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THE TWO-PHOTON DEGENERATE JAYNES–CUMMINGS MODEL WITH AND WITHOUT ROTATING-WAVE APPROXIMATION

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We take into account the two-photon process and generalize the Jaynes–Cummings (JC) model to the case of atomic level degenerate in the projections of the angular momenta, and we establish two-photon degenerate JC models with and without the rotating-wave approximation (RWA) quantum theory. Comparing the atom population inversion of the generalized JC model with that of the original JC model, we found that the revival period of the degenerate JC model becomes longer and the maximum amplitude of atomic inversion decreases with RWA. Without RWA, the quantum chaos of the generalized JC model is much weaker than that of the original JC model.

Keywords: generalized Jaynes–Cummings model, two-photon process, atom population inversion
PACC: 4250, 3280

I. INTRODUCTION

The Jaynes–Cummings (JC) model has been recognized as the simplest and most effective model to describe the interaction of a single linearly-polarized mode of the quantized electromagnetic field with an isolated two-level atom. In addition to its exact solvability within the rotating-wave approximation (RWA), the model provides a number of remarkable properties due to the quantum aspects of the field, such as 'collapse and revival' in the time domain. This model has been studied by many authors, and numerous non-classical effects have been predicted.[1−3] However, if the levels of an isolated atom are degenerate in the projection of the total electronic angular momenta on the quantization axis, the original JC model becomes, in general, invalid. Recently, Reshetov[4] considered the degeneracy of atomic levels and generalized the JC model to the case of atomic level degenerate in the projections of the angular momenta within RWA. On the other hand, the two-photon process is an important process in the interaction between the atom and quantum electromagnetic fields. Many nonclassical effects, for example, the squeezed states of light,[5,6] involve the two-photon process. In this paper, we take into account the two-photon process and the degeneracy of atomic levels and generalize the JC model. We establish a two-photon degenerate JC model with and without RWA quantum theory. Comparing the atomic inversion of the generalized JC model with that of the original JC model, we found that the period of revival increases and the amplitude decreases with RWA. When we introduce counter-rotating terms, the quantum chaos are suppressed by the degeneracy of atomic levels.

This paper is organized as follows: in section II we present the general formulae for the problem and derive the temporal evolution of the wave function with and without RWA; in section III, we numerate the atom population inversion and compare the results of the generalized JC model with that of the original JC model. A brief summary is given in the final section.

II. GENERAL FORMALISM

Taking into account the degeneracy of atomic levels, the full set of the system may be written as

$$|n, J_\alpha, m_\alpha\rangle = |n\rangle \cdot |J_\alpha, m_\alpha\rangle,$$
$$n = 0, 1, ..., m_\alpha = -J_\alpha, ..., J_\alpha, \alpha = b, c,$$

where n is the number of photons in the field mode, and b and c denote the upper and lower atomic levels respectively. The values of J_b and J_c are the total electronic angular momenta of resonant levels, while m_b and m_c are their projections on the quantization axis, the Cartesian axis z, which is directed along the polarization vector of the field mode.

In the two-photon process, some intermediate states are involved, which usually accompany the Stark shift. Here we will only consider the degeneracy of atomic levels and omit Stark shift so as to make the effect of the degeneracy be seen clearly. If the

frequencies ω_1 and ω_2, the intermediate states coupled to the upper and lower levels, respectively, are equal, the system may be treated as a single-mode two-photon process. Brune[7] have successfully made a single-mode two-photon maser. We will consider this circumstance here.

We assume that the interaction between the atom and the field observes the two-photon resonant condition, i.e. $\omega = 2\omega_0$. The Hamiltonian of the system without RWA can be written as

$$\hat{H} = \hat{H}_0 + \hat{V}, \tag{1}$$

with

$$\hat{H}_0 = \hbar\omega_0[\hat{a}^+\hat{a} + (\hat{n}_b - \hat{n}_c)], \tag{2}$$

where $\hat{a}^+$ and $\hat{a}$ are the operators of the creation and annihilation of photons with frequency ω_0 in the field mode, and

$$\hat{n}_\alpha = \sum_{m_\alpha=-J_\alpha}^{J_\alpha} |J_\alpha, m_\alpha\rangle\langle J_\alpha, m_\alpha|, \qquad \alpha = b, c$$

are the operators of total population of resonant atomic levels b and c.

$$\hat{V} = \mathrm{i}g\hbar(\hat{a}^{+2}\hat{p} - \hat{a}^2\hat{p}^+ + \hat{a}^{+2}\hat{p}^+ - \hat{a}^2\hat{p}) \tag{3}$$

denotes the interaction term between degenerate atom and the field, and

$$\hat{p} = \sum_m \alpha_m |J_c,m\rangle\langle J_b,m| \tag{4}$$

is the dipole moment operator of the atomic transition $J_b \to J_c$, where

$$\alpha_m = (-1)^{j_b-m} \begin{pmatrix} J_b & 1 & J_c \\ -m & 0 & m \end{pmatrix} \tag{5}$$

is the matrix element defined through the Wigner $3j$-symbol.

