Vol 10 No 7, July 2001
1009-1963/2001/10(07)/0658-04
Chinese Physics
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and IOP Publishing Ltd

A FORMULA FOR CALCULATING THE ERRORS OF SUPERCONDUCTING MAGNETIZATION CURVE*

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(Received 16 October 2000; revised manuscript received 21 March 2001)

Because of field inhomogeneity in the magnetization measurement system, large errors may exist in the decreasing field superconducting magnetization curves, but not in the increasing field curves. The physical origin of the large errors is proposed here. A simple formula for calculating the errors is given. This formula is consistent with the experimental data.

Keywords: SQUID magnetometer, errors of superconducting magnetization measurement, formula for calculating the errors
PACC: 7460, 0755

I. INTRODUCTION

The magnetization curve data are important for investigating superconductor properties such as flux pinning, critical current, new sub-phases within the mixed states, and transitions among these phases.[1] However, there are certain problems with the decreasing measurement.[2−5] In the magnetic measurement system, for example the MPMS-5 SQUID magnetometer which is an advanced facility and is widely used, the magnetic field in the system is not perfectly uniform over the range of sample translation. The variable field or inhomogeneous field (dH) is proportional to the applied magnetic field and scan length. If the field is 5T and the measurement is performed over scan lengths of 3, 4, 5 and 6 cm, the dH has values of 0.0025, 0.01, 0.03 and 0.07 T, respectively.[6] If the sample is reversible, its measurement is not difficult and the resulting signal reflects the average magnetic field. But for irreversible samples, the dH will produce errors.[6,7] The superconducting irreversible properties may be measured towards reversible.[1] Thus it is ultimately important to know the error and accuracy of the measurement.[1,8,9]

We approach this problem through experiments. The samples used are melted YBCO, Bi(2223)/Ag tape and NbTi wire, and the data were taken with the MPMS-5 SQUID magnetometer. The data obtained and their magnetic hysteresis loop properties are essentially the same as those reported by other authors,[2−4] and, for brevity, are not reproduced here. The data show clearly that the field inhomogeneity in the measurement system greatly affects the superconducting magnetic hysteresis loops. In this paper we will try to find an answer to why the field nonuniformity greatly affects the superconducting magnetic hysteresis loops, and introduce a formula for calculating the errors which agrees well with the experimental data.

II. WHY AN INHOMOGENEOUS FIELD CAN PRODUCE LARGE ERRORS

According to the Bean model, when the field is increased and $H > H_{\rm p}$, ($H_{\rm p}$ is the field at which the flux lines can fully penetrate the superconductor centre), or when the decreasing field step length is larger than $2H_{\rm p}$, the shaded areas in Figs.1 and 2 represent flux profiles of increasing and decreasing fields, respectively. H is the field acting on the sample located under the pick-up coils. H+dH is the field acting on the sample when the sample moves to the centre of the pick-up coils, dH being the inhomogeneous field in the measurement system. The magnetic induction B is proportional to the shaded areas. The measured magnetization M is proportional to the difference of B and H, i.e. $M = (B-H)/4\pi$. For Fig.1 M is negative, and for Fig.2 M is positive.

For the increasing field measurement, owing to

*Project supported by the National High Technology Development Program of China (Grant No. 863-CD090000).

the flux pinning of type II superconductors, the shapes of the flux profiles are shown by Fig.1. Although the flux profile at field H (when the sample is located under the pick-up coils) is lower than the flux profile at field H+dH (when the sample is at the centre of the pick-up coils), the shapes of the flux profiles between the fields H and H+dH are not changed. This means that, when the field acting on the sample changes from H to H+dH, the magnetic field strength H and the magnetic induction B are increased by the same value; the $(B - H)$ remains unchanged. In this case, even if the dH is large, the measured M will maintain the same value. Thus the inhomogeneous field will not cause any error in the measurements performed by increasing field.

