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CALCULATION OF KAON ELECTROMAGNETIC FORM FACTOR*

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The kaon meson electromagnetic form factor is calculated in the framework of coupled Schwinger-Dyson and Bethe-Salpeter formulation in simplified impulse approximation (dressed vertex) with modified flat-bottom potential, which is a combination of the flat-bottom potential taking into consideration the infrared and ultraviolet asymptotic behaviours of the effective quark-gluon coupling. All the numerical results give a good fit to experimental values.

Keywords: Schwinger-Dyson equation, Bethe-Salpeter equation, electromagnetic form factor
PACC: 1340F, 1240Q

I. INTRODUCTION

Many properties of hadrons have been investigated in the framework of Bethe-Salpeter (BS) formulation or coupled Schwinger-Dyson (SD) and BS formulation,[1,2] using simple assumptions and approximations. This has proved compatible with experimental values. In our previous work,[3,4] we assumed a vector-vector type flat-bottom potential (FBP), which is a sum of Yukawa potentials. The potential itself not only satisfies chiral invariance and fully relativistic covariance, but also suppresses the singular point that the Yukawa potential has. It proves to work well in understanding the meson structure, such as electromagnetic form factor, radius and decay constant. Later, to modify the FBP, we combined our previous work with that of Munczek and Jain,[1] taking into consideration the effects of the infrared and ultraviolet behaviours.[4] However, the bare vertex does not satisfies the Ward-Takahashi identity, which plays an essential role in quantum field renormalization. As a lowest order approximation, one can simplify the dressed quark-gluon vertex, and take the first term of the Ball-Chiu vertex.[5] We find it works well in calculating electromagnetic form factor of the π meson in the framework of coupled SD and BS formulation.[6] In this paper, we calculate the kaon meson electromagnetic form factor with the simplified dressed vertex.

II. THEORY

The only way of determining the fully dressed quark-gluon vertex is to simultaneously solve the SD equation for Γ_μ, but we do not pursue this routine here. Instead, we choose the ansatz for the vertex which satisfies the Slavnov-Taylor identity in quantum chromodynamics (QCD),[7]

$$\begin{aligned} &q_\mu \Gamma^\mu(k,p)[1+b(q^2)] \\ =&[1-B(q,p)]S^{-1}(k) - S^{-1}(p)[1-B(q,p)], \end{aligned} \quad (1)$$

where $q = k - p$, $b(q^2)$ is the ghost self-energy, and $B(p,q)$ is the ghost scattering kernel, constraining the longitudinal part of the vertex. Obviously, the often used rainbow approximation, in which Γ_μ is simply taken to be γ_μ , cannot satisfy Eq.(1) in an interacting theory. Neglecting ghosts, this relation reduces to the form of the Ward-Takahashi identity in quantum electrodynamics (QED)

$$q_\mu \Gamma^\mu(k,p) = S^{-1}(k) - S^{-1}(p). \quad (2)$$

The constraint this relation places on the vertex can now be inferred from the QED studies.[8–10] Taking these into account one is led to a vertex of the form

$$\Gamma^\mu(k,p) = \Gamma^\mu_{\rm BC}(k,p) + \Gamma^\mu_{\rm CP}(k,p), \quad (3)$$

where

$$\begin{aligned} \Gamma^\mu_{\rm BC}(k,p) =& \frac{A(p^2)+A(k^2)}{2}\gamma^\mu \\ &+ \frac{(p+k)^\mu}{p^2-k^2}\left\{[A(p^2)-A(k^2)]\frac{\boldsymbol{\gamma}\cdot\boldsymbol{p}+\boldsymbol{\gamma}\cdot\boldsymbol{k}}{2}\right. \\ &\left. - [B(p^2)-B(k^2)]\right\}, \\ \Gamma^\mu_{\rm CP}(k,p) =& \frac{\gamma^\mu(k^2-p^2)-(p+k)^\mu(\boldsymbol{\gamma}\cdot\boldsymbol{k}-\boldsymbol{\gamma}\cdot\boldsymbol{p})}{2d(k,p)} \\ &\cdot [A(k^2)-A(p^2)], \end{aligned} \quad (4)$$

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with

$$d(k,p) = \frac{1}{(k^2+p^2)}(k^2-p^2)^2 + \left[\frac{B^2(k^2)}{A^2(k^2)} + \frac{B^2(p^2)}{A^2(p^2)}\right]^2. \quad (5)$$

