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NONLINEAR PULSE PROPAGATION IN TWIN-CORE FIBRES WITH BIREFRINGENCE

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An equation describing the nonlinear pulse propagation in twin-core fibres with birefringence is derived. Particularly, the properties of the nonlinear coupling equation are analysed, and modulational instability is discussed in detail.

Keywords: twin-core fibre, birefringence, modulational instability
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I. INTRODUCTION

The twin-core fibre is a special fibre which has two parallel cores, and is now used to produce devices widely used in all-optical communication systems, such as optical switches, optical logic gates, gain-flattened erbium-doped fibre amplifiers, optical soliton filters, and other important optical connecting devices.[1,2]

Normal mode theory and coupling mode theory are important methods of investigating pulse transmission in the twin-core fibre. Normal mode theory investigates the interaction between waveguides by looking upon the two cores and the envelope of the twin-core fibre as an integrated entity, and it is accurate in resolving some transmission problems when the fibre refractive index has a special distribution. The coupling mode theory is used to investigate the interaction of optical fields between independent cores of the twin-core fibre. It is direct and simple in resolving many coupling transmission problems, and is now used widely in coupling guides and, therefore, has important applications in many fields.[3−5]

When normal pulses, such as Gaussian pulses, propagate in the twin-core fibre, each pulse may be split into many pulses and the pulse shape distorted because of the coupling interaction between the two cores. Recent research on the twin-core fibre shows that if the soliton pulses are transmitted in the twin-core fibre, pulse splitting and deformation can be avoided.[6,7] In 1994, Vallee and Essiambre[8] discussed numerically the long transmission of solitons in the twin-core fibre with successive amplification. Their results showed that the twin-core fibre can be used as the optimum transmission media.

When pulses propagate in long distance fibre, there is constant or randomly varying birefrigence in the fibre due to the slight core ellipticity and asymmetric stresses of the refractive index.[9,10] Constant birefrigence can be used in pulse multiplex to increase the communication capacity, but the influence of randomly varying birefringence, if present, on soliton transmission in single or twin-core fibre cannot be neglected. It has been thought of as one of the most important limiting factors for capacity in optical fibre communication.[11,12]

In this paper, the equation which describes nonlinear pulse propagation in the twin-core fibre with birefringence is derived, and the properties of the nonlinear coupling equation and the modulational instability are discussed in detail. The influence of randomly varying birefringence on soliton switching, propagation in nonlinear fibre couplers, and polarization-division multiplexed solitons in twin-core fibres with polarization mode dispersion can be investigated with the nonlinear coupling equation in the future.

II. THEORETICAL ANALYSIS

In general, interaction between envelopes of each polarization of electric field in single-core fibres with birefringence takes place in the full fibre core.[13] Also, when the mode coupling is weak, the electric field in twin-core fibres with birefringence can be described as

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$$\begin{aligned}\boldsymbol{E}(r,\theta,z,t) =& U_1(z,t)R_1(r,\theta)\mathrm{e}^{\mathrm{i}(k_{10}z-\omega_{10}t)}\boldsymbol{e}_1 \\ &+ U_1^*(z,t)R_1(r,\theta)\mathrm{e}^{-\mathrm{i}(k_{10}z-\omega_{10}t)}\boldsymbol{e}_1^* \\ &+ V_1(z,t)S_1(r,\theta)\mathrm{e}^{\mathrm{i}(l_{10}z-\omega_{10}t)}\boldsymbol{d}_1 \\ &+ V_1^*(z,t)S_1(r,\theta)\mathrm{e}^{-\mathrm{i}(l_{10}z-\omega_{10}t)}\boldsymbol{d}_1^* \\ &+ U_2(z,t)R_1(r,\theta)\mathrm{e}^{\mathrm{i}(k_{20}z-\omega_{20}t)}\boldsymbol{e}_2 \\ &+ U_2^*(z,t)R_2(r,\theta)\mathrm{e}^{-\mathrm{i}(k_{20}z-\omega_{20}t)}\boldsymbol{e}_2^* \\ &+ V_2(z,t)S_2(r,\theta)\mathrm{e}^{\mathrm{i}(l_{20}z-\omega_{20}t)}\boldsymbol{d}_2 \\ &+ V_2^*(z,t)S_2(r,\theta)\mathrm{e}^{-\mathrm{i}(l_{20}z-\omega_{20}t)}\boldsymbol{d}_2^*, \quad (1)\end{aligned}$$

where U_i and V_i (i=1,2) designate the slowly varying envelopes of each polarization in the ith core of the twin-core fibre, k_{i0} and l_{i0} (i=1,2) are the respective wavenumbers, R_i and S_i are the transverse linear mode contributions, and $\boldsymbol{e}_i$ and $\boldsymbol{d}_i$ (i=1,2) are the unit vectors which vary with coordinates r and θ.

