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NEW MESON MASS RELATION AND LOWEST PSEUDOSCALAR GLUEBALL MASS*

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After considering its mixing with the glueball, we give a new mass relation for the meson nonet. According to this mass relation and the predicted mass of the pseudoscalar glueball given by lattice calculations and the effective Hamiltonian, the expected mass of the mixed pseudoscalar glueball is about 1.7 GeV. This result is helpful in the experimental search for the mixed isoscalar pseudoscalar glueball. $\eta(1760)$ is discussed as a possible candidate for this type of particle.

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There is some inconsistency between theory and experiment about the meson mass spectrum and the lowest glueball mass. For example, the lowest mass of the pseudoscalar glueball which is given by lattice calculation is about $2232 \pm 370 \pm 220\,\mathrm{MeV}$.[1] However, the mass of the experimental candidate is much lower than this value.[2] Another question concerns the Gell-Mann–Okubo formula.[3] It is known that the vector meson nonet and baryon decuplet satisfy the Gell-Mann–Okubo formula well. But the pseudoscalar meson nonet does not satisfy the Gell-Mann–Okubo formula at all. Why does the pseudoscalar meson nonet not satisfy this formula? The third question is about the glueball–meson mixing. It is believed that there is mixing between the meson and the glueball. If there is mixing, how can we determine the masses of the glueball and the meson after mixing? A meson mass relation can be obtained after considering glueball–meson mixing. This mass relation is almost well satisfied by all the meson nonet,[4] and it can help us to determine the masses of some mixed glueballs.

Firstly, let us discuss the general case. Suppose that there is a meson nonet. M_1^0 and M_8^0 are the $SU(3)$ singlet state and the isoscalar of $SU(3)$ nonet states. The nearest glueball which has the same $I^G J^{PC}$ as the nonet is denoted as G^0. Generally, these three states will have mixing. The corresponding states after mixing are denoted as M_1, M_8 and G. Suppose that the mixing matrix is U, then we have

$$\begin{pmatrix} M_1 \\ M_8 \\ G \end{pmatrix} = U \begin{pmatrix} M_1^0 \\ M_8^0 \\ G^0 \end{pmatrix}. \tag{1}$$

In the most general case, the mixing matrix is a 3×3 unitary matrix. However, in the present case, it is a real orthogonal matrix. If the mass matrix before mixing is denoted as M^0, then the mass matrix after mixing is given by:

$$M = UM^0U^t, \tag{2}$$

where $U^t = U^{-1}$ is the transposition matrix of U. The diagonal elements of the mass matrix M^0 are $m_{M_1^0}$, $m_{M_8^0}$ and m_{G^0} respectively, and the diagonal elements of the mass matrix M are m_{M_1}, m_{M_8} and m_G respectively. The trace of equation (2) gives the following relation:

$$m_{M_1^0} + m_{M_8^0} + m_{G^0} = m_{M_1} + m_{M_8} + m_G. \tag{3}$$

The particles that have isospin 1 in the meson nonet are denoted as M_π and the particles have isospin 1/2 are denoted as M_K. Supposed that the Gell-Mann–Okubo mass formula is well satisfied by the meson nonet before mixing, i.e.

$$4m_{M_\mathrm{K}} - m_{M_\pi} = 3m_{M_8^0} \quad 2m_{M_\mathrm{K}} + m_{M_\pi} = 3m_{M_1^0}. \tag{4}$$

Then we have

$$2m_{M_\mathrm{K}} = m_{M_8^0} + m_{M_1^0}. \tag{5}$$

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Thus, equation (3) will change into:

$$2m_{M_K} + m_{G^0} = m_{M_1} + m_{M_8} + m_G. \qquad (6)$$

This is the mass relation for the meson nonet. In the above relation, M_1, M_8 and M_K are mesons which have already been observed in experiments and their masses are already known. G^0 is the pure glueball, and its mass is predicted by theoretical calculations. So, in the above relation, the only unknown is the mass of the mixed glueball G. This can be calculated from the above relation. Therefore, this meson mass relation can help us to determine the mass of the mixed glueball G.

Now, let us use this mass relation to discuss the lowest mass of the pseudoscalar glueball. According to the Particle Data Group (PDG),[2] the two pseudoscalar isoscalar mesons are η and η'. Their masses are

$$m_\eta = 547 \text{ MeV}, \quad m_{\eta'} = 958 \text{ MeV}. \qquad (7)$$

The isospin 1/2 mesons are kaons; their masses are $m_K = 494$ MeV. Then equation (6) changes into:

$$m_G = m_{G^0} - 517(\text{MeV}). \qquad (8)$$

In the above relation, m_{G^0} is the mass of the pure pseudoscalar glueball. Its mass can be obtained from theoretical calculations. The calculated mass of the pure pseudoscalar glueball from lattice calculations is $2.23 \pm 0.37 \pm 0.22$ MeV.[1] Then equation (8) means that the centre value of the lowest mass of the mixed glueball is about 1713 MeV. However, its predicted centre mass from an effective Hamiltonian is about 2.19 MeV.[5] Then the lowest centre mass of the mixed glueball is about 1673 MeV. According to the PDG,[2] there are two pseudoscalar mesons below 2.0 GeV: $\eta(1440)$ and $\eta(1760)$. The mass of $\eta(1760)$ is much closer to the predicted masses given by both lattice calculations and effective Hamiltonian. The width of this resonance is narrow (only about 60 MeV).[6] This resonance is also found in $J/\psi \to \gamma K\bar{K}\pi$ and $J/\psi \to \gamma\eta\pi\pi$.[7] Using all these experimental results, the partial width of $\eta(1760)$ decaying into two gluons can be calculated.[8] It is found that this partial width is large. From these results, especially its mass position, it seems that $\eta(1760)$ is more like a mixed glueball. But in order to confirm the glueball nature of $\eta(1760)$, many things need to be done, particularly the accurate measurement of the total branching ratio of $J/\psi \to \gamma\eta(1760)$, its mass and width, and a detailed study of its decay pattern.

Evidence to support the glueball nature of $\eta(1760)$ can be found from the study of $J/\psi \to \gamma\gamma\rho^0$.[9] For $\eta(1440)$ appears in the invariant mass spectrum of $\gamma\rho^0$, while $\eta(1760)$ does not, and the glueball should have a very weak coupling with photons. Further evidence to support the glueball nature of $\eta(1760)$ can be found from J/ψ radiative decay to $\gamma\eta'$. The mass of η is close to the mass of η_8, but the mass of η' is much higher than the mass of η_1. The glueball content is mainly mixed into η' so as to increase its mass by about 500 MeV. In other words, the meson $\eta'(958)$ should contain large amount of glueball content. This result is consistent with experimental results because η' is abundantly produced in radiative J/ψ decay. The branching ratio of $J/\psi \to \gamma\eta'$ is large,[2] 4.31×10^{-3}. The large sample of J/ψ data in BES provides a good opportunity for the confirmation of the mixed isoscalar pseudoscalar glueball.

Equation (6) is also helpful in the search for the scalar and tensor glueballs.[4] For example, the two isoscalar tensor mesons are $f_2(1270)$ and $f_2'(1525)$, and the isospin 1/2 tensor meson is $K_2^*(1430)$. According to equation (6), we have $m_G \approx m_{G^0}$. This means that the mass of the mixed tensor glueball is very close to the mass of the pure glueball. The mixing between the glueball and ordinary tensor mesons is small. The mass of the tensor glueball should be close to theoretical predictions.

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