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ADAPTIVE CONTROL AND IDENTIFICATION OF CHAOTIC SYSTEMS

Li Zhi(李 智) and Han Chong-zhao(韩崇昭)

School of Electronic and Information Engineering, Xi'an Jiaotong University, Xi'an 710049, China

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A novel adaptive control and identification on-line method is proposed for a class of chaotic system with uncertain parameters. We prove that, using the presented method, a controller and identifier is developed which can remove chaos in nonlinear systems and make the system asymptotically stabilizing to an arbitrarily desired smooth orbit. And at the same time, estimates to uncertain parameters converge to their true values. The advantage of our method over the existing result is that the controller and identifier is directly constructed by analytic formula without knowing unknown bounds about uncertain parameters in advance. A computer simulation example is given to validate the proposed approach.

Keywords: chaotic systems, adaptive chaos control, identification on line, uncertain parameters
PACC: 0545, 4265

I. INTRODUCTION

In recent years, there has been considerable progress made in the study of the control of chaotic systems.[1−3] In this paper, we study this problem using a more general model.

The chaotic system considered in this paper takes the form

$$\dot{x} = f(x) + F(x)\theta + u, \tag{1}$$

where the states of systems are $x \in R^n$, unknown uncertain parameters are $\theta \in R^p$, and $u \in R^n$ is the control input to be determined. The function

$$\dot{x} = f(x) + F(x)\theta \tag{2}$$

represents the autonomous part of Eq.(1).

Let $x_{\rm d}$ be an arbitrarily desired smooth orbit. We choose

$$u = \dot{x}_{\rm d} - f(x_{\rm d}) - F(x_{\rm d})\hat{\theta} + \alpha(x, x_{\rm d}, \hat{\theta}), \tag{3}$$

where $\alpha(x, x_{\rm d}, \hat{\theta})$ is a compensator to be designed, and $\hat{\theta} \in R^p$ is the estimate value of the unknown parameter vector $\theta \in R^p$.

Substituting Eq.(3) into Eq.(1), we obtain the error dynamical system

$$\dot{e} = \tilde{f}(e, x_{\rm d}) + \tilde{F}(e, x_{\rm d})\theta + F(x_{\rm d})\tilde{\theta} + \overline{\alpha}(e, x_{\rm d}, \hat{\theta}), \tag{4}$$

where

$$\tilde{f}(e, x_{\rm d}) = f(x) - f(x_{\rm d}),\ \ \overline{\alpha}(e, x_{\rm d}, \hat{\theta}) = \alpha(x, x_{\rm d}, \hat{\theta}),$$

$$\tilde{F}(e, x_{\rm d}) = F(x) - F(x_{\rm d}),\ \ \tilde{\theta}(t) = \theta - \hat{\theta}(t), e = x - x_{\rm d}.$$

The goal of control is to find a compensator $\overline{\alpha}(e, x_{\rm d}, \hat{\theta})$ and a parameter estimation update law $\dot{\hat{\theta}}$ so that

$$\lim_{t\to\infty} \| e(t) \| = \lim_{t\to\infty} \| x(t) - x_{\rm d}(t) \| = 0, \tag{5}$$

$$\lim_{t\to\infty} \| \tilde{\theta}(t) \| = \lim_{t\to\infty} \| \theta - \hat{\theta}(t) \| = 0. \tag{6}$$

II. DESIGN OF CONTROLLER AND IDENTIFIER

Assumption 1:

For an arbitrarily given $x_{\rm d}, \dot{e} = \tilde{f}(e, x_{\rm d})$, is asymptotically stable at $e = 0$, i.e. $\forall (e, t) \in R^n \times R^+$, there exists a positive number $\gamma > 0$ and a scalar function with well-defined first continuous derivatives

$$V : R^n \times R^+ \to R^+,$$

such that

1) $\lambda_{\min}||e||^2 \le V(e, t) \le \lambda_{\max}||e||^2,$ (7)

2) $\dfrac{{\rm d}V}{{\rm d}t} = \dfrac{\partial V(e,t)}{\partial t} + ({\rm grad}V, \tilde{f}(e, x_{\rm d})) \le -\gamma||e||^2.$ (8)

Theorem 1. *If Eq.(4) satisfies Assumption 1, then there exist a compensator $\overline{\alpha}(e, x_{\rm d}, \hat{\theta})$ and a parameter estimation update law $\dot{\hat{\theta}}$ such that*

$$\lim_{t\to\infty} \| e(t) \| = \lim_{t\to\infty} \| x(t) - x_{\rm d}(t) \| = 0,$$

$$\lim_{t\to\infty} \| \tilde{\theta}(t) \| = \lim_{t\to\infty} \| \theta - \hat{\theta}(t) \| = 0,$$

where

$$\overline{\alpha}(e, x_{\rm d}, \hat{\theta}) = -\tilde{F}(e, x_{\rm d})\hat{\theta},$$

$$\dot{\hat{\theta}} = -(\tilde{F}(e, x_{\rm d}) + F(x_{\rm d}))^T {\rm grad}V - \tilde{\theta}.$$

Proof: For Eq.(4), choose Lyapunov function $W(e, \tilde{\theta}, t) = V(e, t) + \dfrac{1}{2}\tilde{\theta}^T\tilde{\theta}$.

