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THE LAX PAIR FOR C_2-TYPE RUIJSENAARS-SCHNEIDER MODEL*

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We study the C_2 Ruijsenaars-Schneider model with interaction potential of trigonometric type. The Lax pairs for the model with and without spectral parameters are constructed. Also given are the involutive Hamiltonians for the system. Taking a non-relativistic limit, we obtain the Lax pair of C_2 Calogero-Moser model.

Keywords: Lax pair, Ruijsenaars-Schneider model
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I. INTRODUCTION

The Ruijsenaars-Schneider (RS) and Calogero-Moser (CM) models as integrable many-body models recently have attracted remarkable attention and have been extensively studied. They describe one-dimensional n-particle systems with pairwise interaction. Their importance lies in various fields ranging from lattice models in statistical physics[1,2] to the field theory and gauge theory,[3,4] e.g. to the Seiberg-Witten theory,[5] etc. Recently, the Lax pairs for the CM models in various root systems have been given by Olshanetsky *et al.*,[6] Inozemtsev,[7] D'Hoker *et al.*[8] and Bordner *et al.*[9] with or without spectral parameters, respectively. Further, a more general algebra-geometric construction was proposed by Hurtubise *et al.* in Ref.[10], while the commutative operators for the RS model based on various types of Lie algebra were given by Komori,[11,12] Diejen[13,14] and Hasegawa *et al.*[1,15] An interesting result is that in Ref.[16] the authors show that for the sl_2 trigonometric RS and CM models the same non-dynamical r-matrix structure exists compared with the usual dynamical ones. On the other hand, similar to Hasegawa's result that A_{N-1} RS model can be obtained as transfer matrices associated to the Sklyanin algebra, they also reveal that corresponding CM model's integrability can be depicted by sl_N Gaudin algebra.[17]

As for the C_n-type RS model, commuting difference operators acting on the space of functions on the C_2-type weight space have been constructed by Hasegawa *et al.* in Ref.[15]. Extending that work, the diagonalization of elliptic difference system of that type has been studied by Kikuchi in Ref.[18]. Despite the fact that the Lax pairs for CM models have been proposed for general Lie algebra even for all of the finite reflection groups,[19] the Lax integrabilities of the RS model are not clear except only for A_{N-1}-type[20,2,21–24], i.e. the Lax pairs for the RS models other than A_{N-1}-type have not yet been obtained.

In this paper, we concentrate on the C_2-type trigonometric Ruijsenaars-Schneider model. The basic materials about the C_2 RS model are reviewed in section II. In section III, we present the Lax pair without spectral parameters and its integrability in the Liouville sense is also given. In section IV, taking its non-relativistic limit, we recover the system of corresponding CM-type. In section V, we give the Lax pair for the system with spectral parameters, and show that at certain limit it will degenerate to the one without spectral parameters. The last section is a brief summary and some discussion.

II. MODEL AND EQUATIONS OF MOTION

As a relativistic-invariant generalization of the C_n-type Calogero-Moser model, the C_n-type Ruijsenaars-Schneider system is completely integrable whose integrability is shown by Ruijsenaars[25] and Diejen.[13,14] In terms of the canonical variables p_i, $x_j (i,j = 1,\ldots,2)$ satisfying the canonical Poisson bracket $\{p_i,p_j\} = \{x_i,x_j\} = 0$, $\{x_i,p_j\} = \delta_{ij}$, the

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Hamiltonian of C_2 RS system can be of the form

$$H=\sum_{i=1}^{2}\Big\{\mathrm{e}^{p_i}f(2x_i)\prod_{k\neq i}^{2}(f(x_{ik})f(x_i+x_k)) + \mathrm{e}^{-p_i}g(2x_i)\prod_{k\neq i}^{2}(g(x_{ik})g(x_i+x_k))\Big\}, \quad (1)$$

where

$$f(x):=\frac{\sin(x-\gamma)}{\sin(x)},$$
$$g(x):=f(x)|_{\gamma\to-\gamma},$$
$$x_{ik}:=x_i-x_k,$$

and γ denotes the coupling constant. Notice that in Ref.[25] Ruijsenaars used another "gauge" of the momenta such that the two systems are connected by the following canonical transformation:

$$x_i\longrightarrow x_i,$$
$$p_i\longrightarrow p_i+\frac{1}{2}\ln\prod_{j\neq i}^{2}\frac{f(x_{ij})f(x_i+x_j)}{g(x_{ij})g(x_i+x_j)}\frac{f(2x_i)}{g(2x_i)}. \quad (2)$$

