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STATISTICAL ENTROPIES OF THE TAUB–NUT/BOLT AdS SPACES FROM THE HORIZON CONFORMAL FIELD THEORY*

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The covariant phase technique is used to compute the constraint algebra of the four-dimensional space-times which are asymptotic to anti-de Sitter (AdS), such as the planar Taub–NUT AdS and Taub–bolt AdS spaces, and the hyperbolic Taub–bolt AdS space. The standard Virasoro subalgebrae with corresponding central charges for these objects are constructed and the resulting densities of states yield the expected Bekenstein–Hawking entropies.

Keywords: statistical entropy, black hole, conformal field theory
PACC: 0470, 0460

I. INTRODUCTION

By comparing black hole physics and thermodynamics, and from the discovery of black hole evaporation, Bekenstein and Hawking[1−3] have found that the black hole entropy is given by $A_H/4$, where A_H is the area of the black hole. However, the statistical mechanical description of the Bekenstein–Hawking entropy[1−3] in terms of microscopic states is still an outstanding open question and much effort has been concentrated on the problem for some years.[4−22] A major step toward finding a universal mechanism was taken by Strominger and Vafa.[23] By using Brown and Henneaux's result[24] that the asymptotic symmetry group of AdS_3 is generated by the Virasoro algebra and, therefore, any consistent quantum theory of gravity on AdS_3 is conformal theory, Strominger[25] calculated the entropy of the three-dimensional (3D) Bañados–Teitelboim–Zanelli (BTZ) black hole by counting excitations of AdS_3. In the light of this work, one could statistically re-interpret the black hole entropy by establishing a relation conformal field theory on the boundary of related 3D BTZ space. In order to overcome the limitations of Strominger's method, such as the fact that the approach can only be used for (2+1) dimensions and it is based on an algebra of transformations at infinity, Carlip[26] derived the central extension of the constraint algebra of general relativity by using the manifestly covariant phase space method[27−30] and a boundary which is a surface that looks locally like a Killing horizon. A natural set of boundary conditions leads to a Virasoro subalgebra with a calculable central charge. He then studied the statistical entropy of the Rindler space, Taub–NUT and Taub–bolt spaces, and two-dimensional (2D) dilaton gravity. We have recently extended Carlip's study to statistical entropies of the rotating-charged black hole[16] and the stationary dilaton black hole[17] by the covariant phase space method. The main purpose of this paper is to study the entropies of the four-dimensional planar Taub–NUT AdS and Taub–bolt AdS, and the hyperbolic Taub–bolt AdS spaces since all of these objects possess some special properties.

The plan of this paper is as follows. In Section II, by using the covariant phase techniques, we extend Carlip's result[31] for $\boldsymbol{L}_{a_1a_2\cdots a_{n-1}} = \dfrac{1}{16\pi G}\epsilon_{a_1a_2\cdots a_{n-1}}R$ to the case including the cosmological term, $L_{a_1a_2\cdots a_{n-1}} = \dfrac{1}{16\pi G}\epsilon_{a_1a_2\cdots a_{n-1}}(R-2\Lambda)$. In Section III, the standard Virasoro subalgebrae with corresponding central charges are constructed for the planar Taub–NUT AdS and Taub–bolt AdS, and the hyperbolic Taub–bolt AdS. The statistical entropies for these objects are then calculated. The last section is devoted to the summary.

II. CONSTRAINT ALGEBRA AT THE KILLING HORIZON

In Refs.[17–30] the authors showed that the variation of the Lagrangian defines the equation of motion

