Vol 10 No 6, June 2001
1009-1963/2001/10(06)/0490-04
Chinese Physics
© 2001 Chin. Phys. Soc.
and IOP Publishing Ltd

QUANTUM FLUCTUATIONS IN A MESOSCOPIC DAMPED LC PARALLEL CIRCUIT IN DISPLACED SQUEEZED FOCK STATE

Gu Yong-jian(顾永建)

Department of Physics, Weifang University, Weifang 261043, *China*

(Received 15 December 2000; revised manuscript received 18 January 2001)

We study the quantum effects of a damped LC parallel circuit considering its different performance from an RLC series circuit in classical physics. The damped LC parallel circuit with a source is quantized and the quantum fluctuations of magnetic flux and electric charge in the circuit in displaced squeezed Fock state are investigated. It is shown that, as in the RLC series circuit, the fluctuations only depend on the squeezing parameter and the parameters of the circuit components in the damped LC parallel circuit, but the effects of the circuit components on the fluctuations are different in the two circuits.

Keywords: mesoscopic circuit, damped LC parallel circuit, displaced squeezed Fock state, quantum fluctuation

PACC: 0365,7335

I. INTRODUCTION

With the rapid progress in nanotechnology and microelectronics, integrated circuits and devices are being miniaturized towards atomic scale dimensions.[1] Quantum effects must be taken into account when the transport length reaches a characteristic dimension, namely the charge carrier inelastic coherence length. In 1973, Louisell first discussed the quantum effects of an inductor-capacitor (LC) series circuit in vacuum state and gave the quantum noise in the circuit.[2] With the rise of mesoscopic physics,[3,4] more attention is paid to this sort of problem.[5,6] A resistor-inductor-capacitor (RLC) series circuit is a more practical model considering the resistance of the inductor and the internal resistance of the source.[7] Quantization of this circuit was realized[8,9] by generalizing Dekker's quantization scheme for a linearly damped harmonic oscillator[10] and making an analogy between the two models, and the quantum fluctuations in the circuit in the squeezed vacuum state and in the displaced squeezed Fock state were investigated.[11−13]

Another important model is an LC parallel circuit, which performs differently from an LC series circuit in classical physics. More practical is a damped LC parallel circuit in view of the resistance of the inductor and the internal resistance of the source. As far as we know, the quantum effects of this circuit have not been deeply studied. In this paper, we study the quantum fluctuations in a mesoscopic damped LC parallel circuit in the displaced squeezed Fock state. The displaced squeezed Fock state is more general than the vacuum state or squeezed vacuum state. Actually, it can be reduced to the squeezed Fock state, displaced Fock state, squeezed vacuum state, coherent state, squeezed state, Fock state and vacuum state. We quantize the circuit starting from its classical equation of motion and derive expressions for the quantum fluctuations. We also discuss the effects of various parameters on the fluctuations.

II. THEORETICAL ANALYSIS

From Kirchhoff's law, the classical equation of motion for a damped LC parallel circuit with a source (see figure 1) can be written as

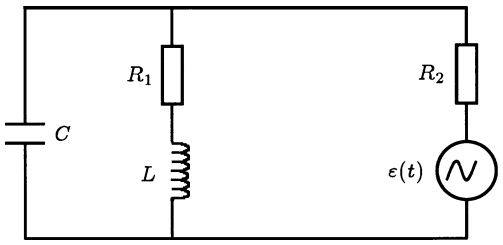


Fig.1. Schematic diagram of a damped LC parallel circuit with a source.