The canonical equations of motion for the Hamiltonian (1) are

$$\dot{x_i}=\{x_i,H\}=\mathrm{e}^{p_i}b_i-\mathrm{e}^{-p_i}b_i^{'}, \quad (3)$$

$$\begin{aligned}\dot{p_i}=\{p_i,H\}=&\sum_{j\neq i}^{2}\Big(\mathrm{e}^{p_j}b_j\big(h(x_{ji})-h(x_j+x_i)\big)\\&+\mathrm{e}^{-p_j}b_j^{'}\big(\hat{h}(x_{ji})-\hat{h}(x_j+x_i)\big)\Big)\\&-\mathrm{e}^{p_i}b_i\Big(2h(2x_i)+\sum_{j\neq i}^{2}\big(h(x_{ij})+h(x_i+x_j)\big)\Big)\\&-\mathrm{e}^{-p_i}b_i^{'}\Big(2\hat{h}(2x_i)+\sum_{j\neq i}^{2}\big(\hat{h}(x_{ij})+\hat{h}(x_i+x_j)\big)\Big),\end{aligned} \quad (4)$$

where

$$h(x):=\frac{\mathrm{d}\ln f(x)}{\mathrm{d}x},\quad \hat{h}(x):=\frac{\mathrm{d}\ln g(x)}{\mathrm{d}x},$$
$$b_i=f(2x_i)\prod_{k\neq i}^{2}\Big(f(x_i-x_k)f(x_i+x_k)\Big),$$
$$b_i^{'}=g(2x_i)\prod_{k\neq i}^{2}\Big(g(x_i-x_k)g(x_i+x_k)\Big). \quad (5)$$

Here, of course $x_i=x_i(t)$, $p_i=p_i(t)$ and the dot on top denotes t-differentiation.

Ⅲ. THE LAX PAIR WITHOUT SPECTRAL PARAMETERS

Let us first mention some results about the integrability of Hamiltonian (1). In Ref.[25] Ruijsenaars demonstrated that the symplectic structure of C_n-type RS system can be proven to be integrable by embedding its phase space to a submanifold of the A_{2n-1}-type RS system, while in Refs.[13,14] and Ref.[12], Diejen and Komori, respectively, gave a series of commuting difference operators which led to its quantum integrability. However, there has been no result about its Lax representation so far. That is, the explicit form of the Lax matrix L, associated with a M which ensures its Lax integrability, has not been proposed up to now. In this section, we restrict our treatment to the formulation of the explicit form for the C_2 RS system, such that some previous, as well as new results, can be obtained in a more straightforward manner by using the Lax pair.

We define one 4×4 Lax matrix for C_2 RS model as follows:

$$L=\begin{pmatrix}A & B\\ C & D\end{pmatrix}, \quad (6)$$

where A, B, C, D are 2×2 matrices (hereafter, we use the indices $i,j=1,2$)

$$\begin{aligned}A_{ij}=&\mathrm{e}^{p_j}b_j\frac{\sin\gamma}{\sin(x_{ij}+\gamma)},\\B_{ij}=&\mathrm{e}^{-p_j}b_j^{'}\frac{\sin\gamma}{\sin(x_i+x_j+\gamma)},\\C_{ij}=&\mathrm{e}^{p_j}b_j\frac{\sin\gamma}{\sin(-x_i-x_j+\gamma)},\\D_{ij}=&\mathrm{e}^{-p_j}b_j^{'}\frac{\sin\gamma}{\sin(x_{ji}+\gamma)}.\end{aligned} \quad (7)$$

For the concise expression for M, we define four auxiliary 2×2 matrices $\widetilde{A}$, $\widetilde{B}$, $\widetilde{C}$, $\widetilde{D}$ as follows:

$$\begin{aligned}\widetilde{A}_{ij}=&\mathrm{e}^{-p_i}b_j^{'}\frac{-\sin\gamma}{\sin(x_{ij}-\gamma)},\\\widetilde{B}_{ij}=&\mathrm{e}^{-p_i}b_j\frac{-\sin\gamma}{\sin(x_i+x_j-\gamma)},\\\widetilde{C}_{ij}=&\mathrm{e}^{p_i}b_j^{'}\frac{-\sin\gamma}{\sin(-x_i-x_j-\gamma)},\\\widetilde{D}_{ij}=&\mathrm{e}^{p_i}b_j\frac{-\sin\gamma}{\sin(x_{ji}-\gamma)},\end{aligned} \quad (8)$$

