Vol 10 No 7, July 2001
1009-1963/2001/10(07)/0599-07

Chinese Physics

© 2001 Chin. Phys. Soc.
and IOP Publishing Ltd

ADDING ONE BIFURCATION IN A ROTOR SYSTEM WITH CLEARANCES*

Sun Zheng-ce(孙政策) and Xu Jian-xue(徐健学)

Institute of Nonlinear Dynamics, Xi'an Jiaotong University, Xi'an, 710049, China

(Received 18 September 2000; revised manuscript received 4 March 2001)

A mathematical model of a rotor system with clearances is analysed by the application of modern nonlinear dynamic theory. From the bifurcation diagrams, it is discovered that the rotor system alternates between periodic and chaotic motions at a supercritical rotational speed, and after undergoing a chaotic region the periodic number of the motion will increase by one. At the same time, the periodic number is equal correspondingly to the integral multiple of the critical rotational speed. At the subcritical rotational speed, it is discovered that the chaotic bands among successive orders of superharmonic responses return to the period one through a reversed period-doubling bifurcation, as a result of a period-doubling bifurcation. It is shown that the increase of damping may reduce the width of the chaotic bands and the amplitude of the periodic response; the increase of nonlinear degree also leads to the reduction of chaotic bandwidth, but makes the amplitude of the subharmonic response increase. So it is suggested that proper damping and correct material selection by considering the dynamic characteristics of the rotor system may reduce the proportion of faults and enhance the dynamic characteristics when designing the rotor system. The working speed should not be selected at N times its natural frequency and should not be set in the chaotic bands among the successive orders of periodic motion for the same purpose.

Keywords: chaos, bifurcation, impacting, transition zone
PACC: 0545

I. INTRODUCTION

Clearance is inevitable in rotational facilities in practice. It may cause the joints of moving parts to wear away and loosen; an accident could even arise because of the impact between them. In a high-speed rotor system, the impacting caused by clearance between the rotor and stator is one of the important causes of faults.

Clearance is also one of the principal factors giving rise to nonlinear elements in a mathematical model of rotor systems. Because of its existence, impacting can arise when the rotor system is working. It can cause the system to get into complicated dynamic phenomena, which include not only periodic elements, such as fractional frequency, synchronous frequency and frequency-doubling responses, but also quasi-periodic and chaotic elements. By systematically studying the dynamic characteristics of rotor systems with clearances, some theoretical basis may be obtained for improving the dynamic properties of the rotor system and properly diagnosing accidents.

Due to the complexity of the dynamic behaviour raised by the clearance, many scholars have already studied the phenomenon. Kobrinsky[1] documented the existence of various periodic regions in vibro-impacting systems. As was noticed later by Thompson and Ghaffari,[2] when periodically excited, the structure may respond in a fairly regular and orderly fashion, as well as entirely chaotic patterns of motion. The chaotic behaviour of rotational structures with clearances and the rubs against stationary elements were discussed by Szczygielski,[3] and Ehrich.[4,5] Goldman[6] studied the supercritical subharmonic phenomena of rotor systems with clearances through an analytical approach associated with numerical calculations. A reversed period adding was reported by Wai Chin *et al.*[7]

In this paper, we have numerically analysed the model of a rotor system with clearances, and drawn a conclusion that the addition of one bifurcation occurs at the supercritical speed, together with the skips of amplitude arising at the subcritical speed. By using bifurcation diagrams, the influence of nonlinear stiffness and damping on the system has been investigated, and then some pertinent advice is given for improving the dynamic characters of the rotor system.

II. MATHEMATICAL MODEL OF A ROTOR SYSTEM

*Project supported by the National Natural Science Foundation of China (Grant Nos. 19990510 and 19972051).

The object studied in this paper is a simplified Jeffcott rotor system having a single point mass with two degrees of freedom mounted on a massless flexible shaft, as shown in Fig.1.