Linear variations of wavenumbers k_{i0} and l_{i0} (i=1,2) can be expanded as the linear dispersion relation

$$\begin{aligned}k_{i0} &= k_{i0}(\omega_0) + k_i'(\omega-\omega_0) + \frac{1}{2}k_i''(\omega-\omega_0)^2, \\ l_{i0} &= l_{i0}(\omega_0) + l_i'(\omega-\omega_0) + \frac{1}{2}l_i''(\omega-\omega_0)^2, \quad i=1,2.\end{aligned} \quad (2)$$

The following result is obtained

$$\begin{aligned}\mathrm{j}\frac{\partial U_i^{(\mathrm{linear})}}{\partial z} &= \mathrm{j}k_i'\frac{\partial U_i}{\partial t} + \frac{1}{2}k_i''\frac{\partial^2 U_i}{\partial t^2}, \\ \mathrm{j}\frac{\partial V_i^{(\mathrm{linear})}}{\partial z} &= \mathrm{j}l_i'\frac{\partial V_i}{\partial t} + \frac{1}{2}l_i''\frac{\partial^2 V_i}{\partial t^2}, \quad i=1,2. \end{aligned} \quad (3)$$

where $\mathrm{j}=\sqrt{-1}$.

The nonlinear and coupling variations due to the Kerr effect can be written as[13]

$$\frac{\partial \boldsymbol{E}^{(\text{nonlinear and coupling})}}{\partial z} = \frac{\mathrm{j}\chi}{3}(\boldsymbol{E}\cdot\boldsymbol{E})\boldsymbol{E} \quad (4)$$

where χ is the usual Kerr coefficient.

We substitute Eq.(1) into Eq.(4), and average over the two transverse dimensions. This average introduces an extra factor which roughly equals 1/2.[13] We keep only the slowly varying terms, and assume that $U_1\boldsymbol{e}_1$ and $U_2\boldsymbol{e}_2$ are in the same direction, as are $V_1\boldsymbol{d}_1$ and $V_2\boldsymbol{d}_2$. Based on the optical waveguide coupling theory investigated by Snyder and Love,[14] the coupling in the same (parallel) direction is strong, but coupling in the orthogonal direction is weak and can be neglected. We find

$$\begin{aligned}&\frac{\partial U_i^{(\text{nonlinear and coupling})}}{\partial z} \\ =&\frac{\mathrm{j}\chi}{2}\left[|U_i|^2U_i + \frac{2}{3}|V_i|^2U_i\right] \\ &+ \frac{\mathrm{j}\chi}{6}V_i^2U_i^*\exp[-2\mathrm{j}(k_{i0}-l_{i0})] \\ &+ \frac{\mathrm{j}\chi}{2}\left[U_iU_{3-i}^* + 2|U_{3-i}|^2\right. \\ &\left.+ \frac{2}{3}(V_iV_{3-i}^* + U_i^*U_{3-i} + |V_{3-i}|^2)\right]U_{3-i} \\ &\frac{\partial V_i^{(\text{nonlinear and coupling})}}{\partial z} \\ =&\frac{\mathrm{j}\chi}{2}\left[|V_i|^2V_i + \frac{2}{3}|U_i|^2V_i\right] \\ &+ \frac{\mathrm{j}\chi}{6}U_i^2V_i^*\exp[-2\mathrm{j}(k_{i0}-l_{i0})] \\ &+ \frac{\mathrm{j}\chi}{2}\left[V_iV_{3-i}^* + 2|V_{3-i}|^2\right. \\ &\left.+ \frac{2}{3}(U_iU_{3-i}^* + V_i^*V_{3-i} + |U_{3-i}|^2)\right]V_{3-i}, \\ &\qquad i=1,2. \end{aligned} \quad (5)$$

