Vol 10 No 7, July 2001
1009-1963/2001/10(07)/0636-03
Chinese Physics
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and IOP Publishing Ltd

TO ENHANCE THE INTENSITY OF SONOLUMINESCENCE*

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(Received 15 September 2000; revised manuscript received 2 March 2001)

The transient resonance of a sonoluminescence bubble has been analysed. When the bubble performs its transient resonance at the nth order harmonics of the standing waves in the liquid, the light intensity strongly depends on the amplitude of the driving pressure (proportional to its $2n$ power, with $n = f_r/f$, where f_r is Minnaert's linear resonant frequency of the bubble and f is the frequency of driving sound). The kinetic energy of a vibrating bubble becomes maximum approximately when it is in its equilibrium size. For example, when the ambient temperature of a bubble decreases from 34°C to 4°C, a huge increase of the light intensity emitted by it can be explained. A suggestion was made that, within the limits permitted by the phase diagrams, as high an increase in driving pressure as possible could enhance the light intensity of sonoluminescence up to four orders of magnitude.

Keywords: sonoluminescence, nonlinear acoustics, transient resonance of a bubble
PACC: 4325

A gas bubble levitated in a strong sound field can emit bursts of light,[1] the duration of which seems to be in the range from several tens to several hundreds of picoseconds.[2,3] As is indicated by the experiment, some noble gas in air is essential for stable sonoluminescence (SL),[4] and a theoretical explanation of the rectified diffusion is also given.[5] Recently, however, we have been concerned with how to enhance the light intensity or the internal temperature of the bubble. The temperature can be predicted to increase up to 10^8K using the ideal shock-wave theory.[6] But as the inevitable dissipation of the gas in the bubble has to be taken into account, the temperature can decrease to several ten thousand degrees Kelvin.[7,8] Particularly, the results of multi-bubble sonoluminescence (MBSL) spectra have given much lower temperatures,[9] which may even reflect only the temperature of the liquid in the neighbourhood of the bubble, as pointed out in Ref.[10] concerning the formation of jets caused by MBSL. A result worth noting is that as the water temperature decreases from 40° to 1°C, the intensity of the light emission increases by a factor of over 200.[11] One explanation of this phenomenon has been given by means of numerical computation[12] whereas the physical picture is not so clear. In this paper, the author believes that a bubble will be in its transient resonance when it is driven by a strong sound. Analytically, we obtain a result that the energy density in the bubble, and therefore the light intensity of its sonoluminescence, depends on the $2n$th power of the amplitude of the driving pressure, where n is the ratio of its resonant frequency to the frequency of the driving pressure. The result shows that it may be possible to enhance the light intensity by up to four orders of magnitude.

When a transducer transmits an exiting sound pressure $p(t) = p_a \sin \omega t$ into a spherical vessel filled with a liquid, a standing-wave field can be created in it. In the linear acoustics, one can easily give an expression of the standing-wave field. As the acoustic amplitude p_a is very high in the nonlinear case, however, there is no expression available. By means of the perturbation method, the author of Ref.[13] obtained a nonlinear solution for a progressive wave and showed that the sound pressure of the nth harmonic wave $p^{(n)}$ is proportional to $(p_a)^n$. In this way, the author found that the standing-wave field in a spherical-symmetric vessel can be expanded as a Fourier series approximately, of which the nth harmonic wave can be denoted as

$$p^{(n)}(r,t) \sim A_n (p_a)^n F(r,t) \sin(n\omega t). \qquad (1)$$

In linear acoustics, $F(r,t) \sim j_0(nk_n r)$, where $j_0(x)$ is the zero-order spherical Bessel function, k_n satisfies $j_0(nk_n R)$=0. In general, $A_n F(r,t)$ are determined by the motion, the continuity and the state equations in the liquid. If a bubble is trapped at the pressure antinode, it will be in transient resonance when its resonant

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frequency is close to that of the nth-order harmonics in Eq.(1). For an infinitesimal vibration, the resonant frequency of a bubble of which the radius is a_0 can be denoted by[14]

$$\omega_r = 2\pi f_r = \frac{C_0}{a_0}\sqrt{\frac{3\rho_0}{\rho}}, \tag{2}$$

where ρ is the density of the liquid, and C_0 and ρ_0 are the sound speed and the density of the gas in the bubble, respectively. For example, if the frequency of the driving pressure f=26.4kHz, a_0=4μm, $[f_r(a)/f]$=31, i.e. the bubble will be in resonance at the 31st-order harmonics of the standing-wave field, where the notation $[x]$ means that only the integer part of x will be taken. Since the damping δ of a bubble is low, its resonant peak is so sharp that this vibration will be predominant in a short period during which $[x] < n+1$ (collapses) or $[x] > n-1$ (expands).