Due to the existence of clearance between the rotor and stator, analysis of the rotor system may be divided into two stages in terms of its impacting situation. The first stage is that, before impacting, the rotor system exerts a spring restitution force from the flexible shaft deformation. The second stage is that, after impacting, the system exerts a spring restitution force from the stator deformation, in addition to that of shaft deformation. The impacting time between the stator and the rotor is so short that we think the stator deformation may be linear, and then the rotor system is analogously acted upon by a bilinear spring force. The bilinear spring force reaction is illustrated in Fig.2, and may be represented by the following relationships:

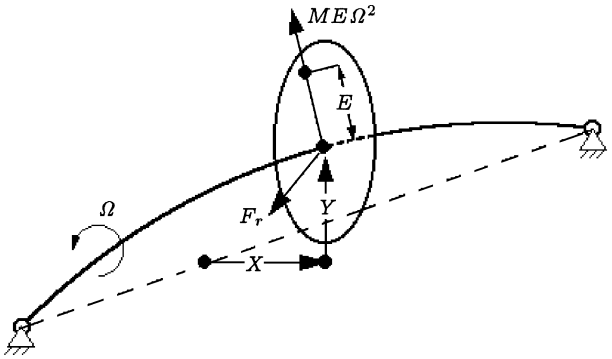


Fig.1. Jeffcott rotor model.

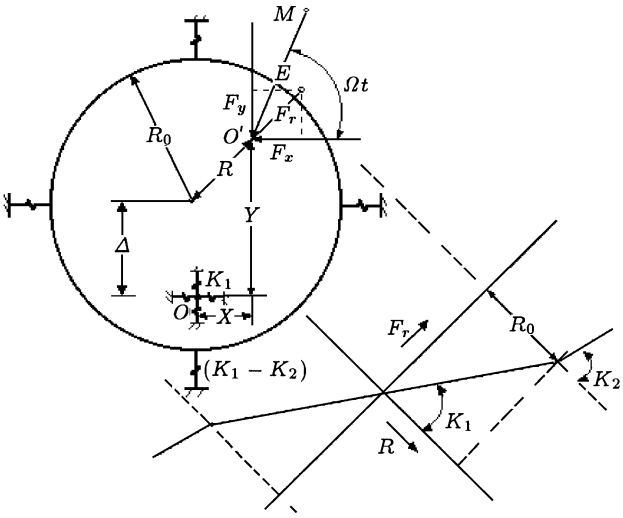


Fig.2. Nonlinear spring system.

$$
\begin{aligned}
&R = [(Y-\Delta)^2 + X^2]^{1/2},\\
&F_r = RK_1 \rightarrow (R < R_0),\\
&F_r = R_0K_1 + (R-R_0)K_2 \rightarrow (R > R_0),\\
&F_x = F_rX/R,\\
&F_y = F_r(Y-\Delta)/R + \Delta K_1 \rightarrow (\Delta < R_0). \qquad (1)
\end{aligned}
$$

In the light of Fig.2, it is assumed that the centre of the coordinate system is the centre of the rotational shaft at the beginning of rotation. In comparison to the restitution forces of the shaft deformation, the gravity of the rotor affects the dynamic characteristics of the rotor system so slightly that it may be neglected. At the same time, the system is also affected by the centrifugal forces resulting from the mass deflection of the rotor. So the governing equations of motion are

$$
\begin{aligned}
&M\mathrm{d}^2X/\mathrm{d}T^2 + C\mathrm{d}X/\mathrm{d}T + F_x = ME\Omega^2\cos\Omega T,\\
&M\mathrm{d}^2Y/\mathrm{d}T^2 + C\mathrm{d}Y/\mathrm{d}T + F_y = ME\Omega^2\sin\Omega T. \qquad (2)
\end{aligned}
$$

