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A NEW SLIDING MODE CONTROL FOR A CLASS OF UNCERTAIN TIME-DELAY CHAOTIC SYSTEMS*

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We propose a new sliding mode control scheme for a class of uncertain time-delay chaotic systems. It is shown that a linear time invariant system with the desired system dynamics is used as a reference model for the output of a time-delay chaotic system to track. A sliding mode controller is then designed to drive the output of the time-delay chaotic system to track the desired linear system. On the sliding mode, the output of the controlled time-delay chaotic system can behave like the desired linear system. A simulation example is given in support of the proposed control scheme.

Keywords: time-delay chaotic systems, sliding mode control, linear desired system
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I. INTRODUCTION

In the past decade, there has been tremendous interest in studying the control of chaotic systems.[1−14] In Refs.[1] and [2], a few control strategies have been developed to stabilize the unstable periodic motion in the chaotic set. In Refs.[3–9], controllers were designed to suppress chaotic behaviours. In Refs.[10–12], adaptive controllers were developed to adaptively eliminate the effects of dynamic chaos, and in Refs.[13,14], sliding mode control schemes were developed to eliminate chaotic behaviours.

Many systems involve time delay. Since Mackey and Glass (M–G)[15] first found chaos in a time-delay system, there has been increasing interest in delayed chaotic systems.[15−19] Recently, Lu and He,[17] and Lu *et al.*[18] have observed and analysed chaotic motion in a first-order differential delayed equation for a piecewise linear delayed function. Tian and Gao[19] showed that a simple delay system governed by a first-order differential delay equation may behave chaotically.

Though there has been some research on the sliding mode control of a delay-free chaotic system,[13−14] little attention has been paid to the sliding mode control of a time-delay uncertain chaotic systems. In this paper, we propose a new sliding mode output tracking control scheme for a class of uncertain time-delay chaotic systems. It is shown that a linear time invariant system with desired system dynamics is used as a reference model for the output of a time-delay chaotic system to track. A sliding mode controller is then designed to drive the output of the time-delay chaotic system to track the desired linear system. On the sliding mode, the output can behave like the desired linear system. A simulation example with the time-delay chaotic system[19] is given in support of the proposed control scheme.

II. A NEW SLIDING MODE CONTROL SCHEME

Consider the following time-delay chaotic systems described by a first-order differential equation:

$$\begin{aligned} \dot{x}(t) =& f(x(t), x(t-\tau)) + \theta g(x(t), x(t-\tau)) + u(t), \\ y =& h(x) \end{aligned} \tag{1}$$

where $x(t)$ is the vector of the state variable, $u(t)$ is the input signal, f and g are nonlinear functions, θ is an unknown parameter, and $y = h(x)$ is the output. We assume the system (1) behaves chaotically without control ($u(t) = 0$). The system (1) can represent a class of chaotic systems, such as the first-order time-delay M–G chaotic system in Refs.[15–16], the first-order piecewise linear time-delay chaotic system in Refs.[17–18] and a class of first-order time-delay chaotic system in Ref.[19].

The idea of this paper is to design a sliding mode controller which can drive the output to track a de-

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sired linear dynamics cx before the sliding variable vector converges to the system origin. On the sliding mode, the output $y = h(x)$ in system (1) will behave the same as the following desired linear system:

$$\dot{x}(t) = cx(t), \tag{2}$$

where $c < 0$.

For the above purpose, we define the following sliding mode variable vector:

$$s = h(x) - cx(t). \tag{3}$$

After differentiating the sliding variable vector s with respect to time t, we have

$$\begin{aligned}\dot{s} =& h_x^{'}(x)\dot{x} - c\dot{x}\\ =&(h_x^{'}(x) - c)\dot{x}\\ =&(h_x^{'}(x) - c)(f(x(t), x(t-\tau))\\ &+ \theta g(x(t), x(t-\tau)) + u(t))\\ =&(h_x^{'}(x) - c)(f(x(t), x(t-\tau))\\ &+ \hat{\theta} g(x(t), x(t-\tau)) + u(t))\\ &+ (-\hat{\theta} + \theta)(h_x^{'}(x) - c)g(x(t), x(t-\tau)).\end{aligned}$$