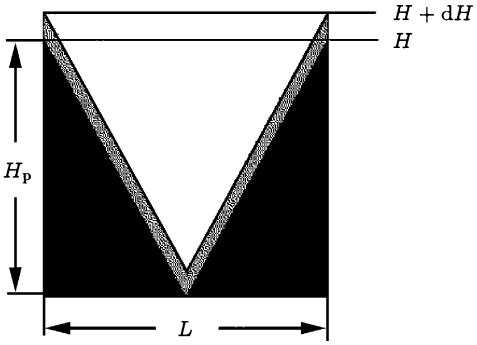


Fig.1. The increasing field superconducting flux profile. L is the diameter or the thickness of the sample. H_p is the field at which the flux lines can fully penetrate the superconductor. The shaded black area represents the flux profile under the field of H. The shaded grey area represents the flux profile variance produced by dH.

Since decreasing field measurement and decreasing field steps are larger than $2H_p$, the flux lines are concentrated to the centre of the superconductor (owing to the flux pinning of type II superconductors); M is positive. When the sample moves to the centre of the pick-up coils (which means the field acting on the sample changes from H to H+dH), the slopes of the flux line distribution in the superconductor are not in a single direction. The shapes of the flux profiles between H and H+dH are very different. These are shown in Fig.2. If dH=0, M will be proportional to the area $LH_p/2$ (L is the diameter or the thickness of the sample) and will be the true magnetization (no error value). Usually, it has a value symmetrical to that of the increasing field measurement.[1] If $0 < \mathrm{d}H < H_{c1}$, then dH can produce a perfect diamagnetic moment as large as $\mathrm{d}H/4\pi$. The dH makes the relative errors equal to $\mathrm{d}H/4\pi M$. Generally, in type II superconductors, this kind of error may be quite large. If $\mathrm{d}H > H_{c1}$, M will be proportional to the shaded black area in Fig.2. This area is much smaller than the area of $LH_p/2$. The larger the dH, the larger the errors. If dH is large enough, the errors can easily become as large as 100%. This is clearly illustrated in Fig.2.

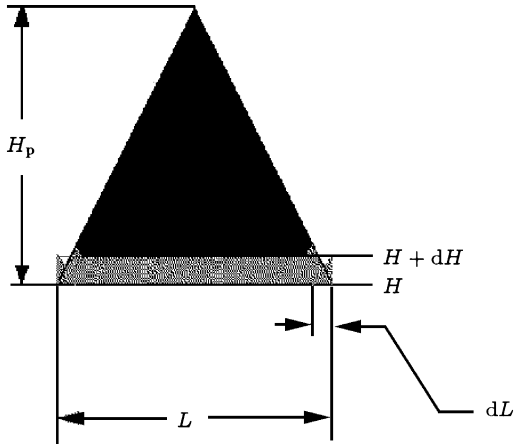


Fig.2. The decreasing field superconducting flux profile when the field change is larger than $2H_p$. L is the diameter or the thickness of the sample. H_p is the field at which the flux lines can fully penetrate the superconductor centre. The shaded black area represents $(B - H)$ under field H+dH.

From the difference of superconducting flux distributions between increasing and decreasing fields shown in Figs.1 and 2, we can understand why an inhomogeneous field in the measurement system will produce large errors on the decreasing field magnetization curves but not on the increasing field magnetization curves.

From the difference of superconducting flux distributions between H and H+dH shown in Fig.2, we will obtain a simple formula to calculate the errors and discuss this in the next section.

Ⅲ. A FORMULA FOR CALCULATING THE ERRORS

From Fig.2, if dH=0, then the measured magnetization M is proportional to $L \times H_p/2$,
i.e. $M_{\mathrm{d}H=0} \propto L \times H_p/2$. The relative error is zero.

If $\mathrm{d}H > 2H_p$, then the shape of the flux distribution is the same as that shown in Fig.1; $M_{\mathrm{d}H>2H_p} = -M_{\mathrm{d}H=0}$. The relative error is 200%.