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n-form E and the symplectic potential $(n-1)$-form Θ via the equation

$$\delta \boldsymbol{L} = \boldsymbol{E}\delta\phi + \mathrm{d}\boldsymbol{\Theta}, \tag{1}$$

where $\boldsymbol{L}$ is an n-form, $\boldsymbol{E}\delta\phi = \boldsymbol{E}_g^{ab}\delta g_{ab} + \boldsymbol{E}_\psi\delta\psi$, $\phi = (g_{ab}, \psi)$ denotes an arbitrary collection of dynamical fields, and the equations of motion are taken to be $\boldsymbol{E}_g^{ab} = 0,\ \boldsymbol{E}_\psi = 0$. Let ξ^a be any smooth vector field on the space-time manifold $\boldsymbol{M}$, i.e. ξ^a is the infinitesimal generator of a diffeomorphism, then a Noether current $(n-1)$-form can be defined as[29,30]

$$\boldsymbol{J}[\xi] = \boldsymbol{\Theta}[\phi, \mathcal{L}_\xi\phi] - \xi\cdot\boldsymbol{L}, \tag{2}$$

where the "central dot" denotes the contraction of the vector field ξ^a into the first index of the differential form $\boldsymbol{L}$. Lee[27] showed that $\boldsymbol{J}$ is closed for all ξ^a, i.e. $\mathrm{d}\boldsymbol{J} = 0$. Then we obtain[29]

$$\boldsymbol{J} = \mathrm{d}\boldsymbol{Q}, \tag{3}$$

where $\boldsymbol{Q}$ is a Noether charge $(n-2)$-form. From the variation of the Noether current $(n-1)$-form, we know that the symplectic current[27] $(n-1)$-form $\omega[\phi, \delta_1\phi, \delta_2\phi] = \delta_2\boldsymbol{\Theta}[\phi, \delta_1\phi] - \delta_1\boldsymbol{\Theta}[\phi, \delta_2\phi]$ can be expressed as

$$\omega[\phi, \delta\phi, \mathcal{L}_\xi\phi] = \delta\boldsymbol{J}[\xi] - \mathrm{d}(\xi\cdot\boldsymbol{\Theta}[\phi, \delta\phi]), \tag{4}$$

and the Hamiltonian equation of motion is

$$\begin{aligned}\delta\boldsymbol{H}[\xi] &= \int_C \omega[\phi, \delta\phi, \mathcal{L}_\xi\phi] \\ &= \int_C [\delta\boldsymbol{J}[\xi] - \mathrm{d}(\xi\cdot\boldsymbol{\Theta}[\phi, \delta\phi])].\end{aligned} \tag{5}$$

Using Eq.(3) and defining a $(n-1)$-form $\boldsymbol{B}$ as $\delta\int_{\partial C}\xi\cdot\boldsymbol{B}[\phi] = \int_{\partial C}\xi\cdot\boldsymbol{\Theta}[\phi, \delta\phi]$, one obtains the Hamiltonian[31]

$$\boldsymbol{H}[\xi] = \int_{\partial C}(\boldsymbol{Q}[\xi] - \xi\cdot\boldsymbol{B}[\phi]). \tag{6}$$

It is well known that the Poisson bracket forms the standard "surface deformation algebra"[24,31]

$$\{H[\xi_1], H[\xi_2]\} = H[\{\xi_1, \xi_2\}] + K[\xi_1, \xi_2], \tag{7}$$

where the central term $K[\xi_1, \xi_2]$ depends on the dynamical fields only through their boundary values.

In this paper, we focus our attention on n-dimensional space which are (locally) asymptotic to anti-de Sitter (AdS) space-time. So we take the Lagrangian n-form as

$$\boldsymbol{L}_{a_1a_2\cdots a_n} = \frac{1}{16\pi G}\epsilon_{a_1a_2\cdots a_n}(R - 2\Lambda), \tag{8}$$

where $\epsilon_{a_1a_2\cdots a_n}$ is a volume element (a continuous non-vanishing n-form) and Λ is the cosmological constant ($\Lambda > 0$ for de Sitter space, and $\Lambda < 0$ for anti-de Sitter space). By using the infinitesimal generator of a diffeomorphism, ξ, the symplectic potential $(n-1)$-form is given by

$$\begin{aligned}&\boldsymbol{\Theta}_{a_1a_2\cdots a_{n-1}}[g, \mathcal{L}_\xi g] \\ =&\frac{1}{8\pi G}\epsilon_{ca_1a_2\cdots a_{n-1}}(\nabla_e\nabla^{[e}\xi^{c]} + R^c_e\xi^e).\end{aligned} \tag{9}$$