$$LC\frac{\mathrm{d}^2 j(t)}{\mathrm{d}t^2}+\left(R_1C+\frac{L}{R_2}\right)\frac{\mathrm{d}j(t)}{\mathrm{d}t}+\left(1+\frac{R_1}{R_2}\right)j(t)=\frac{\varepsilon(t)}{R_2}, \qquad (1)$$

where L and C denote the inductance and capacitance, respectively, R_1 and R_2 are the resistance of

the inductor and the internal resistance of the source, respectively, $j(t)$ is the current through the inductor, and $\varepsilon(t)$ the electromotive force of the source. Let $\Phi = Lj(t)$, which stands for the magnetic flux (also called generalized current in some references). Substituting Φ into Eq.(1) gives

$$C\frac{\mathrm{d}^2\Phi}{\mathrm{d}t^2} + \left(\frac{R_1C}{L} + \frac{1}{R_2}\right)\frac{\mathrm{d}\Phi}{\mathrm{d}t} + \frac{1+R_1/R_2}{L}\Phi = \frac{\varepsilon(t)}{R_2}. \tag{2}$$

We make an analogy between Eq.(2) and that of a linearly damped harmonic oscillator. The canonically conjugate variable of Φ is $Q = C\mathrm{d}\Phi/\mathrm{d}t$. It is clear that the electric charge on the capacitor

$$q = Q + \frac{R_1C}{L}\Phi. \tag{3}$$

To quantize the circuit, look upon Φ and Q as operators, and order $[\Phi, Q] = \mathrm{i}\hbar$. From Eq.(3), q is also an operator, and we have $[\Phi, q] = \mathrm{i}\hbar$. The Hamiltonian of the circuit after quantization is given by [8]

$$H = \hbar(\omega - \mathrm{i}\lambda)a^+a + a^+S(t) + aS^*(t), \tag{4}$$

where

$$a = (2C\hbar\omega)^{-1/2}[Q + (\lambda - \mathrm{i}\omega)C\Phi], \tag{5}$$

$$a^+ = (2C\hbar\omega)^{-1/2}[Q + (\lambda + \mathrm{i}\omega)C\Phi], \tag{6}$$

with

$$\omega^2 = \omega_0^2 - \lambda^2, \quad \omega_0^2 = \frac{1+R_1/R_2}{LC},$$
$$\lambda = \frac{R_1}{2L} + \frac{1}{2R_2C}, \quad S(t) = \mathrm{i}\sqrt{\hbar/2}\frac{\varepsilon(t)}{R_2C}, \tag{7}$$

for the damped LC parallel circuit with a source. From $[\Phi, Q] = \mathrm{i}\hbar$, we get $[a, a^+]$=1. If $S(t)$=0, we can define the vacuum state and Fock states for $[a, a^+]$=1.

From Eqs.(5), (6) and (3) we have

$$\Phi = -\mathrm{i}\sqrt{\frac{\hbar}{2\omega C}}(a^+ - a), \tag{8}$$

$$q = -\mathrm{i}\sqrt{\frac{\hbar C}{2\omega}}[(\kappa + \mathrm{i}\omega)a - (\kappa - \mathrm{i}\omega)a^+], \tag{9}$$

where

$$\kappa = \lambda - \frac{R_1}{L} = \frac{1}{2R_2C} - \frac{R_1}{2L}. \tag{10}$$

We assume the circuit is in the displaced squeezed Fock state[14−16]

$$|\alpha, \xi, n\rangle = D(\alpha)S(\xi)|n\rangle, \tag{11}$$

where $D(\alpha), S(\xi)$ are the displacement operator and squeezing operator, respectively, and can be expressed as

$$D(\alpha) = \exp(\alpha a^+ - \alpha^* a), \tag{12}$$

$$S(\xi) = \exp\left(\frac{1}{2}\xi a^{+2} - \frac{1}{2}\xi^* a^2\right), \tag{13}$$

where $\alpha = |\alpha|\mathrm{e}^{\mathrm{i}\varphi}$ is the displacement parameter and $\xi = r\mathrm{e}^{\mathrm{i}\theta}$ the squeezing parameter. Using the formula

$$\mathrm{e}^{\lambda A}B\mathrm{e}^{-\lambda A} = B + \lambda[A, B] + \frac{\lambda^2}{2!}[A, [A, B]] + \cdots, \tag{14}$$

we can easily prove that

$$D^+(\alpha)aD(\alpha) = a + \alpha, \tag{15}$$

$$D^+(\alpha)a^+D(\alpha) = a^+ + \alpha^*, \tag{16}$$

$$S^+(\xi)aS(\xi) = a\cosh r + a^+\mathrm{e}^{\mathrm{i}\theta}\sinh r, \tag{17}$$

$$S^+(\xi)a^+S(\xi) = a^+\cosh r + a\mathrm{e}^{-\mathrm{i}\theta}\sinh r. \tag{18}$$