Let $N = [f_r(a_\mathrm{m})/f]$ and $n = [f_r(a_0)/f]$ be the orders of harmonics of the standing-wave field, where a_m and a_0 are the maximum and ambient radii of the bubble, respectively. Obviously, $N = [na_0/a_\mathrm{m}]$=3 when a_0 = 4μm, a_m/a_0=10 and f=26.5kHz, which means that the bubble will be in its transient resonance with the third-order harmonics of the standing-wave field during its expansion period. Since the expression $\tau_N = T[1/(N-1) - 1/N]$ denotes the driven time interval when the bubble expands from a radius less than a_m, where $T = 1/f$, during which process the bubble expands, its transient resonant frequency becomes lower and lower, its forcing amplitude becomes larger and larger, and the driven time will be longer and longer. Conversely, during the collapsing process, its transient resonant frequency becomes higher and higher, the forcing amplitudes becomes smaller and smaller, and the bouncing periods will be shorter and shorter. Thus, the larger N is, the longer τ_N is; for example, when N=3, τ_m=$T/6$. For an ideal resonator, the potential energy will be maximum when its amplitude reaches the maximum value a_m. On the other hand, the kinetic energy will be maximum and equals the maximum potential energy when it moves to its equilibrium position. The vibration of the bubble could be compared to a child playing on a swing; it will be accelerated sequentially by such a force that the forcing frequency is m/T with $m = n, n-1, n-2,...$ (expands) or $n, n+1, n+2,...$ (collapses). For a stable SL bubble, we believe that its potential energy can approximately transform into its kinetic energy when the radius of the bubble reaches its equilibrium value a_0, since the larger a_m/a_0 the smaller N, and the higher potential energy. Thus, the kinetic energy of a vibrating bubble with the radius a_0 will define the SL intensity of its emission.

When a bubble is trapped at the centre of the spherical vessel, the standing-wave field makes it pulsate. At the same time, the bubble will scatter the sound wave. Let the velocity potentials outside (the scattered wave) and inside the bubble be[14]

$$\varphi_\mathrm{os} = \sum_n B_n h_n^{(2)}(nk_n r)\sin(n\omega t) \tag{3}$$

and

$$\varphi_\mathrm{is} = \sum_n D_n j_0(nk_0 r)\sin(n\omega t), \tag{4}$$

respectively, where $j_0(x)$ and $h_0^{(2)}(x)$ are respectively the spherical Bessel and the second-kind spherical Hankel functions, and B_n and D_n are constants to be determined by the continuity conditions at the surface of the bubble. It can be seen that, at its surface, the velocity of the standing-wave field is zero. Since the arguments of the functions nk_0a_0 and nk_na_0 are much less than one even for n <60, one can easily obtain

$$D_n \approx \frac{\rho}{\rho_0} A_n (p_a)^n, \tag{5}$$

which means that the vibration of the gas in the bubble is proportional to p^n, and the kinetic energy density in the bubble

$$E_n = \rho_0 \left|\frac{\partial \varphi_\mathrm{is}}{\partial r}\right|^2_{r=a} = \rho_0 \left|\frac{A_n}{a_0}\right|^2 (p_a)^{2n}. \tag{6}$$

These results show that the energy in the bubble, and consequently the SL intensity, depend strongly on the amplitude of the driving sound. For example, as in Ref.[11], f=26.4 kHz, when the temperature decreases from 34°C to 4°C, the relevant quantities a_m/a_0, a_0 and p_a vary from 7, 4μm and 1.4 atm. to 14, 5.5μm and 1.25atm., and the relevant number of the harmonics N, n are accordingly 2, 31 and 3, 23. If n=23, $E_{23}(4°\mathrm{C})/E_{23}(34°)$=372; if n=31, $E_{31}(4°\mathrm{C})/E_{31}(34°\mathrm{C})$=2252. On the other hand, if one could so adjust the standing-wave field that $\mathrm{A}_{31}/\mathrm{A}_{23}$ is not too small, the SL intensity should be increased by a factor of G_nm=$E_{31}(4°\mathrm{C})/E_{23}(34°\mathrm{C})$=$8.4\times10^4 \times (A_{31}/A_{23})^2$. According to the theory of Fourier series' coefficients, one has

$$A_n j_0(nk_n a_0) \approx O\left(\frac{1}{n^{m+1}}\right),$$

where m is the order at which the derivative of the standing-wave field can exist; for example, m=0 is for

an ideal shock wave. Thus, $A_{31}/A_{23} \approx (23/31)^{m+1}$, from which one can see that even m=3 $G_{n3} \approx 10^4$ rather than several hundred times! These results remind us that are there any potentialities to increase the light intensity of sonoluminescence to be exploited. In so far as is permissible by the stable equilibrium, for instance, one can adjust the standing-wave field in order to increase $a_{\rm m}/a_0$, and increase p_a etc, in order to enhance the SL intensity further.

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