such that M can be of the form

$$M=\begin{pmatrix}\mathcal{A} & \mathcal{B}\\ \mathcal{C} & \mathcal{D}\end{pmatrix}, \quad (9)$$

where entries of M are

$$\begin{aligned}
\mathcal{A}_{ij} =& \cot(x_{ij})(A_{ij} - \widetilde{A}_{ij}),\\
\mathcal{D}_{ij} =& \cot(x_{ji})(D_{ij} - \widetilde{D}_{ij}) \qquad (i \neq j),\\
\mathcal{B}_{ij} =& \cot(x_i + x_j)(B_{ij} - \widetilde{B}_{ij}),\\
\mathcal{C}_{ij} =& \cot(-x_i - x_j)(C_{ij} - \widetilde{C}_{ij}),\\
\mathcal{A}_{ii} =& -\sum_{k\neq i}^{2} \frac{A_{ik} - \widetilde{A}_{ik}}{\sin(x_{ik})} - \sum_{k=1}^{2} \frac{B_{ik} - \widetilde{B}_{ik}}{\sin(x_i + x_k)},\\
\mathcal{D}_{ii} =& \sum_{k\neq i}^{2} \frac{D_{ik} - \widetilde{D}_{ik}}{\sin(x_{ik})} + \sum_{k=1}^{2} \frac{C_{ik} - \widetilde{C}_{ik}}{\sin(x_i + x_k)}. \qquad (10)
\end{aligned}$$

We have checked with the help of a computer that L, M satisfies the Lax equation

$$\dot{L} = \{L, H\} = [M, L], \qquad (11)$$

which is equivalent to the equations of motion (3) and (4). The Hamiltonian H can be rewritten in the following form:

$$H = \sum_{j=1}^{2} (\mathrm{e}^{p_j} b_j + \mathrm{e}^{-p_j} b_j^{'}) = \mathrm{Tr} L. \qquad (12)$$

The characteristic polynomial of the Lax matrix L is

$$\begin{aligned}
\det(L - vId) =& \sum_{j=0}^{4} (-v)^{4-j} H_j\\
=& v^4 - Hv^3 + H_2 v^2 - Hv + 1, \qquad (13)
\end{aligned}$$

where $H_0 = H_4 = 1$, $H_1 = H_3 = H$. The function-independent Hamiltonian flows H and H_2 are

$$\begin{aligned}
H =& \mathrm{e}^{p_1} f(2x_1)\, f(x_{12}) f(x_1 + x_2)\\
&+ \mathrm{e}^{-p_1} g(2x_1)\, g(x_{12}) g(x_1 + x_2)\\
&+ \mathrm{e}^{p_2} f(2x_2)\, f(x_{21}) f(x_2 + x_1)\\
&+ \mathrm{e}^{-p_2} g(2x_2)\, g(x_{21}) g(x_2 + x_1), \qquad (14)\\
H_2 =& \mathrm{e}^{p_1 + p_2} f(2x_1)\, (f(x_1 + x_2))^2 f(2x_2)\\
&+ \mathrm{e}^{-p_1 - p_2} g(2x_1)\, (g(x_1 + x_2))^2 g(2x_2)\\
&+ \mathrm{e}^{p_1 - p_2} f(2x_1)\, (f(x_{12}))^2 f(-2x_2)\\
&+ \mathrm{e}^{p_2 - p_1} g(2x_1)\, (g(x_{12}))^2 g(-2x_2)\\
&+ 2 f(x_{12})\, g(x_{12})\, f(x_1 + x_2) g(x_1 + x_2). \qquad (15)
\end{aligned}$$

We verify that H and H_2 Poisson commute each other

$$\{H, H_2\} = 0, \qquad (16)$$

which ensures the complete integrability of the C_2 RS model (in the Liouville sense).