From Eqs.(8), (9) and (15)−(18), we get

$$S^+(\xi)D^+(\alpha)\Phi D(\alpha)S(\xi) = -\mathrm{i}\sqrt{\frac{\hbar}{2\omega C}}[A(\xi)a - A^*(\xi)a^+ + B(\alpha)], \tag{19}$$

$$S^+(\xi)D^+(\alpha)qD(\alpha)S(\xi) = -\mathrm{i}\sqrt{\frac{\hbar C}{2\omega}}[E(\xi)a - E^*(\xi)a^+ + F(\alpha)], \tag{20}$$

where

$$A(\xi) = \mathrm{e}^{-\mathrm{i}\theta}\sinh r - \cosh r, \tag{21}$$

$$B(\alpha) = -2\mathrm{i}|\alpha|\sin\varphi, \tag{22}$$

$$E(\xi) = (\kappa + \mathrm{i}\omega)\cosh r - (\kappa - \mathrm{i}\omega)\mathrm{e}^{-\mathrm{i}\theta}\sinh r, \tag{23}$$

$$F(\alpha) = 2\mathrm{i}|\alpha|(\kappa\sin\varphi + \omega\cos\varphi). \tag{24}$$

From Eqs.(19) and (20), and using the formula

$$\mathrm{e}^{\lambda A}B^m\mathrm{e}^{-\lambda A} = (\mathrm{e}^{\lambda A}B\mathrm{e}^{-\lambda A})^m, \tag{25}$$

we obtain

$$\begin{aligned} &S^+(\xi)D^+(\alpha)\Phi^2D(\alpha)S(\xi) \\ =&\frac{\hbar}{2\omega C}[|A(\xi)|^2(2a^+a+1) \\ &- B^2(\alpha) - A^2(\xi)a^2 - A^{*2}(\xi)a^{+2} \\ &- 2B(\alpha)A(\xi)a + 2B(\alpha)A^*(\xi)a^+], \end{aligned} \tag{26}$$

$$\begin{aligned} &S^+(\xi)D^+(\alpha)q^2D(\alpha)S(\xi) \\ =&\frac{\hbar C}{2\omega}[|E(\xi)|^2(2a^+a+1) \\ &- F^2(\alpha) - E^2(\xi)a^2 - E^{*2}(\xi)a^{+2} \\ &- 2F(\alpha)E(\xi)a + 2F(\alpha)E^*(\xi)a^+]. \end{aligned} \tag{27}$$

From Eqs.(19), (20) and (26), (27) we get the average and the mean-square values of magnetic flux and charge in displaced squeezed Fock state as follows:

$$\langle \Phi \rangle = -\mathrm{i}\sqrt{\frac{\hbar}{2\omega C}}B(\alpha), \tag{28}$$

$$\langle q \rangle = -\mathrm{i}\sqrt{\frac{\hbar C}{2\omega}}F(\alpha), \tag{29}$$

$$\langle \Phi^2 \rangle = \frac{\hbar}{2\omega C}[(2n+1)|A(\xi)|^2 - B^2(\alpha)], \tag{30}$$

$$\langle q^2 \rangle = \frac{\hbar C}{2\omega}[(2n+1)|E(\xi)|^2 - F^2(\alpha)]. \tag{31}$$

Therefore the quantum fluctuations of magnetic flux and charge and their product are

$$\langle (\Delta\Phi)^2 \rangle = \frac{\hbar}{2\omega C}|A(\xi)|^2(2n+1), \tag{32}$$

$$\langle (\Delta q)^2 \rangle = \frac{\hbar C}{2\omega}|E(\xi)|^2(2n+1), \tag{33}$$

$$\langle (\Delta\Phi)^2 \rangle \langle (\Delta q)^2 \rangle = \frac{\hbar^2}{2\omega^2}|A(\xi)|^2|E(\xi)|^2(2n+1)^2. \tag{34}$$