We can separate the nonlinear variation and the coupling variation from Eq.(5), and find

$$\begin{aligned}&\frac{\partial U_i^{(\text{nonlinear})}}{\partial z} \\ =&\frac{\mathrm{j}\chi}{2}\left[|U_i|^2U_i + \frac{2}{3}|V_i|^2U_i\right] \\ &+ \frac{\mathrm{j}\chi}{6}V_i^2U_i^*\exp[-2\mathrm{j}(k_{i0}-l_{i0})] \\ &\frac{\partial V_i^{(\text{nonlinear})}}{\partial z} \\ =&\frac{\mathrm{j}\chi}{2}\left[|V_i|^2V_i + \frac{2}{3}|U_i|^2V_i\right] \\ &+ \frac{\mathrm{j}\chi}{6}U_i^2V_i^*\exp[2\mathrm{j}(k_{i0}-l_{i0})], \\ &\qquad i=1,2; \end{aligned} \quad (6)$$

$$\begin{aligned}&\frac{\partial U_i^{(\text{coupling})}}{\partial z} \\ =&\frac{\mathrm{j}\chi}{2}\left[U_iU_{3-i}^* + 2|U_{3-i}|^2\right. \\ &\left.+ \frac{2}{3}(V_iV_{3-i}^* + U_i^*U_{3-i} + |V_{3-i}|^2)\right]U_{3-i} \\ &\frac{\partial V_i^{(\text{coupling})}}{\partial z} \\ =&\frac{\mathrm{j}\chi}{2}\left[V_iV_{3-i}^* + 2|V_{3-i}|^2\right. \\ &\left.+ \frac{2}{3}(U_iU_{3-i}^* + V_i^*V_{3-i} + |U_{3-i}|^2)\right]V_{3-i}, \\ &\qquad i=1,2. \end{aligned} \quad (7)$$

The coupling nonlinear Schrödinger equation, which describes pulse propagation in twin-core fibres with birefringence, can be derived by combining Eqs.(3), (6) and (7):

$$
\begin{aligned}
&\mathrm{j}\left(\frac{\partial U_i}{\partial z}+k_i'\frac{\partial U_i}{\partial t}\right)-\frac{1}{2}k_i''\frac{\partial^2 U_i}{\partial t^2}\\
&+\frac{\chi}{2}\left[|U_i|^2U_i+\frac{2}{3}|V_i|^2U_i\right]\\
&+\frac{\chi}{6}V_i^2U_i^*\exp[-2\mathrm{j}(k_{i0}-l_{i0})]\\
&+\frac{\chi}{2}\left[U_iU_{3-i}^*+2|U_{3-i}|^2\right.\\
&\left.+\frac{2}{3}(V_iV_{3-i}^*+V_i^*V_{3-i}+|V_i|^2)\right]U_{3-i}=0,\\
&\mathrm{j}\left(\frac{\partial V_i}{\partial z}+l_i'\frac{\partial V_i}{\partial t}\right)-\frac{1}{2}l_i''\frac{\partial^2 V_i}{\partial t^2}\\
&+\frac{\chi}{2}\left[|V_i|^2V_i+\frac{2}{3}|U_i|^2V_i\right]\\
&+\frac{\chi}{6}U_i^2V_i^*\exp[2\mathrm{j}(k_{i0}-l_{i0})]\\
&+\frac{\chi}{2}\left[V_iV_{3-i}^*+2|V_{3-i}|^2\right.\\
&\left.+\frac{2}{3}(U_iU_{3-i}^*+U_i^*U_{3-i}+|U_i|^2)\right]V_{3-i}=0,\\
&\qquad i=1,2. \qquad (8)
\end{aligned}
$$

We can normalize Eq.(8) by assuming

$$
\begin{aligned}
&k_i''=l_i''=-\frac{\lambda_i}{2\pi c^2}D(\lambda),\qquad i=1,2,\\
&k_1'=k_2',\quad l_1'=l_2'. \qquad (9)
\end{aligned}
$$

We define

$$
\begin{aligned}
&\xi=\frac{\pi z}{2z_0},\quad z_0=\frac{\pi^2c^2t_0^2}{D(\lambda)\lambda_0},\\
&t_0=0.568\tau_0,\quad t=\frac{1}{t_0}\left(t-\frac{z}{\overline{\nu}_g}\right),\\
&\overline{\nu}_g=\frac{2}{k_i'+l_i'},\quad \rho=\frac{8\pi c}{\lambda_0}t_0,\quad \delta_i=\frac{\pi c\Delta n_i}{D(\lambda)\lambda_0},\\
&u_i=\left(\frac{\chi}{2}\right)^{1/2}U_i,\quad \nu_i=\left(\frac{\chi}{2}\right)^{1/2}V_i,\quad i=1,2.
\end{aligned}
$$