IV. NON-RELATIVISTIC LIMIT TO THE CALOGERO-MOSER SYSTEM

The non-relativistic limit can be achieved by rescaling $p_i \longmapsto \beta p_i$, $\gamma \longmapsto \beta\gamma$ while letting $\beta \longmapsto 0$, and making a canonical transformation

$$p_i \longmapsto p_i + \gamma\left(\cot(2x_i) + \sum_{k\neq i}^{2} (\cot(x_{ik}) + \cot(x_i + x_k))\right), \qquad (17)$$

such that

$$L \longmapsto Id + \beta L_{\mathrm{CM}} + O(\beta^2), \qquad (18)$$

$$M \longmapsto 2\beta M_{\mathrm{CM}} + O(\beta^2), \qquad (19)$$

and

$$H \longmapsto 4 + 2\beta^2 H_{\mathrm{CM}} + O(\beta^2). \qquad (20)$$

L_{CM} can be expressed as

$$L_{\mathrm{CM}} = \begin{pmatrix} A_{\mathrm{CM}} & B_{\mathrm{CM}} \\ -B_{\mathrm{CM}} & -A_{\mathrm{CM}} \end{pmatrix}, \qquad (21)$$

where

$$\begin{aligned}
(A_{\mathrm{CM}})_{ii} =& p_i,\\
(B_{\mathrm{CM}})_{ij} =& \frac{\gamma}{\sin(x_i + x_j)},\\
(A_{\mathrm{CM}})_{ij} =& \frac{\gamma}{\sin(x_{ij})} \qquad (i \neq j). \qquad (22)
\end{aligned}$$

M_{CM} is

$$M_{\mathrm{CM}} = \begin{pmatrix} \mathcal{A}_{\mathrm{CM}} & \mathcal{B}_{\mathrm{CM}} \\ \mathcal{B}_{\mathrm{CM}} & \mathcal{A}_{\mathrm{CM}} \end{pmatrix}, \qquad (23)$$

where

$$\begin{aligned}
(\mathcal{A}_{\mathrm{CM}})_{ii} =& -\sum_{k\neq i}^{2} \left(\frac{\gamma}{\sin^2 x_{ik}} + \frac{\gamma}{\sin^2(x_i + x_k)}\right)\\
&- \frac{\gamma}{\sin^2(2x_i)},\\
(\mathcal{B}_{\mathrm{CM}})_{ij} =& \frac{\gamma \cos(x_i + x_j)}{\sin^2(x_i + x_j)},\\
(\mathcal{A}_{\mathrm{CM}})_{ij} =& \frac{\gamma \cos(x_{ij})}{\sin^2 x_{ij}} \quad (i \neq j), \qquad (24)
\end{aligned}$$

which coincide with the form given in Ref.[6] with the difference of a constant diagonalized matrix.

The Hamiltonian of the C_2-type CM model can be given by

$$\begin{aligned}
H_{\mathrm{CM}} =& \frac{1}{2}\sum_{k=1}^{2} p_k^2 - \frac{\gamma^2}{2}\sum_{k\neq i}^{2}\left(\frac{1}{\sin^2 x_{ik}}\right.\\
&\left. + \frac{1}{\sin^2(x_i + x_k)} + \frac{1}{\sin^2(2x_i)}\right)\\
=& \frac{1}{4}\mathrm{Tr} L^2. \qquad (25)
\end{aligned}$$

The $L_{\rm CM}$, $M_{\rm CM}$ satisfies the Lax equation

$$\dot{L}_{\rm CM} = \{L_{\rm CM}, H_{\rm CM}\} = [M_{\rm CM}, L_{\rm CM}]. \qquad (26)$$

V. THE LAX PAIR WITH SPECTRAL PARAMETERS

Also, we can give the Lax pair which include spectral parameters. We define the Lax matrix for the trigonometric RS model as follows:

$$L = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \qquad (27)$$

where A, B, C, D are 2×2 matrices$(i,j=1,2)$

$$\begin{aligned} A_{ij} &= \mathrm{e}^{p_j} b_j \frac{\sin(x_{ij}+\gamma+\lambda)\sin\gamma}{\sin(x_{ij}+\gamma)\sin(\gamma+\lambda)}, \\ B_{ij} &= \mathrm{e}^{-p_j} b_j' \frac{\sin(x_i+x_j+\gamma+\lambda)\sin\gamma}{\sin(x_i+x_j+\gamma)\sin(\gamma+\lambda)}, \\ C_{ij} &= \mathrm{e}^{p_j} b_j \frac{\sin(-x_i-x_j+\gamma+\lambda)\sin\gamma}{\sin(-x_i-x_j+\gamma)\sin(\gamma+\lambda)}, \\ D_{ij} &= \mathrm{e}^{-p_j} b_j' \frac{\sin(x_{ji}+\gamma+\lambda)\sin\gamma}{\sin(x_{ji}+\gamma)\sin(\gamma+\lambda)}. \end{aligned} \qquad (28)$$