For a systematic study, the system is simplified and made dimensionless by some assumptions and parameter normalization.[4] The last equations of motion are as follows:

$$
\begin{aligned}
&\mathrm{d}^2x/\mathrm{d}t^2 = \omega^2\cos\omega t - 2\xi(\mathrm{d}x/\mathrm{d}t) - f_x,\\
&\mathrm{d}^2y/\mathrm{d}t^2 = \omega^2\sin\omega t - 2\xi(\mathrm{d}y/\mathrm{d}t) - f_y. \qquad (3)
\end{aligned}
$$

The nonlinear spring forces are

$$
\begin{aligned}
&r = [(y-\delta)^2 + x^2]^{1/2},\\
&f_r = r\beta \rightarrow (r < r_0),\\
&f_r = r_0\beta + (r-r_0) \rightarrow (r > r_0),\\
&f_x = f_rx/r,\\
&f_y = f_r(y-\delta)/r + \delta\beta \rightarrow (\delta < r_0). \qquad (4)
\end{aligned}
$$

In Eqs.(1)–(4), R is the radial deflection from the stator centreline, R_0 is the radial clearance between the stator and rotor, T and t denote time, X and x are horizontal deflections from the rotational centreline, Y and y are vertical deflections from the rotational centreline, β is the stiffness ratio, Δ and δ are the rotor centreline offset in the vertical direction from the stator centreline, E is the mass eccentricity, ξ is the damping ratio referred to stiff linear system, Ω and ω are rotational speeds, K_1 is the support stiffness within the clearance, K_2 is the total support stiffness beyond the clearance, M is the rotor point mass, F_r and f_r are radial force components, F_x and f_x are horizontal support reaction force components, and F_y and f_y are vertical support force components.

Ⅲ. ALTERNATIONS BETWEEN CHAOS AND PERIOD, PERIOD ADDING PHENOMENA AT THE SUPER-CRITICAL ROTATIONAL SPEED

After the dimensionless equations of motion have been obtained, they are converted to first-order equation sets $\dot{\boldsymbol{u}} = f(\boldsymbol{u})$, and then integrated numerically by the fourth-order Runge–Kutta–Gill method in order to compute x and y as functions of t for fixed rotational speed ω, damping ratio ξ, stiffness ratio β, clearance r_0 and offset δ.

The rotational speed is one of the most important factors affecting the dynamic characteristics of a rotor system. It may be divided into two parts in terms of its ratios referring to the critical rotational speed of the stiff linear system for $\beta = K_1/K_2 = 1$.

Figure 3 is a bifurcation diagram of the motion in the Poincaré sector when $\xi = 0.012$, $\beta = 0.001$, over a range of m from 1 to 10.1. The horizontal and vertical coordinate axes of the diagram represent rotational speed ratio m and vertical displacement of the rotor centre y respectively, in which the rotational speed ratio $m = \omega/\omega_\nu, \omega_\nu$ is the vertical natural frequency of the rotor system. In Fig.3, it can be found that clear peaks of steady-state response in the y direction occur at the proximity of integral multiples of the critical rotational speed, where the value of corresponding m in the horizontal direction is exactly or nearly an integral number. At the same time, when m=1, the corresponding periodic number of the motion in Fig.3 is 1; when m=2, the number is 2, until m=10, the periodic number of the steady-state periodic response is equal to the integral part of the m value. That is to say, when m=1, 2, ..., n, n+1, the corresponding steady-state responses of the system are period 1, 2, ..., n, n+1, which are called first-, second-, ..., nth-, (n+1)th-order subharmonic responses, respectively. As m increases, the magnitudes of all subharmonic response peaks in Fig.3 increase orderly, and chaotic bands occur among successive orders of subharmonic responses. Then, after the motion undergoes a chaotic band, the period number increases by one. Chin *et al.*[7] reported a reversed period adding by analysis of impact oscillators. Shaw and Holmes[8] presented a period adding phenomenon in a two-dimensional (2D) system, but the magnitudes of harmonic response peaks decrease with the increase of the parameters in motion equations. Thompson and Ghraffari[9] showed an analogous phenomenon by a 2D mathematical model of a moored marine system at nearly the same time. So the period adding with periodic number being equal to integral multiples of the critical rotational speed and the increase of magnitudes of the steady-state response peak with the increase of rotational speed ratio is a new and important bifurcation phenomenon.