If $2H_p > \mathrm{d}H > H_{c1}$, then the measured magnetization M is proportional to the shaded black area which is equal to $L \times H_p/2 - (L \times \mathrm{d}H - \mathrm{d}H \times \mathrm{d}L/4)$,
i.e. $M_{\mathrm{d}H\neq 0} \propto L \times H_p/2 - (L \times \mathrm{d}H - \mathrm{d}H \times \mathrm{d}L/4)$.

Since $\mathrm{d}L/L=\mathrm{d}H/H_\mathrm{p}$,

$$M_{\mathrm{d}H\neq 0} \propto L \times H_\mathrm{p}/2 - \{L \times \mathrm{d}H - 0.25(\mathrm{d}H/H_\mathrm{p})^2\}.$$

The relative errors are represented by

$$\begin{aligned}\mathrm{d}M =& (M_{\mathrm{d}H\neq 0} - M_{\mathrm{d}H=0})/M_{\mathrm{d}H=0} \\ =& -2\mathrm{d}H/H_\mathrm{p} + 0.5(\mathrm{d}H/H_\mathrm{p})^2.\end{aligned}$$

The formula for calculating the relative errors is

$$\mathrm{d}M = -2\mathrm{d}H/H_\mathrm{p} + 0.5(\mathrm{d}H/H_\mathrm{p})^2$$

$$(\text{when } \mathrm{d}H < 2H_\mathrm{p})$$

$$\mathrm{d}M = -2 \quad (\text{when } \mathrm{d}H \geq 2H_\mathrm{p}).$$

The above formula is based on some suppositions such as: the Bean model, H_{c1}=0, M is independent of H, the SQUID output signal of the near centre position dominates the measured signal (from the curve of SQUID output versus position, the output signal at the near centre position is much larger than the signal at the near bottom or top positions), and the field in the central part is perfectly uniform (the inhomogeneity of the field in the central part within 1cm is very small, the field uniformity being less than 0.0005%[6]), etc. The parameter H_p in the above formula is proportional to the critical current. The smaller the critical current, the larger the errors. However, H_p is almost independent of the initial diamagnetic moment and demagnetization coefficient, and the errors produced by dH come from the diamagnetic moment of superconductors. Thus H_p is not a good parameter for calculating the errors. We must look for another parameter.

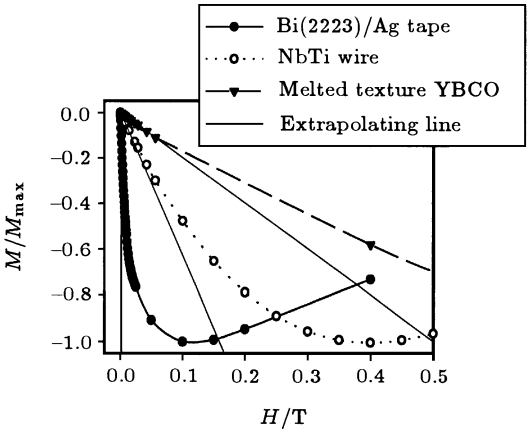


Fig.3. The relative initial diamagnetic curves of Bi(2223)/Ag tape (when H//C), NbTi wire and melt-textured YBCO.

For different samples, there are different superconducting initial diamagnetic curves as shown in Fig.3. H_m is the field obtained by linearly extrapolating the superconducting initial diamagnetic curve. The values of H_m for Bi(2223)/Ag tape, NbTi single filament wire and melt textured YBCO are 0.025T, 0.16T and 0.5T, respectively. H_m is not only closely related to the critical current and demagnetization coefficient, but also represents the superconducting diamagnetic property. Since the errors discussed here come from the diamagnetic moment of superconductors, H_m is a better parameter than H_p for characterizing the errors.