M is

$$M = \begin{pmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{pmatrix}, \qquad (29)$$

where entries of M are

$$\begin{aligned} \mathcal{A}_{ij} =& \mathrm{e}^{p_j} b_j \frac{\sin(x_{ij}+\lambda)}{\sin\lambda\sin x_{ij}} - \mathrm{e}^{-p_i} b_j' \frac{\sin(x_{ij}+\lambda+4\gamma)}{\sin(\lambda+4\gamma)\sin x_{ij}}, \\ \mathcal{D}_{ij} =& \mathrm{e}^{-p_j} b_j' \frac{\sin(x_{ji}+\lambda)}{\sin\lambda\sin x_{ji}} - \mathrm{e}^{p_i} b_j \frac{\sin(x_{ji}+\lambda+4\gamma)}{\sin(\lambda+4\gamma)\sin x_{ji}} \qquad (i\neq j), \\ \mathcal{B}_{ij} =& \mathrm{e}^{-p_j} b_j' \frac{\sin(x_i+x_j+\lambda)}{\sin\lambda\sin(x_i+x_j)} - \mathrm{e}^{-p_i} b_j \frac{\sin(x_i+x_j+\lambda+4\gamma)}{\sin(\lambda+4\gamma)\sin(x_i+x_j)}, \\ \mathcal{C}_{ij} =& \mathrm{e}^{p_j} b_j \frac{\sin(x_i+x_j-\lambda)}{\sin\lambda\sin(x_i+x_j)} - \mathrm{e}^{p_i} b_j' \frac{\sin(x_i+x_j-\lambda-4\gamma)}{\sin(\lambda+4\gamma)\sin(x_i+x_j)}, \\ \mathcal{A}_{ii} =& (\cot(\gamma)+\cot(\lambda))\mathrm{e}^{p_i} b_i - (\cot(\lambda+\gamma)-\cot(\gamma))\mathrm{e}^{-p_i} b_i' \\ &+ \sum_{k\neq i}^{2} \Bigg((\cot(x_{ik}+\gamma)-\cot(x_{ik}))\,\mathrm{e}^{p_k} b_k \\ &+ \frac{\sin(x_{ik}+\lambda+4\gamma)\sin(x_{ki}+\lambda+\gamma)\sin\gamma}{\sin(\lambda+4\gamma)\sin x_{ik}\sin(x_{ki}+\gamma)\sin(\lambda+\gamma)} \mathrm{e}^{-p_i} b_k' \Bigg) \\ &+ \sum_{k=1}^{2} \Bigg((\cot(x_i+x_k+\gamma)-\cot(x_i+x_k))\mathrm{e}^{-p_k} b_k' \\ &+ \frac{\sin(x_i+x_k+\lambda+4\gamma)\sin(x_i+x_k-\lambda-\gamma)\sin\gamma}{\sin(\lambda+4\gamma)\sin(x_i+x_k)\sin(x_i+x_k-\gamma)\sin(\lambda+\gamma)} \mathrm{e}^{-p_i} b_k \Bigg), \\ \mathcal{D}_{ii} =& (\cot(\gamma)+\cot(\lambda))\mathrm{e}^{-p_i} b_i' - (\cot(\lambda+\gamma)-\cot(\gamma))\mathrm{e}^{p_i} b_i \\ &+ \sum_{k\neq i}^{2} \Bigg((\cot(x_{ki}+\gamma)-\cot(x_{ki}))\mathrm{e}^{-p_k} b_k' \\ &+ \frac{\sin(x_{ki}+\lambda+4\gamma)\sin(x_{ik}+\lambda+\gamma)\sin\gamma}{\sin(\lambda+4\gamma)\sin x_{ki}\sin(x_{ik}+\gamma)\sin(\lambda+\gamma)} \mathrm{e}^{p_i} b_k \Bigg) \\ &+ \sum_{k=1}^{2} \Bigg((\cot(x_i+x_k)-\cot(x_i+x_k-\gamma))\mathrm{e}^{p_k} b_k \\ &+ \frac{\sin(x_i+x_k-\lambda-4\gamma)\sin(x_i+x_k+\lambda+\gamma)\sin\gamma}{\sin(\lambda+4\gamma)\sin(x_i+x_k)\sin(x_i+x_k+\gamma)\sin(\lambda+\gamma)} \mathrm{e}^{p_i} b_k' \Bigg). \end{aligned} \qquad (30)$$

The L, M satisfies the Lax equation Eq.(11) and the Hamiltonian H can also be rewritten in the form of Eq.(12).