And $V(e,t)$ satisfies Assumption 1. We have

$$\begin{aligned}\frac{\mathrm{d}W}{\mathrm{d}t} =&\frac{\partial V(e,t)}{\partial t} + (\mathrm{grad}V, \tilde{f}(e,x_\mathrm{d})) \\ &+ (\mathrm{grad}V, \tilde{F}(e,x_\mathrm{d})\theta + F(x_\mathrm{d})\tilde{\theta}) \\ &+ (\mathrm{grad}V, \overline{\alpha}(e,x_\mathrm{d},\hat{\theta})) + \dot{\tilde{\theta}}^T\tilde{\theta} \\ \le& -\gamma \parallel e \parallel^2 + (\mathrm{grad}V, (\tilde{F}(e,x_\mathrm{d}) + F(x_\mathrm{d}))\tilde{\theta}) \\ &+ \dot{\tilde{\theta}}^T\tilde{\theta} \\ =& -\gamma \parallel e \parallel^2 - \parallel \tilde{\theta} \parallel^2 .\end{aligned}$$

This completes the proof.

From the process of anterior deduction and Theorem 1, we obtain

Theorem 2. *If Assumption 1 holds, then for an arbitrarily given desired smooth orbit x_d, there exists an adaptive controller and identifier $u(x,x_\mathrm{d},\hat{\theta})$ that makes system*

1) satisfy

$$\lim_{t\to\infty} \parallel e(t) \parallel = \lim_{t\to\infty} \parallel x(t) - x_\mathrm{d}(t) \parallel = 0,$$

$$\lim_{t\to\infty} \parallel \tilde{\theta}(t) \parallel = \lim_{t\to\infty} \parallel \theta - \hat{\theta}(t) \parallel = 0.$$

Ⅲ. SIMULATION AND CONCLUSION

As an example, we apply the result to the Rössler system with unknown parameters

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & -1 & -1\\ 1 & a & 0\\ x_3 & 0 & -c\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ b\end{bmatrix} + u. \tag{9}$$

We first translated Eq.(9) into the form of Eq.(1)

$$\dot{x} = f(x) + F(x)\theta + u, \tag{10}$$

where

$$f(x) = \begin{bmatrix}-x_2 - x_3\\ x_1\\ x_1x_3\end{bmatrix}, \quad F(x) = \begin{bmatrix}0 & 0 & 0\\ x_2 & 0 & 0\\ 0 & 1 & -x_3\end{bmatrix},$$

u is an attached control input, and $\theta = [a\ b\ c]^T$ is unknown parameter vector of the system.

It has been proved that Rössler system presents chaos when a=b=0.2, c=5.7. Given the desired orbit $x_\mathrm{d} = [0\ 0\ \sin t]^T$, the true value of "unknown" parameter vector, initial state of system and initial value of estimate for "unknown" parameter vector are taken as, $\theta = [0.2 \quad 0.2 \quad 5.7]^T, x(0) = [5 \quad -6 \quad -2]$ and $\hat{\theta}(0) = [0.1 \quad -0.5 \quad 2]^T$, respectively. Construct the controller and identifier $u(x,x_\mathrm{d},\hat{\theta})$ according to Eq.(3).

From Theorem 1 we obtain the compensator and update law for parameters estimation as

$$\overline{\alpha}(e,x_\mathrm{d},\hat{\theta}) = u_r(e,x_\mathrm{d}) - \tilde{F}(e,x_\mathrm{d})\hat{\theta},$$

$$\dot{\tilde{\theta}} = -(\tilde{F}(e,x_\mathrm{d}) + F(x_\mathrm{d}))^T\mathrm{grad}V - \tilde{\theta},$$

where

$$u_r(e,x_\mathrm{d}) = \begin{bmatrix}-2 & 1 & 1\\ -1 & -2 & 0\\ -\sin t & 0 & -2-e_1\end{bmatrix} e.$$

Choose $V(e,t) = \mathrm{e}^T P e$,

where

$$P = \begin{bmatrix}3 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 5\end{bmatrix}, \lambda_\mathrm{min} = 3, \quad \lambda_\mathrm{max} = 5, \quad \gamma = 12.$$

Using Theorem 1 and Theorem 2, we conclude that there exists an adaptive controller and identifier $u(x,x_\mathrm{d},\hat{\theta})$ which makes the Rössler system satisfy

$$\lim_{t\to\infty} \parallel e(t) \parallel = \lim_{t\to\infty} \parallel x(t) - x_\mathrm{d}(t) \parallel = 0,$$

$$\lim_{t\to\infty} \parallel \tilde{\theta}(t) \parallel = \lim_{t\to\infty} \parallel \theta - \hat{\theta}(t) \parallel = 0.$$

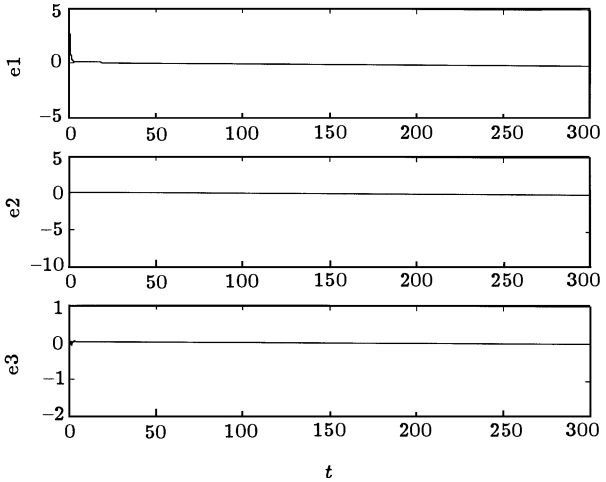


Fig.1. Time of response error states.