According to the above analysis, using H_m instead of H_p, we could rewrite the formula for calculating the relative errors as

$$\begin{aligned}&\mathrm{d}M = -2\mathrm{d}H/H_\mathrm{m} + 0.5(\mathrm{d}H/H_\mathrm{m})^2 \\ &(\text{when } \mathrm{d}H < 2H_\mathrm{m}), \\ &\mathrm{d}M = -2 \quad (\text{when } \mathrm{d}H > 2H_\mathrm{m}),\end{aligned} \tag{1}$$

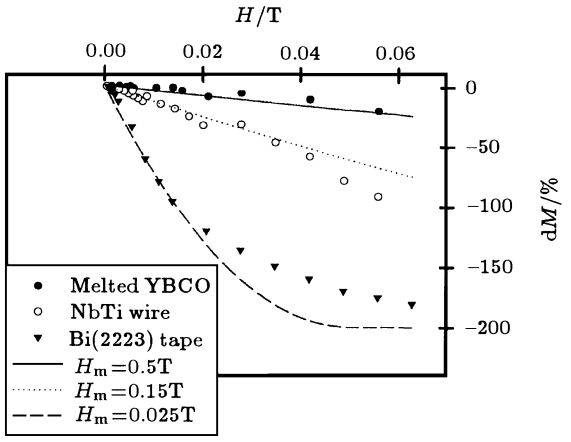


Fig.4. The relative errors produced by the inhomogeneous field. The curves are calculated by Eq.(1). The scattered points are experimental data.

Equation (1) is in good agreement with the experimental results. Figure 4 shows the relative errors produced by the inhomogeneous field. The curves are calculated by Eq.(1). The scattered points are experimental data. The negative sign of dM means that the measured data are smaller than the real data. If dH <0.02T, Eq.(1) is in agreement with the experimental results. If dH >0.02T, Eq.(1) can give a qualitative estimate of the effect of field nonuniformity on the decreasing field measurement. When the field is 5T and the measurement is performed with 4cm scan lengths, the dH is 0.01T. Thus for general measurement (a scan length less than 4cm), we can use Eq.(1) to calculate the errors.

H_m is an important parameter for discussing and calculating the errors. From dH, H_m and Eq.(1), we can easily find the errors produced by dH. H_m is proportional to the demagnetization coefficient and inversely proportional to the critical current. Thus the demagnetization coefficient and critical current determine their errors. Different superconducting samples have different errors. The smaller the critical current, the larger the errors. The larger the demagnetization coefficient, the larger the errors. For Bi(2223)/Ag tape used in this paper, when dH is 0.016T and $H \perp ab$, the error produced by dH is larger than 100%. But for melt-textured YBCO presented in this paper, when dH is 0.056T, the error is about 20%. Because the errors may be very large, when we report experimental superconducting magnetic hysteresis loops, we should also report the field nonuniformity or dH in the measuring system and the field H_m (especially when H_m is small and dH is large).

IV. CONCLUSIONS

In the measurement of type II superconducting magnetic hysteresis loops, there are large errors caused by the field nonuniformity in the measurement system on the decreasing field magnetization curve. For some samples, the errors can easily become as large as 100%. The errors are determined by the demagnetization coefficient and the critical current of the superconductor. The smaller the critical current, the larger the error, and the larger the demagnetization coefficient, the larger the error.

Using the difference of superconducting flux distributions between the cases of increasing and decreasing fields, we have not only explained why there are large errors in the decreasing field measurement but we have also found a formula to calculate the errors. In this formula, $dM = -2dH/H_m + 0.5(dH/H_m)^2$, H_m is obtained by linearly extrapolating the superconducting initial diamagnetic curve. When we report experimental superconducting magnetic hysteresis loops, we should also report the dH and H_m (especially when H_m is small and dH is large). H_m and dH are important parameters for discussing and calculating the errors.

ACKNOWLEDGMENTS

The authors are grateful to Professor H W Weber and Professor F M Sauerzopf of Atominstitut der Oester. Univ. Vienna, Austria, for helpful and fruitful discussions.

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