From Eqs.(21) and (23), $|A(\xi)|^2$ and $|E(\xi)|^2$ can be written as

$$|A(\xi)|^2 = \cosh 2r - \sinh 2r\cos\theta, \tag{35}$$

$$\begin{aligned}|E(\xi)|^2 =& \frac{1}{LC}\cosh 2r + \sinh 2r[2\omega\kappa\sin\theta \\ &+ (\omega^2-\kappa^2)\cos\theta].\end{aligned} \tag{36}$$

From Eqs.(28)–(34) we see that, besides the parameters of the circuit components, the average values of both the magnetic flux and the charge only depend on the displacement parameter, the mean-square values depend on both the displacement parameter and the squeezing parameter, while the quantum fluctuations and their product only depend on the squeezing parameter. To be more specific, consider the effects of the squeezing parameter on the fluctuations when $R_1 \to 0$ and $R_2 \to \infty$. Eqs.(32) and (33) are then reduced to

$$\langle (\Delta\Phi)^2 \rangle = \frac{\hbar}{2}\sqrt{\frac{L}{C}}(2n+1)(\cosh 2r - \cos\theta\sinh 2r), \tag{37}$$

$$\langle (\Delta q)^2 \rangle = \frac{\hbar}{2}\sqrt{\frac{C}{L}}(2n+1)(\cosh 2r + \cos\theta\sinh 2r). \tag{38}$$

As seen from the above equations, when θ takes different values, the circuit exhibits different quantum fluctuations. When the fluctuation of Φ reaches its minimum, the fluctuation of q reaches its maximum (θ=0), and vice versa ($\theta = \pi$). So one can choose a suitable squeezing angle to lower the quantum noise of a needed physical quantity.

On the other hand, the quantum fluctuations are affected by the circuit components. For the sake of simplicity, consider a special case that the squeezing parameter ξ=0. Eqs.(32) and (33) then reduce to

$$\begin{aligned}\langle (\Delta\Phi)^2 \rangle =& \frac{\hbar}{2}\sqrt{\frac{L}{C}}(2n+1) \\ &\cdot \left[1 + \frac{R_1}{R_2} - \frac{C}{4L}\left(R_1 + \frac{L}{R_2 C}\right)^2\right]^{-1/2},\end{aligned} \tag{39}$$

$$\begin{aligned}\langle (\Delta q)^2 \rangle =& \frac{\hbar}{2}\sqrt{\frac{C}{L}}(2n+1) \\ &\cdot \left[1 + \frac{R_1}{R_2} - \frac{C}{4L}\left(R_1 + \frac{L}{R_2 C}\right)^2\right]^{-1/2}.\end{aligned} \tag{40}$$

Eqs.(39) and (40) show how the circuit components affect the fluctuations in the damped LC parallel circuit. For the RLC series circuit, Ref.[12] gives

$$\langle (\Delta q)^2 \rangle = \frac{\hbar}{2}\sqrt{\frac{C}{L}}(2n+1)\left(1 - \frac{R^2 C}{4L}\right)^{-1/2}, \tag{41}$$

$$\langle (\Delta\Phi)^2 \rangle = \frac{\hbar}{2}\sqrt{\frac{L}{C}}(2n+1)\left(1 - \frac{R^2 C}{4L}\right)^{-1/2}, \tag{42}$$

which are different from Eqs.(39) and (40).

If the parameters α, ξ and n take special values, we can obtain the quantum fluctuations in special quantum states, which include the squeezed Fock state $|0,\xi,n\rangle$, squeezed vacuum state $|0,\xi,0\rangle$, squeezed state $|\alpha,\xi,0\rangle$, displaced Fock state $|\alpha,0,n\rangle$, coherent state $|\alpha,0,0\rangle$, Fock state $|0,0,n\rangle$ and vacuum state $|0,0,0\rangle$. We give these in table 1.