From Eqs.(2) and (9) we know that the Noether current $(n-1)$-form can be expressed as

$$\begin{aligned}J_{a_1a_2\cdots a_{n-1}} =&\frac{1}{8\pi G}\epsilon_{ca_1a_2\cdots a_{n-1}}(\nabla_e\nabla^{[e}\xi^{c]} \\ &+ R^c_e\xi^e) - \xi\cdot\boldsymbol{L} \\ =&\frac{1}{8\pi G}\epsilon_{ca_1a_2\cdots a_{n-1}}\Big[\nabla_e\nabla^{[e}\xi^{c]} \\ &+ (R^c_e - \frac{1}{2}\delta^c_e R + \delta^c_e\Lambda)\xi^e\Big].\end{aligned} \tag{10}$$

Denoting by δ_ξ the variation corresponding to the diffeomorphism generated by ξ, for the Noether current $\boldsymbol{J}[\xi]$ we have

$$\begin{aligned}\delta_{\xi_2}\boldsymbol{J}[\xi_1] &= \xi_2\mathrm{d}\boldsymbol{J}[\xi_1] + \mathrm{d}(\xi_2\cdot\boldsymbol{J}[\xi_1]) \\ &= \mathrm{d}[\xi_2(\boldsymbol{\Theta}[\phi, \mathcal{L}_{\xi_1}\phi] - \xi_1\cdot\boldsymbol{L})].\end{aligned} \tag{11}$$

Substituting Eq.(11) into Eq.(5) and using Eq.(10), we arrive at

$$\begin{aligned}\delta_{\xi_2}H[\xi_1] =&\frac{1}{16\pi G}\int_{\partial C}\epsilon_{bca_1\cdots a_{n-2}}[\xi_2^b\nabla_e(\nabla^e\xi_1^c \\ &- \nabla^c\xi_1^e) - \xi_1^b\nabla_e(\nabla^e\xi_2^c - \nabla^c\xi_2^e)] \\ &+ \frac{1}{8\pi G}\int_{\partial C}\epsilon_{bca_1\cdots a_{n-2}} \\ &\cdot\left(R^c_e - \frac{1}{2}\delta^c_e R + \delta^c_e\Lambda\right)\xi_1^e\xi_2^b \\ &- \frac{1}{8\pi G}\int_{\partial C}\epsilon_{bca_1\cdots a_{n-2}}R^c_e\xi_2^e\xi_1^b.\end{aligned} \tag{12}$$

Since the "bulk" part of the generator $H[\xi_1]$ on the left-hand side vanishes on the shell, we can interpret the left-hand side of Eq.(12) as the variation of the boundary term J, i.e. $\delta_{\xi_2}J[\xi_1]$. On the other hand, the change in $J[\xi_1]$ under a surface deformation generated by $J[\xi_2]$ can be precisely described by the Dirac bracket $\{J[\xi_1], j[\xi_2]\}^*$, that is,

$$\delta_{\xi_2}J[\xi_1] = \{J[\xi_1], J[\xi_2]\}^*. \tag{13}$$

Then from Eq.(7) we obtain the following relation on the shell

$$\{J[\xi_1], J[\xi_2]\}^* = J[\{\xi_1, \xi_2\}] + K[\xi_1, \xi_2], \tag{14}$$

where $J[\xi]$ is defined by Eq.(10) and the Dirac bracket is given by

$$\begin{aligned}\{J[\xi_1], J[\xi_2]\}^* =& \frac{1}{16\pi G}\int_{\partial C}\epsilon_{bca_1\cdots a_{n-2}}[\xi_2^b\nabla_e(\nabla^e\xi_1^c \\ &- \nabla^c\xi_1^e) - \xi_1^b\nabla_e(\nabla^e\xi_2^c - \nabla^c\xi_2^e)] \\ &+ \frac{1}{8\pi G}\int_{\partial C}\epsilon_{bca_1\cdots a_{n-2}} \\ &\cdot\left(R_e^c - \frac{1}{2}\delta_e^c R + \delta_e^c\Lambda\right)\xi_1^e\xi_2^b \\ &- \frac{1}{8\pi G}\int_{\partial C}\epsilon_{bca_1\cdots a_{n-2}}R_e^c\xi_2^e\xi_1^b.\end{aligned} \tag{15}$$