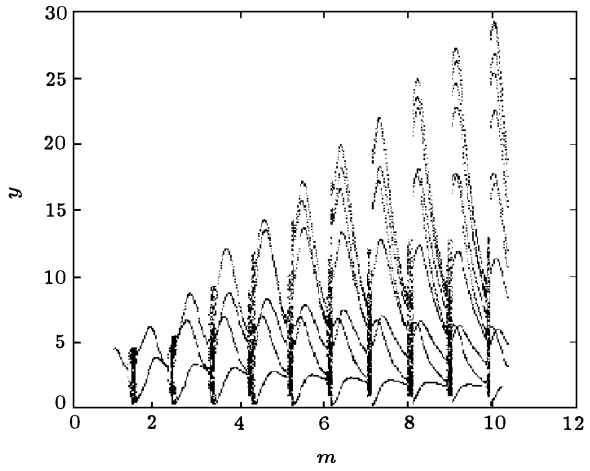


Fig.3. Bifurcation diagram for (ξ, β)=(0.012, 0.001) at the supercritical rotational speed.

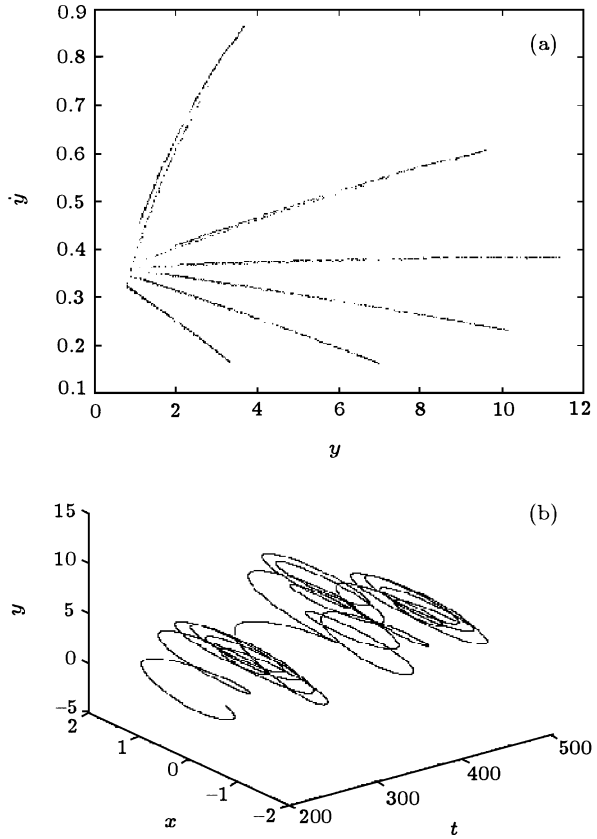


Fig.4. (a) Strange attractor in chaotic bands between the sixth-order and seventh-order subharmonic responses. (b) Chaotic trajectory of the rotor centre in the chaotic bands between the sixth-order and seventh-order subharmonic responses.

In the Poincaré sector, a steady-state map in the space of $(y, \dot{y})$ for m=6.14 is shown in Fig.4(a), then a hand-like Poincaré map is observed. The hand-like region of development in the phase projection is reminiscent of the strange attractor mappings of Henon,[10] and this is apparently not a stable periodic state. The ensuing chaotic trajectory of the rotor centre is shown in Fig.4(b), which is in the three-dimensional space of (x, y, t).