We can find the normalized coupling nonlinear Schrödinger equation

$$
\begin{aligned}
&\mathrm{j}\left(\frac{\partial u_i}{\partial \xi}+\delta_i\frac{\partial u_i}{\partial \tau}\right)-\frac{1}{2}\frac{\partial^2 u_i}{\partial \tau^2}+\left[|u_i|^2u_i+\frac{2}{3}|\nu_i|^2\nu_i\right]\\
&+\frac{1}{3}\nu_i^2u_i^*\exp[-2\mathrm{j}\rho\delta\xi]+C_{ii}u_{3-i}=0,\\
&\mathrm{j}\left(\frac{\partial \nu_i}{\partial \xi}-\delta_i\frac{\partial \nu_i}{\partial \tau}\right)+\frac{1}{2}\frac{\partial^2 \nu_i}{\partial \tau^2}+\left[|\nu_i|^2\nu_i+\frac{2}{3}|u_i|^2u_i\right]\\
&+\frac{1}{3}u_i^2\nu_i^*\exp[2\mathrm{j}\rho\delta\xi]+C_{i3-i}\nu_{3-i}=0,\\
&\qquad i=1,2, \qquad (10)
\end{aligned}
$$

where C_{ii} and C_{i3-i} (i=1,2) are the coupling coefficients, and

$$
\begin{aligned}
C_{ii}=&\left[u_iu_{3-i}^*+2|u_{3-i}|^2\right.\\
&\left.+\frac{2}{3}(\nu_i\nu_{3-i}^*+\nu_i^*\nu_{3-i}+|\nu_i|^2)\right],\\
C_{i3-i}=&\left[\nu_i\nu_{3-i}^*+2|\nu_{3-i}|^2\right.\\
&\left.+\frac{2}{3}(u_iu_{3-i}^*+u_i^*u_{3-i}+|u_i|^2)\right],\\
&\qquad i=1,2. \qquad (11)
\end{aligned}
$$

From Eq.(11), we can see that the coupling coefficients between two cores vary with the intensities of the propagation pulses in the twin-core fibre. In many research papers on twin-core fibres, the coupling coefficients are thought of as average values,[1−8] but in fact they vary along the transmission line.

We discuss the properties of the nonlinear coupling equation. If $u_1\approx u_2$, and $\nu_1\approx\nu_2$, then $C_{11}\approx C_{22}$, and $C_{12}\approx C_{21}$. Furthermore, assuming $u_1\approx u_2\approx\nu_1\approx\nu_2$, then $C_{11}\approx C_{22}\approx C_{12}\approx C_{21}$, the twin-core fibre with polarization can be used in the polarization multiplex, and its properties are important for the polarization multiplexed soliton.[15−17]

In twin-core fibres with randomly varying birefringence, $u_1\approx u_2$, and $|u_i\gg|\nu_i|$ (i=1,2), then $C_{ii}\approx 3C_{i3-i}/2$, and the properties are important for nonlinear pulse (soliton) transmission in twin-core fibres.[12]

Neglecting the oscillating and coupling terms, the solutions to Eq.(10) are

$$
\begin{aligned}
&u_i(\xi,\tau)=\left(\frac{3}{5}\right)^{1/2}\exp\left[\frac{\mathrm{j}}{2}(1+\delta_i^2)\xi-\mathrm{j}\delta\tau\right]\sec h\tau,\\
&\nu_i(\xi,\tau)=\left(\frac{3}{5}\right)^{1/2}\exp\left[\frac{\mathrm{j}}{2}(1+\delta_i^2)\xi+\mathrm{j}\delta\tau\right]\sec h\tau,\\
&\qquad i=1,2. \qquad (12)
\end{aligned}
$$

Now we can analyse the modulational instability of the normalized coupling nonlinear Schrödinger equations (10). The solutions to the equation neglecting the oscillating terms are

$$
\begin{aligned}
&u_i(\xi,\tau)=u_{i0}\exp(\mathrm{j}\beta_{ii}\xi),\\
&\nu_i(\xi,\tau)=\nu_{i0}\exp(\mathrm{j}\beta_{i3-i}\xi),\\
&\qquad i=1,2. \qquad (13)
\end{aligned}
$$