Table 1. Products of the quantum fluctuations of magnetic flux and electric charge in various states.

States	Products of the fluctuations
$\|0,\xi,n\rangle$	$\frac{\hbar^2}{4\omega^2}\|A(\xi)\|^2\|E(\xi)\|^2(2n+1)^2$
$\|0,\xi,0\rangle$	$\frac{\hbar^2}{4\omega^2}\|A(\xi)\|^2\|E(\xi)\|^2$
$\|\alpha,\xi,0\rangle$	$\frac{\hbar^2}{4\omega^2}\|A(\xi)\|^2\|E(\xi)\|^2$
$\|\alpha,0,n\rangle$	$\frac{\hbar^2}{4}(2n+1)^2\left[1+\frac{R_1}{R_2}-\frac{C}{4L}\left(R_1+\frac{L}{R_2C}\right)^2\right]^{-1}$
$\|\alpha,0,0\rangle$	$\frac{\hbar^2}{4}\left[1+\frac{R_1}{R_2}-\frac{C}{4L}\left(R_1+\frac{L}{R_2C}\right)^2\right]^{-1}$
$\|0,0,n\rangle$	$\frac{\hbar^2}{4}(2n+1)^2\left[1+\frac{R_1}{R_2}-\frac{C}{4L}\left(R_1+\frac{L}{R_2C}\right)^2\right]^{-1}$
$\|0,0,0\rangle$	$\frac{\hbar^2}{4}\left[1+\frac{R_1}{R_2}-\frac{C}{4L}\left(R_1+\frac{L}{R_2C}\right)^2\right]^{-1}$

Ⅲ. CONCLUSIONS

The damped LC parallel circuit with a source is quantized by making an analogy between its equation of motion with that of a linearly damped harmonic oscillator. The quantum fluctuations of the magnetic flux and the charge in the circuit in the displaced squeezed Fock state are investigated. Our results show that, as in the RLC series circuit, besides the parameters of the circuit components, the average values of both the magnetic flux and the charge only depend on the displacement parameter, the mean-square values depend on both the displacement parameter and the squeezing parameter, while the quantum fluctuations and their product only depend on the squeezing parameter. On the other hand, as shown in Eqs.(39)–(42), the effects of the circuit components on the fluctuations are different in the two circuits.

References

[1] Garcia R G 1992 *Appl. Phys. Lett.* **60** 1960
[2] Louisell W H 1973 *Quantum Statistical Properties of Radiation* (New York: Wiley)
[3] Caldeira A Q and Leggett A J 1983 *Ann. Phys.* (NY) **149** 374
[4] Zurek W H 1991 *Phys. Today* **44** 36
[5] Srivastava Y and Widom A 1987 *Phys. Rep.* **148** 1
[6] Buot F A 1993 *Phys. Rep.* **234** 73
[7] Chen B, Gao S E and Jiao Z K 1995 *Acta Phys. Sin.* **44** 1480 (in Chinese)
[8] Chen B, Li Y Q, Fang H, Jiao Z K and Zhang Q R 1995 *Phys. Lett.* A **205** 121
[9] Zhang Z M, He L S and Zhou S K 1998 *Phys. Lett.* A **244** 196
[10] Dekker H 1979 *Physica* A **95** 311
[11] Yu Z X, Zhang D X and Liu Y H 1997 *Acta Phys. Sin.* (*Overseas Edition*) **6** 522
[12] Wang J S and Sun C Y 1998 *Int. J. Theor. Phys.* **37** 4
[13] Wang X G and Pan S H 2000 *Chin. Phys. Lett.* **17** 171
[14] Moller K B, Jorgensen T G and Dahl J P 1996 *Phys. Rev.* A **54** 5378
[15] Szabo S, Adam P, Janszky J and Domokos P 1996 *Phys. Rev.* A **53** 2698
[16] Nieto M M 1997 *Phys. Lett.* A **229** 135