The amplitude peaks of subharmonic vibration occurring in the proximity of integral multiples of the critical rotational speed, as shown in Fig.3, will give the rotor the worst influence in practice, in addition to chaotic bands among successive orders of subharmonic responses. When the phenomena occur, even a slight change of rotational speed ratio can give rise to changes of motion characteristics in the rotor system. Collisions between the rotor and stator will be strengthened suddenly, and impair the system quickly. So the discovery of the phenomena has an important significance for the design of a rotor system and reduction of faults.

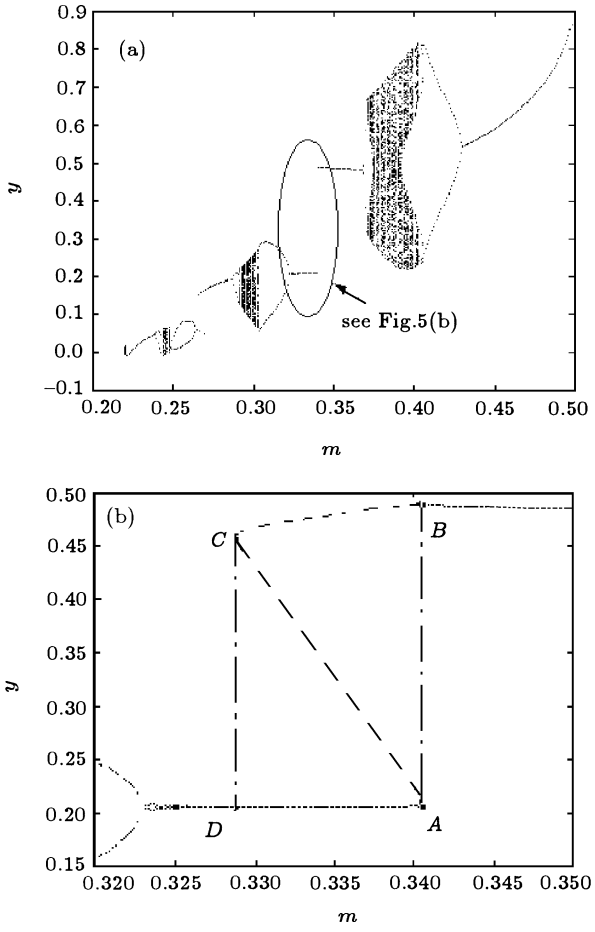


Fig.5. (a) Bifurcation diagram for (ξ, β)=(0.005, 0.005) at the subcritical rotational speeds. (b) Amplitude skip.