In these equations, $\Gamma_{\rm BC}$ is the Ball-Chiu vertex of Ref.[8] and $\Gamma_{\rm CP}$ is the additional piece suggested and studied by Curtis and Pennington in Ref.[9]. Equation (3) specifies our vertex ansatz which has the property that it satisfies the Ward-Takahashi identity, free of kinematic singularities, and is reduced to the bare vertex in the absence of interactions. The power of our computer being limited, for the most part in the following we take the first term of the Ball-Chiu vertex, the simplified vertex as Williams *et al.*[5]

$$\Gamma^\mu(k,p) = \frac{A(p^2)+A(k^2)}{2}\gamma^\mu. \quad (6)$$

The phenomenological potential takes the following form:[4]

$$G(k^2) = \frac{16\pi^2}{3}\left[\frac{d}{(k^2+x_0)\ln\left(\frac{k^2}{\Lambda^2_{\rm QCD}}+x_0\right)} \cdot \left[1 + b\frac{\ln\left[\ln\left(\frac{k^2}{\Lambda^2_{\rm QCD}}+x_0\right)\right]}{\left(\frac{k^2}{\Lambda^2_{\rm QCD}}+x_0\right)}\right] + \sum_{j=0}^{n}\frac{a_j}{k^2+(N+{\rm j}\rho)^2} + (2\pi\eta)^2\delta^4(k)\right]. \quad (7)$$

The first term stands for the two loop renormalization group result for the ultraviolet asymptotic behaviour of $G(k^2)$. In this paper, it just plays the role of ultraviolet modification, where

$$d = \frac{12}{33-2f}, \quad b = \frac{2\beta_2}{\beta_1^2},$$
$$\beta_1 = \frac{1}{3}f - \frac{11}{2}, \quad \beta_2 = \frac{19}{12}f - \frac{55}{4},$$

and f is the number of quark flavours, $\Lambda_{\rm QCD}$ is the QCD scale parameter. The second term, standing for the FBP, which is a sum of Yukawa potentials analogous to the exchange of a series of particles and ghosts with different mass, is determined from the flat-bottom condition.[4,3] In its three-dimensional form, the singularity at the point $r = 0$ is suppressed, $V(0)$=constant and $V(r)$ varies slowly around $r = 0$, which is compatible with the asymptotic behaviour. N stands for the minimum value of the mass, ρ is their mass difference, and a_j is their relative coupling constant. The last term stands for confinement,[11] playing the role of infrared modification to the FBP. $\Lambda_{\rm QCD} \approx 200$MeV, n is set to be 9 and f is set to be 6.

In order to dress the quark-gluon vertex, we take the simplified impulse vertex. The SD equation takes the following form:

$$S^{-1}(p) = \boldsymbol{\gamma}\cdot\boldsymbol{p} - \hat{m} + \frac{\rm i}{(2\pi)^4} \cdot \int d^4k\gamma_\mu S(k)\Gamma_\nu G_{\mu\nu}(k-p), \quad (8)$$

where

$$S^{-1}(p) = A(p^2)\boldsymbol{\gamma}\cdot\boldsymbol{p} - B(p^2) \equiv A(p^2)[\boldsymbol{\gamma}\cdot\boldsymbol{p} - m(p^2)],$$
$$G_{\mu\nu}(k) = -\left[g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right]G(k^2), \quad (9)$$

where $\hat{m}$ stands for an explicit quark mass-breaking term. In this paper, we assume that a Wick rotation to Euclidean variables is allowed for both SD and BS equations, and perform a rotation analytically continuing p and k into the Euclidean region where they can be denoted by $\bar{p}$ and $\bar{k}$, respectively. The wavefunctions of the bound state of the quark-antiquark system should be the solutions to the BS equation. In simplified impulse approximation, it takes the following form:

$$S^{-1}\left(p+\frac{1}{2}Q\right)\chi(Q,p)S^{-1}\left(p-\frac{1}{2}Q\right) = -\frac{\rm i}{(2\pi)^4}\int d^4k\Gamma_\mu\chi(Q,k)\Gamma_\nu G_{\mu\nu}(k-p), \quad (10)$$

where $S(p)$ is the quark propagator function, $G_{\mu\nu}(k)$ is the gluon propagator, Q_μ is the four-momentum of the centre of mass of the kaon meson, p_μ is the relative four-momentum between quark and antiquark in the kaon meson. $\chi(Q,p)$ is the BS wavefunction of the bound state. We perform a Wick rotation analytically continuing k, p into Euclidean region, denoted by $\bar{p}$ and $\bar{k}$ as in the SD equation. Correspondingly, we define