The function-independent Hamiltonian flows can be generated by calculating the characteristic polyno-

mial of that Lax matrix L

$$\det(L - vId) = \sum_{j=0}^{4} \frac{(\sin\lambda)^{(j-1)} \sin(\lambda + j\gamma)}{(\sin(\gamma+\lambda))^j} (-v)^{4-j} H_j, \quad (31)$$

where $H_0 = H_4 = 1$, $H_1 = H_3 = H$. H and H_2 have the same forms as Eqs.(14) and (15).

Remarks:

1. As far as the forms of the Lax pair for the rational-type systems are concerned, we can obtain these by making the following substitutions:

$$\sin x \to x,$$

$$\cos x \to 1,$$

for all the above statements.

2. The Lax pair given in Eqs.(6)–(10) which are without spectral parameters can be derived from the one with spectral parameters (see Eqs.(27)–(30)) by taking the following limit:

$$\lambda \to \mathrm{i}\infty,$$

up to an appropriate gauge transformation of the Lax matrix with a diagonal matrix.

VI. SUMMARY AND DISCUSSION

In this paper, we propose the Lax pairs for the trigonometric C_2 RS model together with its rational limit and show their integrability. Involutive Hamiltonians are shown to be generated by the characteristic polynomial of the Lax matrix. In the non-relativistic limit, the system leads to CM system associated with the root system of C_2 which has been known previously. It is expected that, in the general case of C_n for $n \geq 2$, the explicit expressions of L and M must have similar forms as those presented here. So it would be interesting to make some progress in this respect in the very near future.

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References

[1] Hasegawa K 1997 *Commun. Math. Phys.* **187** 289
[2] Nijhoff F W, Kuznetsov V B, Sklyanin E K and Ragnisco O 1996 *J. Phys.* A **29** L333
[3] Gorsky A and Marshakov A 1996 *Phys. Lett.* B **375** 127
[4] Nekrasov N 1998 *Nucl. Phys.* B **531** 323
[5] Braden H W, Marshakov A, Mironov A and Morozov A 1999 *Nucl. Phys.* B **558** 371
[6] Olshanetsky M A and Perelomov A M 1981 *Phys. Rep.* **71** 314
[7] Inozemtsev V I 1989 *Lett. Math. Phys.* **17** 11
[8] D'Hoker E and Phong D H 1998 *Nucl. Phys.* B **530** 537
[9] Bordner A J, Corrigan E and Sasaki R 1998 *Prog. Theor. Phys.* **100** 1107
[10] Hurtubise J C and Markman E 1999 *e-print* math/9912161
[11] Komori Y and Hikami K 1998 *J. Math. Phys.* **39** 6175
[12] Komori Y 1999 *e-print* math QA/9910003
[13] van Diejen J F 1994 *J. Math. Phys.* **35** 2983
[14] van Diejen J F 1995 *Compositio. Math.* **95** 183
[15] Hasegawa K, Ikeda T and Kikuchi T 1999 *J. Math. Phys.* **40** 4549
[16] Chen K, Hou B Y, Yang W L and Zhen Y 1999 *Chin. Phys. Lett.* **16** 1
Chen K *et al* 1999 *High Energy Phys. Nucl. Phys.* **23** 854 (in Chinese)
[17] Chen K, Fan H, Hou B Y, Shi K J, Yang W L and Yue R H 1999 *Prog. Theor. Phys. Suppl.* **135** 149
[18] Kikuchi T 1999 *e-print* math/9912114
[19] Bordner A J, Corrigan E and Sasaki R 1999 *Prog. Theor. Phys.* **102** 499
[20] Ruijsenaars S N M 1987 *Commun. Math. Phys.* **110** 191
[21] Bruschi M and Calogero F 1987 *Commun. Math. Phys.* **109** 481
[22] Krichever I and Zabrodin A 1995 *Usp. Math. Nauk* **50**(6) 3
[23] Suris Y B 1996 *e-print* hep-th/9602160
[24] Suris Y B 1997 *Phys. Lett.* A **225** 253
[25] Ruijsenaars S N M 1988 *Commun. Math. Phys.* **115** 127