IV. BIFURCATION PHENOMENON AT THE SUBCRITICAL ROTATIONAL SPEED

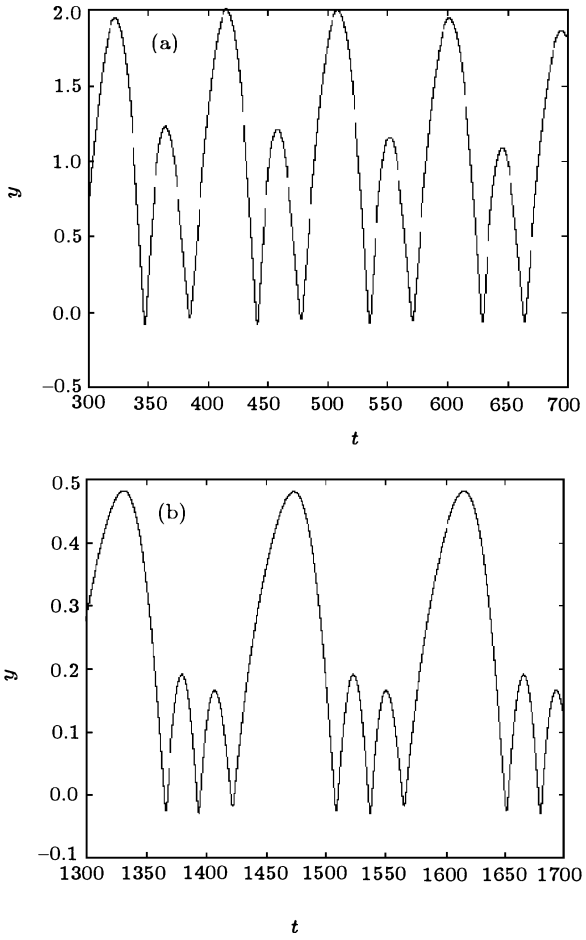


Fig.6. (a) Waveforms of the superharmonic response for (ξ, β, m)=(0.005, 0.005, 1/2). (b) Waveforms of the superharmonic response for (ξ, β, m)=(0.005, 0.005, 1/3).

When m <1, the rotational speed of the rotor system is below the critical rotational speed. Figure 5(a) in coordinates of m and y is a bifurcation diagram for ξ=0.005, β=0.005, where chaotic bands can be observed arising directly from the period-doubling bifurcation lying in the intervals between neighbouring values of m for m=1/2, 1/3, 1/4, 1/5. When m decreases from 0.5 to 0.2, the chaotic motion of the system returns directly to the periodic motion by a reversed period-doubling bifurcation. At the same time, amplitude skips exist in all steady-state regions lying between two of the three chaotic bands. Proceeding in this way, we have found regions approximately centred at speed ratios of 1/2, 1/3, 1/4 and 1/5, at which the motion is regular and repeatable, with two, three, four and five bounces per revolution, respectively. The motions within the regions are superharmonic responses exactly or nearly N times the

forcing frequency excited by mass unbalance. Figures 6(a) and 6(b) are waveforms of the superharmonic response in the vertical direction when m=1/2 and 1/3 respectively. It has been found that there are two amplitude peaks in Fig.6(a) for m=1/2, and three amplitude peaks in Fig.6(b) for m=1/3. Thus, this proves that the collision numbers between the stator and rotor are two and three, respectively. Childs,[11] Muszynska[12] and Ehrich[4] all reported on the superharmonic response of a rotor system at the subcritical rotational speed, with frequency exactly or nearly $1/n$ times its natural frequency. Motions within the chaotic bands between regular regions are irregular and unrepeatable. Figure 7(a) shows the trajectory of the rotor centre in the chaotic band between second-order and third-order superharmonic responses for m=0.382, ξ_2=0.005, β=0.005, in which the trajectory is observed to be irregular with the increase of time t. Figure 7(b) shows a record of the chaotic motion in the format of the Poincaré sector, which is a strange attractor with a fractal structure.

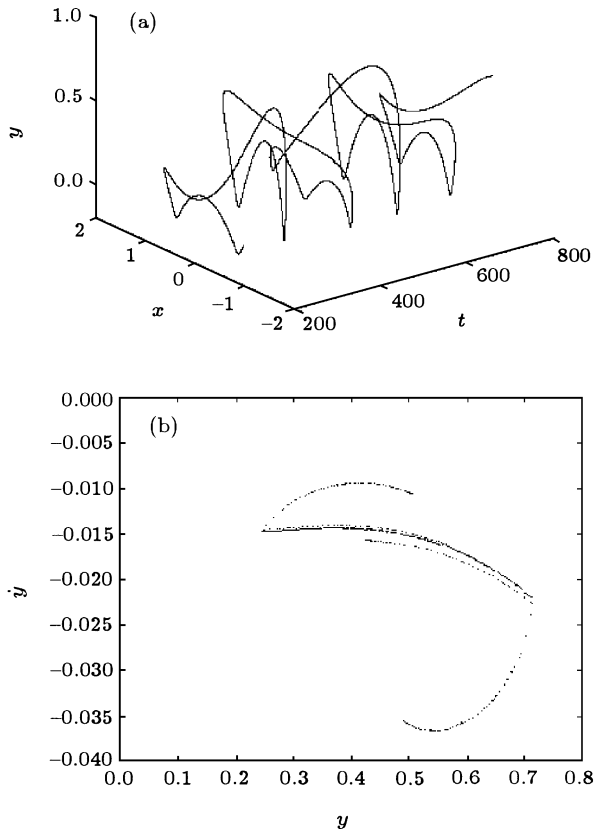


Fig.7. (a) Chaotic trajectory of the rotor centre in the chaotic bands between the second-order and third-order superharmonic responses. (b) Strange attractor between the second-order and third-order superharmonic responses.