$$\bar{Q}_\mu = -{\rm i}Q_\mu. \quad (11)$$

As in our previous work, the lowest order Euclidean pseudo-scalar BS wavefunction $\chi(\bar{Q},\bar{p})$ is expanded in Lorentz invariant functions which can be approximated by

$$\chi(\bar{Q},\bar{p}) = \gamma_5\chi_0^{(1)} + \gamma_5\gamma\cdot\bar{Q}\chi_0^{(2)} - \gamma_5\gamma\cdot\bar{p}\bar{p}\cdot\bar{Q}\chi_1^{(3)} + \gamma_5[\gamma\cdot\bar{p},\gamma\cdot\bar{Q}]\chi_0^{(4)}. \quad (12)$$

Since the phenomenological potential does not depend on the total momentum of kaon meson, the normalization of the wavefunction is given by

$$\frac{\mathrm{i}}{(2\pi)^4}\mathrm{Tr}\int d^4q\bar{\chi}(Q,q)\Big[\Big[\frac{\partial}{\partial Q_\mu}\cdot S^{-1}\Big(q+\frac{1}{2}Q\Big)\Big]\chi(Q,q)S^{-1}\Big(q-\frac{1}{2}Q\Big)+S^{-1}(q+\frac{1}{2}Q)\chi(Q,q)\frac{\partial}{\partial Q_\mu}\cdot S^{-1}\Big(q-\frac{1}{2}Q\Big)\Big]=2Q_\mu, \tag{13}$$

where $\bar{\chi}(Q,q)\equiv\gamma_0\chi(Q,q)^+\gamma_0$.

As we know, the relationship between the electromagnetic form factor and the matrix element of the electromagnetic current between two bound states is

$$F(k^2)(P_\mathrm{i}+P_\mathrm{f})_\mu=\langle P_\mathrm{f}|J_\mu(0)|P_\mathrm{i}\rangle, \tag{14}$$

where J_μ is the operator of the electromagnetic current, $|P_\mathrm{i}\rangle$ and $|P_\mathrm{f}\rangle$ are two bound states. The two bound states are defined by $\bar{\chi}(x,y)=\langle P_\mathrm{f}|\mathrm{T}[\bar{q}(x)q(y)]|0\rangle$ and $\chi(x,y)=\langle 0|\mathrm{T}[q(x)\bar{q}(y)]|P_\mathrm{i}\rangle$.

In momentum space, $\langle P_\mathrm{f}|J_\mu(0)|P_\mathrm{i}\rangle$ takes the following form:

$$\begin{aligned}&\langle P_\mathrm{f}|J_\mu(0)|P_\mathrm{i}\rangle\\=&\frac{1}{(2\pi)^4}\int \mathrm{d}^4q\mathrm{Tr}\bar{\chi}(P_\mathrm{f},q-k)\\&\cdot\Big[\mathrm{i}e_1\Gamma_\mu\chi(P_\mathrm{i},q)S^{-1}\Big(q-\frac{1}{2}P_\mathrm{i}\Big)\\&+\mathrm{i}e_2\bar{\chi}(P_\mathrm{f},q+k)S^{-1}\Big(q+\frac{1}{2}P_\mathrm{i}\Big)\chi(P_\mathrm{i},q)\Gamma_\mu\Big].\end{aligned} \tag{15}$$

Here we take the simplified impulse vertex, $e_1=\frac{2}{3}e$ for u quark and $e_2=-\frac{1}{3}e$ for s quark. Under the space-like condition $k^2>0$, it is convenient to choose the kinematic variables to be $k=(k,0,0,0)$, $P_\mathrm{i}=(k,0,0,\mathrm{i}\sqrt{k^2+M^2})$ and $P_\mathrm{f}=(-k,0,0,\mathrm{i}\sqrt{k^2+M^2})$. M is the mass of the kaon meson. The physical quantities of Eq.(15) are in Minkowski space, and we must perform a Wick rotation into Euclidean variables since the wavefunctions are calculated in the Euclidean region. From Fig.1, we see that $\chi_0^{(1)}(=F_1)$ and $\bar{Q}\chi_0^{(2)}(=F_2)$ are dominant, $\bar{Q}(\bar{p})^2\chi_1^{(3)}(=F_3),\bar{Q}\bar{p}\chi_0^{(4)}(=F_4)\ll\chi_0^{(1)},\bar{Q}\chi_0^{(2)}$, just as in our previous works [4,3,6,12] and the works of other physicists,[1] so we can ignore the contribution from $\bar{Q}(\bar{p})^2\chi_1^{(3)}$ and $\bar{Q}\bar{p}\chi_0^{(4)}$. In the PhD thesis of Wan,[12] the Euclidean BS is expanded into a series of $SO(4)$ eigenfunctions; there may be some other components, however, the first two components are dominant, the others are negligible. Finally we obtain the electromagnetic form factor of the kaon, which is shown in Fig.2, and compared with experimental values.[13−15] In order to obtain the propagator functions and mass functions for the quarks and the wavefunctions of bound state, electromagnetic form factor which are best fitted to the experimentical values and other theoretical results, the parameters we take are $x_0=10.0$, $N=2.1\Lambda_\mathrm{QCD}$, $V(0)=-15.0\Lambda_\mathrm{QCD}$, $\rho=4.6\Lambda_\mathrm{QCD}$, $\eta=390$ MeV , $m_u=8$ MeV , $m_d=150$ MeV and the large momentum cut-off $L=630\Lambda_\mathrm{QCD}$.