Substituting Eq.(13) into Eq.(10), neglecting the oscillating terms, we can find

$$\beta_{11} = u_{10}^2 + \frac{2}{3}\nu_{10}^2 + c_{11}u_{20}\exp[\mathrm{j}(\beta_{22} - \beta_{11})],$$
$$\beta_{12} = \nu_{10}^2 + \frac{2}{3}u_{10}^2 + c_{12}\nu_{20}\exp[\mathrm{j}(\beta_{21} - \beta_{12})],$$
$$\beta_{22} = u_{20}^2 + \frac{2}{3}\nu_{20}^2 + c_{22}u_{10}\exp[\mathrm{j}(\beta_{11} - \beta_{22})],$$
$$\beta_{21} = \nu_{20}^2 + \frac{2}{3}u_{20}^2 + c_{22}\nu_{10}\exp[\mathrm{j}(\beta_{12} - \beta_{21})]. \quad (14)$$

Considering the oscillating terms, the solutions of Eq.(10) can be written as

$$u_i(\xi,\tau) = (u_{i0} + \overline{u}_i)\exp(\mathrm{j}\beta_{ii}\xi),$$
$$\nu_i(\xi,\tau) = (\nu_{i0} + \overline{\nu}_i)\exp(\mathrm{j}\beta_{i3-i}\xi),$$
$$i = 1,2. \quad (15)$$

where $\overline{u}_i$ and $\overline{\nu}_i$ (i=1,2) are small perturbations.

Substituting Eq.(15) into Eq.(10), we can find the linear equation

$$\mathrm{j}\left(\frac{\partial\overline{u}_i}{\partial\xi} + \delta_i\frac{\partial\overline{u}_i}{\partial\tau}\right) - \frac{1}{2}\frac{\partial^2\overline{u}_i}{\partial\tau^2} + u_{i0}^2(\overline{u}_i + \overline{u}_i^*) + \frac{2}{3}\nu_{i0}^2(\overline{\nu}_i + \overline{\nu}_i^*) + C_{ii}u_{3-i}\exp[\mathrm{j}(\beta_{3-i3-i} - \beta_{ii})] = 0,$$
$$\mathrm{j}\left(\frac{\partial\overline{\nu}_i}{\partial\xi} + \delta_i\frac{\partial\overline{\nu}_i}{\partial\tau}\right) - \frac{1}{2}\frac{\partial^2\overline{\nu}_i}{\partial\tau^2} + \nu_{i0}^2(\overline{\nu}_i + \overline{\nu}_i^*) + \frac{2}{3}u_{i0}^2(\overline{u}_i + \overline{u}_i^*) + C_{ii}\nu_{3-i}\exp[\mathrm{j}(\beta_{3-ii} - \beta_{i3-i})] = 0,$$
$$i = 1,2. \quad (16)$$

Approximately, we assume that the propagating pulses in two cores of the twin-core fibre have the same propagation constants (β_{11}=β_{22} and β_{12}=β_{21}), and $\overline{u}_i = m_i + \mathrm{j}n_i$, $\overline{\nu}_i = r_i + \mathrm{j}s_i$ (i=1,2), and also assume

$$\overline{u}_i = (m_{i0} + \mathrm{j}n_{i0})\exp[\mathrm{j}(K\xi - \Omega\tau)],$$
$$\overline{\nu}_i = (r_{i0} + \mathrm{j}s_{i0})\exp[\mathrm{j}(K\xi - \Omega\tau)],$$
$$i = 1,2. \quad (17)$$

Substituting Eq.(17) into Eq.(16), we can find the linearized equations

$$\frac{\partial m_i}{\partial\xi} + \delta_i\frac{\partial m_i}{\partial\tau} + \frac{1}{2}\frac{\partial^2 n_i}{\partial\tau^2} + C_{ii}n_{3-i} = 0,$$
$$-\frac{\partial n_i}{\partial\xi} - \delta_i\frac{\partial n_i}{\partial\tau} + \frac{1}{2}\frac{\partial m_i}{\tau^2} + 2m_iu_{i0}^2 + \frac{4}{3}\nu_{i0}u_{i0}r_i + C_{ii}m_{3-i} = 0,$$
$$\frac{\partial r_i}{\partial\xi} - \delta_i\frac{\partial r_i}{\partial\tau} + \frac{1}{2}\frac{\partial^2 s_i}{\partial\tau^2} + C_{i3-i}s_{3-i} = 0,$$
$$-\frac{\partial s_i}{\partial\xi} + \delta_i\frac{\partial s_i}{\partial\tau} + \frac{1}{2}\frac{\partial^2 r_i}{\tau^2} + 2r_i\nu_{i0}^2 + \frac{4}{3}u_{i0}\nu_{i0}m_i + C_{i3-i}r_{3-i} = 0,$$
$$i = 1,2. \quad (18)$$