The multiple value of the solution caused by the nonlinear element in Eqs.(2) results in an amplitude skip in the light of physics. There is a part of Fig.5(a), as shown in Fig.5(b), where the dashed and dot-dashed lines represent unstable solutions of y and the route of the amplitude skip, respectively. In the DA sector of m, a value of m corresponds to multiple values of y, the dashed line in Fig.5(b) corresponds to unstable solutions; as m increases, the y value will skip to the B point along the dot-dashed line AB when y arrives at the A point.

Amplitude skips and chaotic motion at subcritical rotational speeds are all dangerous phenomena, because the amplitude of vibration will change quickly when they occur, and the collision between the stator and the rotor will be enhanced too. So these phenomena should be avoided in order to reduce faults when the rotor system is working.

V. INFLUENCE OF CLEARANCE AND NONLINEAR ELEMENT−BILINEAR SPRING ON THE DYNAMIC CHARACTERISTICS OF THE ROTOR SYSTEM

Due to the existence of clearance, the nonlinear element−bilinear spring occurs in the rotor system. This is one of the important factors affecting dynamic characteristics of the rotor system with clearances.

Figure 8 is a bifurcation diagram of motion at the subcritical rotational speed with the degree of nonlinearity β=0.005, which is five times that in Fig.3. The values of the remaining parameters are the same as those in Fig.3.

In comparison to the width of chaotic bands among the successive orders of subharmonic responses in Fig.3, the width of those in Fig.8 decreases clearly; in contrast, the height of the amplitude peaks of the subharmonic response increases. According to the analysis of the phenomena, it is demonstrated that the increase of the degree of nonlinearity could effectively suppress the width of the chaotic bands between successive orders of subharmonic responses and, at the same time, correspondingly improve the amplitude of the harmonic responses. So the degree of nonlinearity should be changed exactly according to the physical circumstances. The rotor material that determines the nonlinear degree of the rotor system must be rethought to improve the dynamic characteristics of the system when starting to design the rotor structure.

Damping also is one of the important factors affecting the dynamic characteristics of a rotor system. In order to show the characteristics, all of the parame-

ter values used in Fig.9 are the same as those in Fig.3 except damping ξ=0.005. From the comparison, we indicate that:

a) Due to the decrease of damping, the amplitude of the subharmonic responses in Fig.9 is nearly two times that in Fig.3. Nevertheless, the width of the chaotic bands between the successive orders of periodic response increases correspondingly.

b) In comparison to the location of the rotational speed ratio m corresponding to the amplitude peaks of the subharmonic responses in Fig.3, those in Fig.9 are clearly less deflective. These indicate the hysteresis of damping.

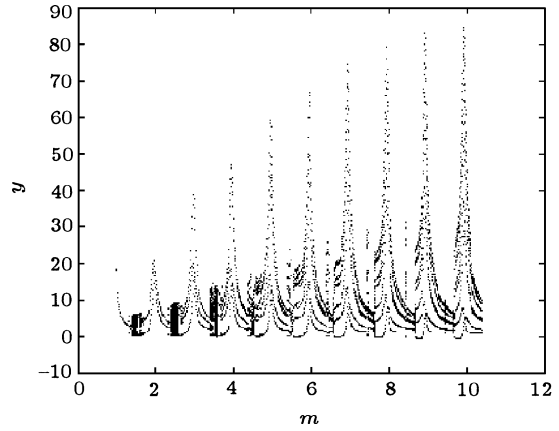


Fig.8. Bifurcation diagram for (ξ, β)=(0.012, 0.005) at the supercritical rotational speed.