If we choose the Lyapunov function

$$V = \frac{1}{2}s^2 + \frac{1}{2}(\theta - \hat{\theta})^2. \tag{4}$$

and differentiate V with respect to time t, we have

$$\begin{aligned}\dot{V} =& s(h_x^{'}(x) - c)(f(x(t), x(t-\tau))\\ &+ \hat{\theta} g(x(t), x(t-\tau)) + u(t))\\ &+ s(\theta - \hat{\theta})(h_x^{'}(x) - c)g(x(t), x(t-\tau)) - (\theta - \hat{\theta})\dot{\hat{\theta}}\\ =& Z(f(x(t), x(t-\tau)) + \hat{\theta} g(x(t), x(t-\tau)) + u(t))\\ &+ s(\theta - \hat{\theta})((h_x^{'}(x) - c)g(x(t), x(t-\tau)) - s^{-1}\dot{\hat{\theta}}),\end{aligned}$$

where $Z = s(h_x^{'}(x) - c)$.

If we choose the control signal as follows:

$$u(t) = -f(x(t), x(t-\tau)) - \hat{\theta} g(x(t), x(t-\tau)) - kZ, \tag{5}$$

where $k > 0$, and

$$\dot{\hat{\theta}} = s(h_x^{'}(x) - c)g(x(t), x(t-\tau)), \tag{6}$$

then

$$\dot{V} \leq 0.$$

According to the Lyapunov stability theory, the sliding variable vector s will converge to zero when $t \to \infty$. On the sliding mode $s = h(x) - cx = 0$, we see that $h(x) = cx(t)$.

That is, the output of nonlinear system (1) behaves like the desired linear system(2).

Ⅲ. A SIMULATION EXAMPLE

Consider the following time-delay chaotic system[19]

$$\begin{aligned}\dot{x} =& \theta x + 104x(t-\tau)(1 - x(t-\tau)),\\ y =& x,\end{aligned} \tag{7}$$

where θ is an unknown parameter and $\tau = 0.5$ is the delayed time. In order to simulate, we assume $\theta = -26$. The corresponding chaotic behaviour of this system is shown in Fig.1.

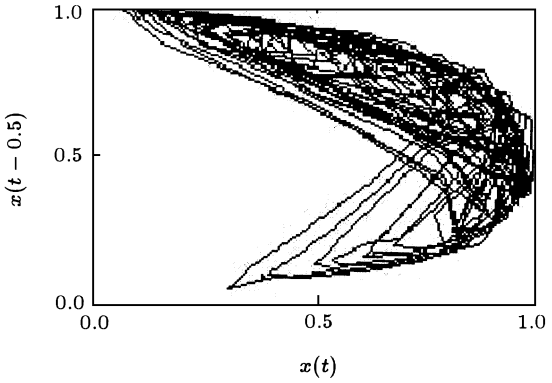


Fig.1. Chaotic behaviour of system (7).

The desired reference linear system is

$$\dot{x} = -2x.$$

The sliding variable vector is defined as

$$s = x - (-2x) = 3x.$$

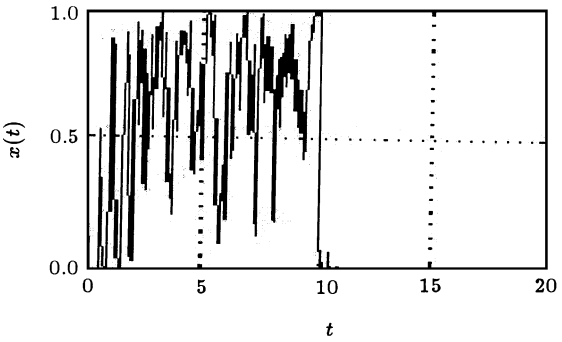


Fig.2. The state variable of $y = x$.

We choose $k = 50$ and begin to control at $t = 10$. Figure 2 shows the convergence property of the output variables. It is seen that by using the proposed control scheme, the output of the controlled time-delay chaotic system is like the desired linear system in

Eq.(2) and the chaotic features are eliminated within a few seconds.

VI. CONCLUSIONS

Based on the Lyapunov stability theory, this paper proposes a new sliding mode control scheme for a class of uncertain time-delay chaotic systems. On the sliding mode, the output of the controlled time-delay chaotic system can behave like the desired linear system. A simulation example is given in support of the proposed control scheme.

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