulation, the parameters used are:[10] κ_{ma}, κ_{mb}=1000; g_{ma}=10500; g_{mb}=4800; η_0=0.2; I_{pa}, I_{pb}=4; τ_2=10ms. The result of numerical simulation shows that, when the coupling coefficient is greater than 0.03, the two systems could inversely synchronize to each other, in which E_{1a}=-E_{2a}, E_{1b}=-E_{2b}. Figs.5(a) and 5(b) respectively show the attractors of the two chaotic systems and the projection of the flow onto E_{1a}-E_{2a} before and after mutual coupling at ρ_{mn}=0.06. (In addition, the projection of the flow onto E_{1b}-E_{2b} is the same.) The line and its direction in Fig.5(b3) shows that the lasing fields of the two systems are inversely synchronized.

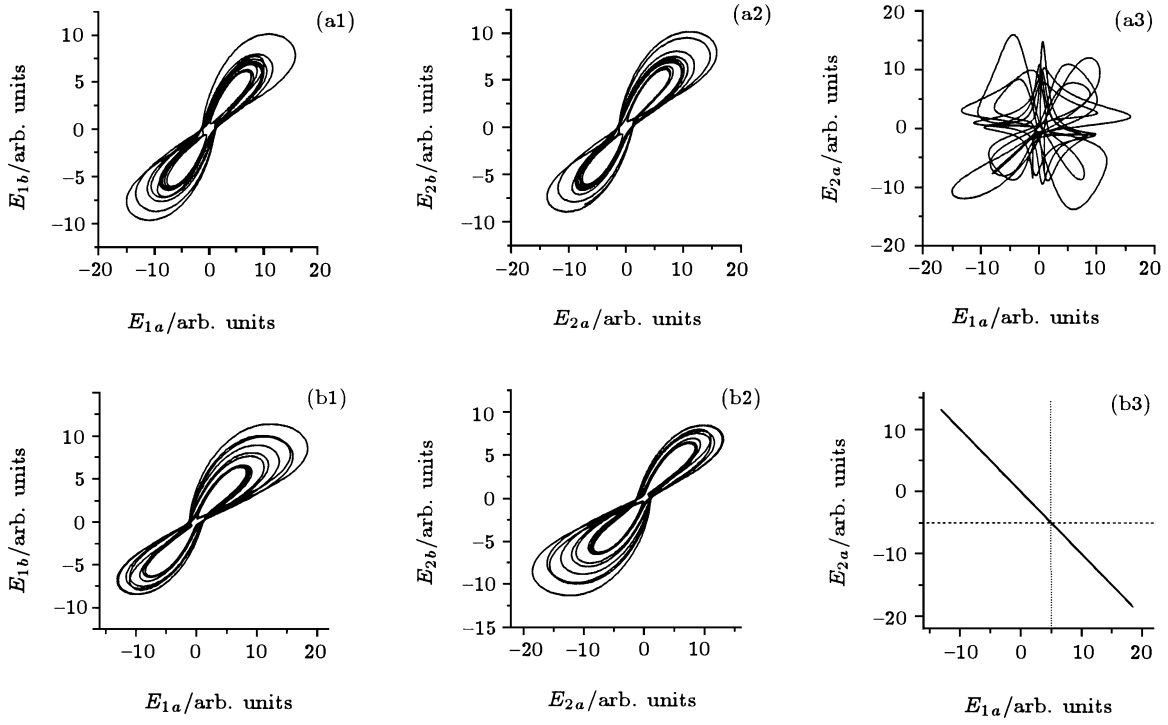


Fig.5. The attractors in the phase plane E_{ma}-E_{mb} and the projections onto the E_{1a}-E_{2a} plane: (a) without coupling; (b) with the coupling at $|\rho_{mn}$=0.06.

Then we investigate the case when the two systems are different or even in different states. Here, we keep one system in developed chaos as above, and change the other system by changing the gain coefficient of g_{1b} or the decay rate. We let the coupling coefficient ρ_{mn}=0.06. Firstly, we regard g_{1b} as the changing parameter and let it change from 4800 to 4200. The numerical result shows that with the decrease of g_{1b}, the coupled system could go into a periodic state through inverse bifurcation. Then we change g_{1b} from 4800 to 5300, and we find that when g_{1b} is greater than 5140, the coupled systems go directly into a complicated periodic state, of which the attractor and sequence are similar to that shown in Fig.3. The process of bifurcation is shown in Fig.6.

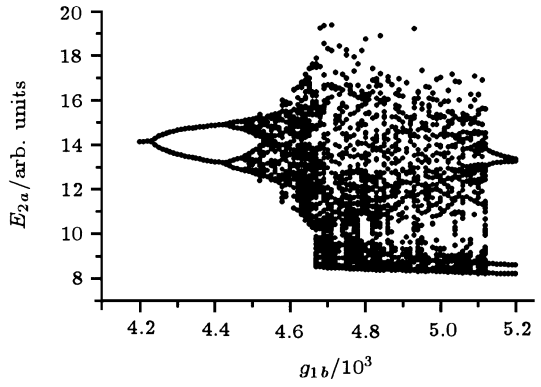


Fig.6. The bifurcation diagrams of the system via g_{1b}.

Secondly, we regard the decay rates (κ_{1a}=κ_{1b}) as the changing parameter and take g_{1b}=4800. We let the decay rate change from 1000 to 1250, and then from 1000 to 850. Similarly, we get the same result as in the first case; that is, they could go into periodic states through inverse bifurcation and directly, respectively, and their attractors are similar, accordingly. In this case, the coupled systems go into the more complicated periodic state as the decay rates are smaller than 940.

Moreover, as we investigate the case when the two systems are in different states, we find that, when the difference of one parameter between the two systems is greater than 3%, the two systems will lose synchronization. But because we can obtain some periodic states at different conditions, we can still use this method to control the chaotic systems.

IV. CONCLUSION

We have investigated the chaos generation in an erbium-doped fibre dual-ring laser by changing the parameters of the gain coefficient and the decay rate. We have also proposed a scheme for inverse synchronization of the two chaotic systems by use of the method of mutual coupling. The numerical result shows that: (1) two independent identical chaotic systems can go into inversely synchronized chaotic oscillation with coupling coefficient greater than 0.03; (2) when one of the systems varies by changing its gain coefficient or decay rate, we can obtain some periodic states. Hence, using the same method, we can either synchronize two identical chaotic systems or control chaotic systems under different conditions. This is significant in practice.

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