After adding $u(x,x_\mathrm{d},\hat{\theta})$ to system (9), the time response of error states and the time response of states are shown in Figs.1 and 2, respectively. The time response of error estimates between unknown parameters and their true values is shown in Fig.3. And the time response of parameter estimation for unknown parameters can be seen in Fig. 4.

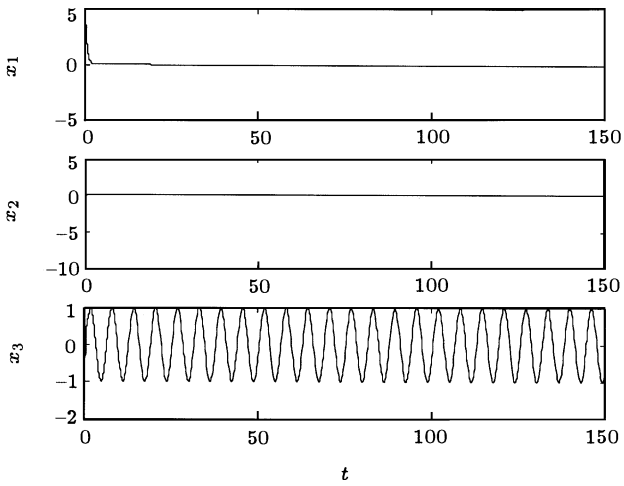


Fig.2. Time response of states.

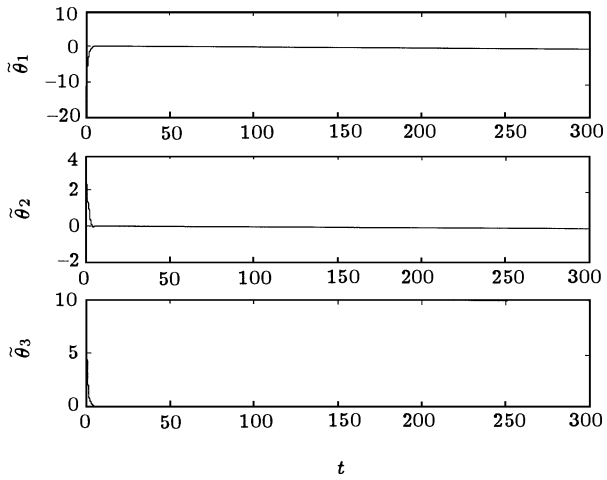


Fig.3. Time response of error estimates for θ.

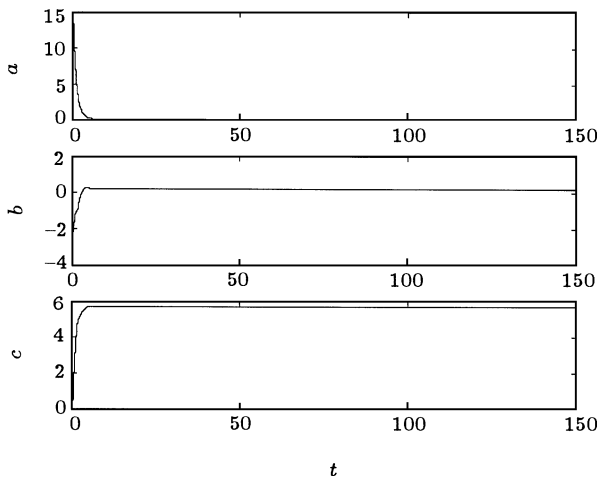


Fig.4. Estimation $\hat{\theta}(t)$ for unknown parameter θ.

Based on the Lyapunov function, a novel adaptive control method is proposed for a class of chaotic systems with uncertain parameters. Using the presented method, an adaptive controller and identifier is developed that can remove chaos in nonlinear systems and make the system asymptotically stabilizing to an arbitrarily desired smooth orbit. And at the same time, estimates of uncertain parameters converge to their true values aysmptotically. The advantage of our method over the existing result is that the controller and identifier is directly constructed by an analytical formula without knowing unknown bounds about uncertain parameters in advance. A computer simulation example is given to validate the proposed approach.

References

[1] Ott E, Grebogy C and Yorke J A 1990 *Phys. Rev. Lett.* **64** 1196

[2] Nitsche G and Dressler U 1992 *Physica* D **58** 153

[3] Chen G and Dong X 1993 *IEEE Trans. Circuits Syst.* **40** 591