For vacuum general relativity ($T_e^c = 0$), the Einstein equation shows

$$R_e^c - \frac{1}{2}\delta_e^c R + \delta_e^c\Lambda = 0. \tag{16}$$

We should note that R_e^c does not vanish since we have introduced the cosmological term, which is different from Carlip's case.[30] Inserting Eq.(16) into Eq.(10), we obtain

$$J_{a_1a_2\cdots a_{n-1}} = \frac{1}{8\pi G}\epsilon_{ca_1a_2\cdots a_{n-1}}\nabla_e\nabla^{[e}\xi^{c]}. \tag{17}$$

From Eqs.(3) and (17) we know that the Noether charge $(n-2)$-form $\boldsymbol{Q}$ can be expressed as

$$\boldsymbol{Q}_{a_1a_2\cdots a_{n-2}} = -\frac{1}{16\pi G}\epsilon_{bca_1a_2\cdots a_{n-2}}\nabla^b\xi^c, \tag{18}$$

and the Dirac bracket (15) can now be reduced to

$$\begin{aligned}\{J[\xi_1], J[\xi_2]\}^* =& \frac{1}{16\pi G}\int_{\partial C}\epsilon_{bca_1\cdots a_{n-2}}[\xi_2^b\nabla_e(\nabla^e\xi_1^c \\ &- \nabla^c\xi_1^e) - \xi_1^b\nabla_e(\nabla^e\xi_2^c - \nabla^c\xi_2^e)] \\ &- \frac{1}{8\pi G}\int_{\partial C}\epsilon_{bca_1\cdots a_{n-2}}R_e^c\xi_2^e\xi_1^b.\end{aligned} \tag{19}$$

Comparing Eq.(19) with Eq.(3.6) in Ref.[30], we know that we have a surplus term $-\frac{1}{8\pi G}\int_{\partial C}\epsilon_{bca_1\cdots a_{n-2}}R_e^c\xi_2^e\xi_1^b$.

We now introduce Carlip's boundary conditions[31]

$$\begin{aligned}&\delta\chi^2 = 0, \quad \chi^a t^b\delta g_{ab} = 0, \\ &\delta\rho_a = -\frac{1}{2\kappa}\nabla_a(\delta\chi^2) = 0, \quad \text{at} \quad \chi^2 = 0,\end{aligned} \tag{20}$$

where $\chi^2 = g_{ab}\chi^a\chi^b$, χ^a is a Killing vector, ρ^a is a vector orthogonal to the orbits of χ^a, $\rho^a \to \chi^a$ at the horizon, and t^a is a unit space-like vector tangent to the boundary ∂M of the space-time $\boldsymbol{M}$. The surface deformation vector is suggested as in the following form

$$\xi^a = \mathcal{R}\rho^a + \mathcal{T}\chi^a, \tag{21}$$

where functions $\mathcal{R}$ and $\mathcal{T}$ satisfy[30]

$$\begin{aligned}&\mathcal{R} = \frac{1}{\kappa}\frac{\chi^2}{\rho^2}\chi^a\nabla_a\mathcal{T}, \\ &\rho^a\nabla_a\mathcal{T} = 0, \quad \text{(at the horizon)}.\end{aligned} \tag{22}$$

For a one-parameter group of diffeomorphism such that $D\mathcal{T}_\alpha = \lambda_\alpha\boldsymbol{T}_\alpha$, ($D \equiv \chi^a\partial_a$), Carlip introduced an orthogonality relation[31]

$$\int_{\partial C}\hat{\epsilon}\ \mathcal{T}_\alpha\mathcal{T}_\beta \sim \delta_{\alpha+\beta}. \tag{23}$$

The technical role of the condition (23) is to guarantee the existence of generators $H[\xi]$. By using the other future-directed null normal vector $N^a = k^a - \alpha\chi^a - t^a$, with $k^a = -\frac{1}{\chi^2}\left(\chi^a - \frac{|\chi|}{\rho}\rho^a\right)$ and a normalization $N_a\chi^a = -1$, the volume element can be expressed as

$$\epsilon_{bca_1\cdots a_{n-2}} = \hat{\epsilon}_{a_1\cdots a_{n-2}}(\chi_b N_c - \chi_c N_b) + \cdots\cdots, \tag{24}$$

where the omitted terms do not contribute to the integral.