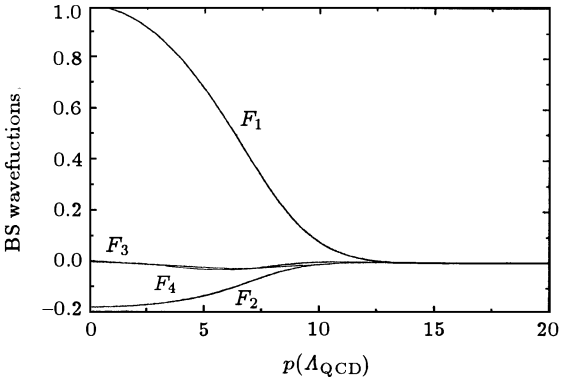


Fig.1. BS wavefunctions.

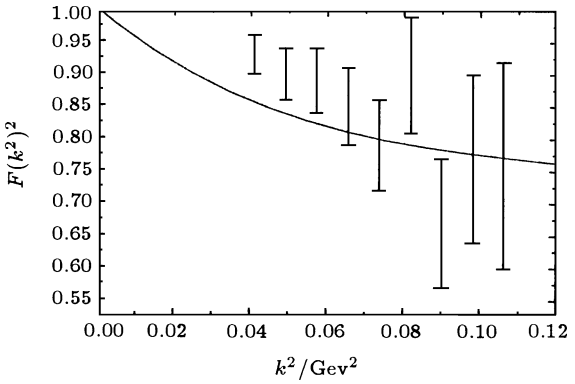


Fig.2. Electromagnetic form factor of the kaon is compared with experimental values.

Ⅲ. CONCLUSION

The coupled integral equations for SD and BS are solved numerically by simultaneous iterations. The iterations converge rapidly to a unique solution of wavefunctions independent of the choice of initial wavefunctions. Numerical results show that, at small k^2 , $A(k^2)$ differs from value 1 appreciably, while it tends to 1 for large k^2. The mass function $m(k^2)=B(k^2)/A(k^2)$ is greatly renormalized at small k^2,

while it takes asymptotic behaviour at large momentum. For u quark, $m(0) = 1683$MeV and for s quark, $m(0) = 1875$MeV. Neither are close to the constituent quark masses; the connection of $m(k^2)$ to constituent masses is somewhat less direct and is precise only for heavy quarks. For heavy quarks, $m_{\rm constituent}(k) = m$ $(k = 2m_{\rm constituent}(k))$, for light quarks, it only gives crude estimates.[16] Figure 1 shows graphs for the wavefunctions $F_1 = \chi_0^{(1)}$, $F_2 = \bar{Q}\chi_0^{(2)}$, $F_3 = \bar{Q}(\bar{p})^2\chi_1^{(3)}$ and $F_4 = \bar{Q}\bar{p}\chi_0^{(4)}$, respectively. Furthermore, the mass of the kaon meson is given by $M = 498$MeV. From Fig.2, we can see that at small k^2 the calculated electromagnetic form factor reaches the lower boundary of experimental data, while it gives a good fit to the experimental values at larger k^2. In contrast, the rainbow SD equation and ladder BS equation give a very good fit to the experimental values,[17] which may be due to the coarse approximation of the simplified impulse vertex. In general, the modified FBP is a good model and works well in the lowest order approximation. This can be extended to other systems, such as the $q\bar{q}$ system, $q\bar{Q}$ system, vector meson and qqq system, or by applying the results to the calculation of quantities and processes requiring detailed knowledge of the quark propagators and the BS wavefunction.

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