We assume

$$\begin{bmatrix} m_1 \\ n_1 \\ r_1 \\ s_1 \\ m_2 \\ n_2 \\ r_2 \\ s_2 \end{bmatrix} = \begin{bmatrix} m_{10} \\ n_{10} \\ r_{10} \\ s_{10} \\ m_{20} \\ n_{20} \\ r_{20} \\ s_{20} \end{bmatrix} \exp[\mathrm{j}(K\xi - \Omega\tau)],$$

$$i = 1,2. \quad (19)$$

The dispersion relation can be obtained as

$$(K - \delta_1\Omega)^2(K + \delta_1\Omega)^2(K - \delta_2\Omega)^2(K + \delta_2\Omega)^2 - \frac{\Omega^4}{4}C_{12}C_{21}\left(2u_{10}^2 - \frac{\Omega^2}{2}\right)\left(2u_{20}^2 - \frac{\Omega^2}{2}\right) - \frac{\Omega^4}{4}C_{11}C_{22}\left(2\nu_{10}^2 - \frac{\Omega^2}{2}\right)\left(2\nu_{20}^2 - \frac{\Omega^2}{2}\right) = 0. \quad (20)$$

We can expand the solution to Eq.(20) as different powers of δ, and find

$$K = K_0 + K_1,$$
$$K_0 = -\mathrm{j}\left\{\frac{\Omega^4}{16}\left[C_{12}C_{21}\left(u_{10}^2 - \frac{\Omega^2}{4}\right)\left(u_{20}^2 - \frac{\Omega^2}{4}\right) + C_{11}C_{22}\left(\nu_{10}^2 - \frac{\Omega^2}{4}\right)\left(\nu_{20}^2 - \frac{\Omega^2}{4}\right)\right]\right\}^{1/8},$$
$$K_1 = \left[\frac{\delta_1^2\delta_2^2}{\delta_1^2 + \delta_2^2}\right]^{1/2}\Omega. \quad (21)$$

We can find the maximum growth rate of instability

at

$$\Omega = \left\{ \frac{3}{8} \times \frac{C_{12}C_{21}(u_{10}^2+u_{20}^2)+C_{11}C_{22}(\nu_{10}^2+\nu_{20}^2)}{C_{12}C_{21}+C_{11}C_{22}} + \left\{ \left[\frac{8}{3} \times \frac{C_{12}C_{21}(u_{10}^2+u_{20}^2)+C_{11}C_{22}(\nu_{10}^2+\nu_{20}^2)}{C_{12}C_{21}+C_{11}C_{22})} \right]^2 - \left(\frac{1}{2} \times \frac{C_{12}C_{21}u_{10}^2u_{20}^2+C_{11}C_{22}\nu_{10}^2\nu_{20}^2}{C_{12}C_{21}+C_{11}C_{22}} \right) \right\}^{1/2} \right\}^{1/2}. \quad (22)$$

In twin-core fibres with randomly varying birefringence, namely $u_1 \approx u_2$, and $|u_i| \gg |\nu_i|$, (i=1,2) the maximum growth rate of instability is located at

$$\Omega = \left\{ \frac{3}{8} \times (u_{10}^2+u_{20}^2) + \left\{ \left[\frac{3}{8} \times (u_{10}^2+u_{20}^2) \right]^2 - \left(\frac{1}{2} \times u_{10}^2u_{20}^2 \right) \right\}^{1/2} \right\}^{1/2}. \quad (23)$$

When the twin-core fibre with polarization can be used in polarization multiplex, namely $u_1 \approx u_2 \approx \nu_1 \approx \nu_2 = \eta$, and $C_{11} \approx C_{22} \approx C_{12} \approx C_{21}$, the maximum growth rate of instability is located at $\Omega = |\eta|$. Based on the equation about the nonlinear pulse propagation in twin-core fibres with birefringence, we have discussed soliton train transmission in twin-core fibres with randomly varying birefringence.[12] In the future, we shall investigate the influence of randomly varying birefringence on soliton switching, soliton propagation in nonlinear fibre couplers, and polarization-division multiplexed solitons in twin-core fibres with polarization mode dispersion.

III. CONCLUSION

In this paper, an equation which describes nonlinear pulse propagation in twin-core fibres with birefringence is derived by separating the linear, nonlinear and coupling variations. The properties of the nonlinear coupling equation are analysed particularly and modulational instability, both when the twin-core fibre with polarization can be used in the polarization multiplex and when the twin-core fibre is of randomly varying birefringence, is discussed in detail.

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