By making use of the conditions listed above, we now calculate the last term on the right-hand side of Eq.(19)

$$\begin{aligned}&-\frac{1}{8\pi G}\int_{\partial C}\epsilon_{bca_1a_2\cdots a_{n-2}}R_e^c\xi_2^e\xi_1^b \\ =&-\frac{1}{8\pi G}\int_{\partial C}\epsilon_{bca_1a_2\cdots a_{n-2}}\left(\frac{1}{2}R - \Lambda\right)\delta_e^c\xi_2^e\xi_1^b \\ =&-\frac{1}{8\pi G}\int_{\partial C}\hat{\epsilon}_{a_1a_2\cdots a_{n-2}}\left(\frac{1}{2}R - \Lambda\right) \\ &\cdot(\chi_b N_c - \chi_c N_b)(\mathcal{T}_1\chi^b + \mathcal{R}_1\rho^b)(\mathcal{T}_2\chi^c + \mathcal{R}_2\rho^c) \\ =&-\frac{1}{8\pi G}\int_{\partial C}\hat{\epsilon}_{a_1a_2\cdots a_{n-2}}\left(\frac{1}{2}R - \Lambda\right) \\ &\cdot(\rho|\chi|\mathcal{T}_1\mathcal{R}_2) + O(\chi^2) \\ =&-\frac{1}{8\pi G}\int_{\partial C}\hat{\epsilon}_{a_1a_2\cdots a_{n-2}}\left(\frac{1}{2}R - \Lambda\right) \\ &\cdot\frac{|\chi|}{\rho}\frac{\chi^2}{\kappa}\mathcal{T}_1D\mathcal{T}_2 + O(\chi^2) \\ =&0.\end{aligned} \tag{25}$$

In the calculation we have used Eqs.(16), (21), (22) and (24). By using the result and Eqs.(21), (22) and (24), after Carlip's discussion we can show that the

second term of Eq.(6) has no contribution to $H[\xi]$, and Eq.(19) can be cast into

$$\begin{aligned}&\{J[\xi_1],J[\xi_2]\}^*\\=&\frac{1}{16\pi G}\int_{\partial C}\epsilon_{bca_1\cdots a_{n-2}}[\xi_2^b\nabla_e(\nabla^e\xi_1^c\\&-\nabla^c\xi_1^e)-\xi_1^b\nabla_e(\nabla^e\xi_2^c-\nabla^c\xi_2^e)]\\=&-\frac{1}{16\pi G}\int_{\partial C}\hat{\epsilon}_{a_1\cdots a_{n-2}}\Big[\frac{1}{\kappa}(\mathcal{T}_1D^3\mathcal{T}_2\\&-\mathcal{T}_2D^3\mathcal{T}_1)-2\kappa(\mathcal{T}_1D\mathcal{T}_2-\mathcal{T}_2D\mathcal{T}_1)\Big].\end{aligned}\tag{26}$$

It is also easy to show that

$$\begin{aligned}\{\xi_1,\xi_2\}^a=&(\mathcal{T}_1D\mathcal{T}_2-\mathcal{T}_2D\mathcal{T}_1)\chi^a\\&+\frac{1}{\kappa}\frac{\chi^2}{\rho^2}D(\mathcal{T}_1D\mathcal{T}_2-\mathcal{T}_2D\mathcal{T}_1)\rho^a.\end{aligned}\tag{27}$$