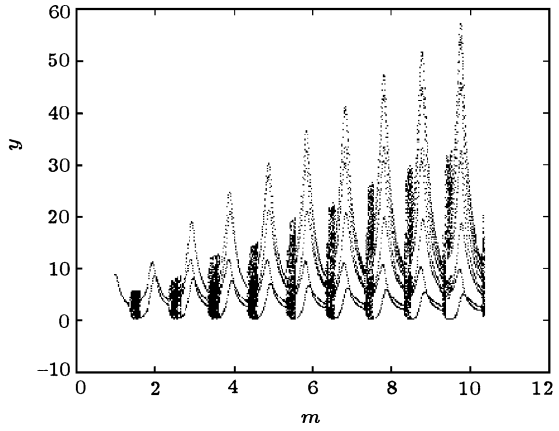


Fig.9. Bifurcation diagram for (ξ, β)=(0.006, 0.001) at the supercritical rotational speed.

Consequently, correct damping not only causes the amplitude of periodic responses to decrease but also suppresses the width of chaotic bands. This is an efficient approach to optimize the dynamic characteristics of the system and to reduce faults.

VI.CONCLUSIONS

Applying the modern nonlinear dynamic theory to analyse the mathematical model of a rotor system with clearance, we find that, when the rotational speed is above the critical rotational speed, the motion of the system alternates between periodic and chaotic responses. After undergoing a chaotic region, the period number increases by one. At the same time, the amplitudes of the subharmonic responses increase orderly as the rotational speed ratio increases. When the rotational speed is below the critical speed, the chaotic bands resulting directly from a period-doubling bifurcation occur between the successive orders of the superharmonic responses. Then, these can return directly to a periodic motion by a reversed period-doubling bifurcation. Moreover, amplitude skips exist in all periodic regions among three chaotic bands at the subcritical speeds.

At supercritical rotational speeds, the damping of the system not only clearly reduces the amplitude of the subharmonic responses but also effectively suppresses the width of the chaotic bands. The nonlinear element–bilinear spring not only makes the amplitude of the periodic responses increase, but suppresses the width of the chaotic bands among the successive orders of the periodic responses. Therefore, when designing a rotor system, correct damping and comprehensive thought about the rotor material which determines the nonlinear stiffness will benefit the improvement of the dynamic characteristics and clearly decrease the number of faults, on the basis of not affecting the efficiency of the system. At the same time, it is suggested that it is better not to locate the working speed of a rotor system in the integral multiples of the critical rotational speed and the chaotic bands in between successive orders of periodic responses in order to reduce faults.

References

[1] Kobrinsky A A and Kobrinsky A E 1973 *Vibroimpacting Systems* (Moska: Nauka) p 33

[2] Thompson J M T and Ghaffari R 1982 *Phys. Lett.* **91A** 46

[3] Szczygielski W M 1987 *Proc. ASME 11th Biennial Conf. of Vibration and Noise (Boston, MA)* p 319

[4] Ehrich F F 1988 *Trans. ASME, J. Vib. Acoust. Stress Reliab. Des.* **110** 9

[5] Ehrich F F 1992 *Trans. ASME, J. Vib. Acoust.* **114** 93

[6] Goldman P and Muszynska A 1994 *Trans. ASME, J. Vib. Acoust.* **116** 541

[7] Chin W, Edward Ott *et al* 1994 *Phys. Rev.* E **50** 4427

[8] Shaw S W and Holmes P J 1983 *J. Sound Vib.* **90** 129

[9] Thompson J M T and Ghaffari R 1983 *Phys. Rev.* A **27** 1741

[10] Henon M 1976 *Commun. Math. Phys.* **50** 69

[11] Childs D W 1982 *Trans. ASME, J. Eng. Power* **104** 533

[12] Muszynska A 1984 Institution of Mechanical Engineers Paper No.C281 84