By making use of Eqs.(3) and (18) and replacing ξ^a in Eq.(18) by $\{\xi_1,\xi_2\}^a$, we have

$$\begin{aligned}J[\{\xi_1,\xi_2\}]=&\frac{1}{16\pi G}\int_{\partial C}\hat{\epsilon}_{a_1a_2\cdots a_{n-2}}\Big[2\kappa(\mathcal{T}_1D\mathcal{T}_2-\mathcal{T}_2D\mathcal{T}_1)\\&-\frac{1}{\kappa}D(\mathcal{T}_1D^2\mathcal{T}_2-\mathcal{T}_2D^2\mathcal{T}_1)\Big].\end{aligned}\tag{28}$$

The central term can then be obtained from Eqs.(14), (26) and (28), which is explicitly given by

$$\begin{aligned}K[\xi_1,\xi_2]=&\frac{1}{16\pi G}\int_{\partial C}\hat{\epsilon}_{a_1a_2\cdots a_{n-2}}\\&\cdot\frac{1}{\kappa}(D\mathcal{T}_1D^2\mathcal{T}_2-D\mathcal{T}_2D^2\mathcal{T}_1).\end{aligned}\tag{29}$$

In the next section we will study the statistical entropy for some well-known AdS solutions by using the constraint algebra described by Eqs.(7), (26), (28) and (29).

Ⅲ. STATISTICAL ENTROPY

A. Planar Taub–NUT AdS and Taub–bolt AdS

Both the four-dimensional planar Taub–NUT AdS and Taub–bolt AdS are described by the line element[30,31]

$$\begin{aligned}\mathrm{d}S^2=&V(r)\left[\mathrm{d}\tau+\frac{n}{l^2}(x\mathrm{d}y-y\mathrm{d}x)\right]^2\\&+\frac{\mathrm{d}r^2}{V(r)}+\frac{r^2-n^2}{l^2}(\mathrm{d}x^2+\mathrm{d}y^2),\end{aligned}\tag{30}$$

where

$$V(r)=\frac{-2mr+l^{-2}(r^4-6n^2r^2-3n^4)}{r^2-n^2},\tag{31}$$

and $l^2=-\Lambda/3$ ($\Lambda<0$).

The Taub–NUT AdS will appear as fixed-point sets at $r=n$. From Eq.(31) one knows that the mass parameter must take the value

$$m_{pn}=-\frac{4n^3}{l^2}.\tag{32}$$

Then Eq.(31) can be rewritten as

$$V_{pn}=\frac{(r-n)^2(r+3n)}{l^2(r+n)}.\tag{33}$$

For the Taub–bolt AdS solutions, where ∂_τ has a fixed point set at $r=r_{pb}>n$. Then one finds that the mass is

$$m_{pb}=\frac{1}{2l^2}\left(r_{pb}^3-6n^2r_{pb}-\frac{3n^4}{r_{pb}}\right).\tag{34}$$

Both the Taub–NUT AdS and Taub–bolt AdS have a time-like Killing vector $\chi^a\frac{\partial}{\partial x^a}=\partial_\tau$. Thus, a one-parameter group of diffeomorphism can be taken as

$$\mathcal{T}_n=\frac{1}{\kappa}\exp\left[in(\kappa\tau)\right].\tag{35}$$

Substituting Eq.(35) into Eq.(29), we obtain

$$K[\mathcal{T}_m,\mathcal{T}_n]=-\frac{iA_{\mathrm{bolt}}}{8\pi G}m^3\delta_{m+n,0},\tag{36}$$

where A_{bolt} is the area of the event horizon. Thus, Eq.(14) takes a standard form of a Virasoro subalgebra

$$i\{J[\mathcal{T}_m],J[\mathcal{T}_n]\}=(m-n)J[\mathcal{T}_{m+n}]+\frac{c}{12}m^3\delta_{m+n,0},\tag{37}$$

with a central charge

$$\frac{c}{12}=\frac{1}{8\pi G}\frac{L^2(r_{pb}^2-n^2)}{l^2}=\frac{A_{\mathrm{bolt}}}{8\pi G},\tag{38}$$

where L^2 accounts for the area of the (x,y) plane, $-L/2\le x,y\le L/2$. The boundary term $J[\mathcal{T}_0]$ can easily be obtained by Eqs.(3) and (18), which is given by $J[\mathcal{T}_0]=\frac{A_{\mathrm{bolt}}}{8\pi G}$. It is well known that a conformal field theory is characterized by a pair of Virasoro algebras, one for left-moving modes and one for right-moving modes. Then, the corresponding densities of the quantum states with a given eigenvalue $\triangle$ of $J[\mathcal{T}_0]$ grow asymptotically for large $\triangle$

$$\rho(\triangle)\sim\exp\left\{2\pi\sqrt{\frac{c}{6}\left(\triangle-\frac{c}{24}\right)}\right\}=\exp\left[\frac{A_{\mathrm{bolt}}}{4G}\right].\tag{39}$$

The statistical entropy is thus given by

$$S_{SE}=\frac{1}{4G}\frac{L^2(r_{pb}^2-n^2)}{l^2}=\frac{A_{\mathrm{bolt}}}{4G},\tag{40}$$

which is equal to the Bekenstein–Hawking entropy.[32]

B. Hyperbolic Taub–bolt AdS

There is also the possibility of having the hyperbolic, instead of planar, fixed-point sets of ∂_τ. The metric to be used is[32]

$$\begin{aligned} \mathrm{d}S^2 =& V(r)\left[\mathrm{d}\tau + 2n(\cosh\theta - 1)\mathrm{d}\varphi\right]^2 \\ &+ \frac{\mathrm{d}r^2}{V(r)} + (r^2 - n^2)(\mathrm{d}\theta^2 + \sinh^2\theta \mathrm{d}\varphi^2), \end{aligned} \tag{41}$$

where

$$V(r) = \frac{-(r^2 + n^2) - 2mr + l^{-2}(r^4 - 6n^2r^2 - 3n^4)}{r^2 - n^2}. \tag{42}$$

It was found in Ref.[32] that there is no hyperbolic Taub–NUT AdS. Nevertheless, hyperbolic Taub–bolt AdS solutions can be constructed by setting the mass as

$$\begin{aligned} m_{hb} =& \frac{1}{2}\left[\frac{1}{l^2}\left(r_{hb}^3 - 6n^2 r_{pb} - \frac{3n^4}{r_{pb}}\right)\right. \\ &\left. - \left(r_h + \frac{n^2}{r_{hb}}\right)\right], \end{aligned} \tag{43}$$

where r_{hb} is the radial position of the fixed point set. Noting that the Taub–bolt AdS has a Killing vector $\chi^a \dfrac{\partial}{\partial x^a} = \partial_\tau$ we obtain

$$\mathcal{T}_n = \frac{1}{\kappa}\exp\left[in(\kappa\tau)\right]. \tag{44}$$

The Virasoro subalgebra also takes the same form as Eq.(37) with the central charge

$$\frac{c}{12} = \frac{1}{8\pi G}\frac{\sigma(r_{hb}^2 - n^2)}{l^2} = \frac{A_{\mathrm{bolt}}}{8\pi G}, \tag{45}$$

where σ is the area of the hyperbolic space.[31] Then, the corresponding statistical entropy is shown explicitly by

$$S_{\mathrm{SE}} = \frac{1}{4G}\frac{\sigma(r_{hb}^2 - n^2)}{l^2} = \frac{A_{\mathrm{bolt}}}{4G}, \tag{46}$$

which is equal to the Bekenstein–Hawking entropy of the hyperbolic Taub–bolt AdS.[31]

IV. SUMMARY

By using the covariant phase techniques and Carlip's boundary constraint conditions,[30] the central extension of the constraint algebra has been worked out and the standard Virasoro subalgebrae with the corresponding central charge are constructed for the planar Taub–NUT AdS and Taub–bolt AdS spaces and the hyperbolic Taub–bolt AdS space. The densities of states calculated by using the Cardy formula yield the statistical entropies for these objects, which agree with their Bekenstein–Hawking entropies. The results support Carlip's supposition: regardless of the details of a quantum theory of gravity, symmetries inherited from the classical theory may be sufficient to determine the asymptotic behaviour of the density of states.

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