A. The wavefunction with RWA

In the interaction representation, the interaction operator with RWA can be written as

$$\hat{V}_\mathrm{I} = \mathrm{i}g\hbar(\hat{a}^{+2}\hat{p} - \hat{a}^2\hat{p}^+). \tag{6}$$

We assume the atom to be initially in the degenerate level c, and the field initially in the coherent state, then the wavefunction of the system prior to the interaction is

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2J_c+1}} \sum_{n,m} F_n |n, J_c, m\rangle, \tag{7}$$

where $F_n = \exp\left(-\frac{|\alpha|^2}{2}\right)\frac{\alpha^n}{\sqrt{n!}}$. We postulate that the wavefunction of the total system at any time $t > 0$ has the form

$$|\Psi(t)\rangle = \sum_{n,m}(a_{n,m}|n, J_b, m\rangle + b_{n+2,m}|n+2, J_c, m\rangle), \tag{8}$$

by solving the Schrödinger equation, we have

$$\begin{cases} a_{n,m} = -\dfrac{F_{n+2}}{\sqrt{2J_c+1}} \sin\Omega_{n+2,m}t, \\ b_{n+2,m} = \dfrac{F_{n+2}}{\sqrt{2J_c+1}} \cos\Omega_{n+2,m}t, \end{cases} \tag{9}$$

where

$$\Omega_{n,m} = \alpha_m g\sqrt{n(n-1)}.$$

Now, we have derived the temporal evolution of the wavefunction, we may obtain the average value of any operator.

B. The temporal evolution of wavefunction without RWA

The validity of the RWA must be under certain conditions and in a certain range. Within the range, the RWA is a good approximation. Once the condition could not meet, the JC model without RWA[8−11] leads to somewhat different conclusions from those of the JC model with RWA, and under such a circumstance we should import the counter-rotating terms. In this section, we introduce the counter-rotating term and give the system the wavefunction at any time. By considering the counter-rotating term, the interaction Hamiltonian is

$$\hat{V}_\mathrm{I} = \mathrm{i}g\hbar(\hat{a}^{+2}\hat{p} - \hat{a}^2\hat{p}^+ + \hat{a}^{+2}\hat{p}^+\mathrm{e}^{4\mathrm{i}\omega_0 t} - \hat{a}^2\hat{p}\mathrm{e}^{-4\mathrm{i}\omega_0 t}), \tag{10}$$

the Schrödinger equation has no exact solution. Using the perturbation theory,[8] we treat the wavefunction with RWA as zero-order approximation and only retain terms up to the first order in g/ω_0; then we obtain

$$a_{n,m} = \frac{A}{\sqrt{2J_c+1}} F_{n+2}\Bigg\{ -\sin\Omega_{n+2,m}t - \mathrm{i}\Omega_{n+2,m}\frac{n(n-1)}{2N^2} \times \left[\frac{\exp\mathrm{i}(\Omega_{n-2,m}+4\omega_0)t}{\Omega_{n-2,m}+4\omega_0} + \frac{\exp\mathrm{i}(4\omega_0-\Omega_{n-2,m})t}{4\omega_0-\Omega_{n-2,m}}\right]\Bigg\},$$

$$b_{n+2,m} = \frac{A}{\sqrt{2J_c+1}} F_{n+2} \Big\{ \cos \Omega_{n+2,m} t + \frac{\alpha_m N^2}{2\sqrt{(n+5)(n+6)}} \times \left[\frac{\exp i(\Omega_{n+6,m} - 4\omega_0)t}{4\omega_0 - \Omega_{n+6,m}} - \frac{\exp i(-4\omega_0 - \Omega_{n+6,m})t}{4\omega_0 + \Omega_{n+6,m}} \right] \Big\}, \quad (11)$$

where A is normalization factor and satisfies the condition

$$\sum_m (|a_{n,m}|^2 + |b_{n+2,m}|^2) = |F_{n+2}|^2.$$

The expressions of $a_{n,m}$ and $b_{n+2,m}$ in Eq.(9) with RWA are similar to those in Ref.[8], the only difference is just that the frequency $\Omega_n = g\sqrt{n(n-1)}$ becomes $\Omega_{n,m} = \alpha_m g\sqrt{n(n-1)}$ and they have the extra factor $\frac{1}{\sqrt{2J_c+1}}$. The forms of $a_{n,m}$ and $b_{n+2,m}$ without RWA in Eq.(11) also have similar forms to that of the original two-photon JC model.[8] But it is the difference that results in some marvellous novel properties. In the next section, we will numerate some results and compare these novel properties with those of the original JC model.

III. RESULTS OF CALCULATIONS WITH AND WITHOUT RWA

In order to observe the results coming from the atomic degeneracy levels, we study the atom population inversion with RWA. From Eq.(9) we obtain the atom population inversion, that is, the population difference between the upper and lower levels,

$$< \hat{n}_b - \hat{n}_c >= -\sum_{n,m} \frac{F_{n+2}^2}{2J_c+1} \cos(2\Omega_{n+2,m} t). \quad (12)$$

We see again from Eq.(12) that the form of population inversion is the same as the original JC model, and the difference is just in the sum of 'm'. The resonant levels b and c in the experiments[7,12] were Rydberg states of the rubidium atom with the angular momenta $J_b = 3/2$ and $J_c = 3/2$ or $J_c = 5/2$. Here we take both J_b and J_c to be $3/2$, and in this case,

$$\alpha_{1/2} = \alpha_{-1/2} = \frac{1}{2\sqrt{15}}, \quad \alpha_{3/2} = \alpha_{-3/2} = \frac{3}{2\sqrt{15}}. \quad (13)$$

In Fig.1(a) we plot the atomic population inversion in the generalized JC model, and in Fig.1(b) that in the original JC model. We also observe 'collapse and revival' phenomena with the generalized JC model, but the behaviour differs from that in the original JC model. Comparing Fig.1(a) with Fig.1(b), we find the revival period of generalized JC model becomes longer than that of the original JC model and

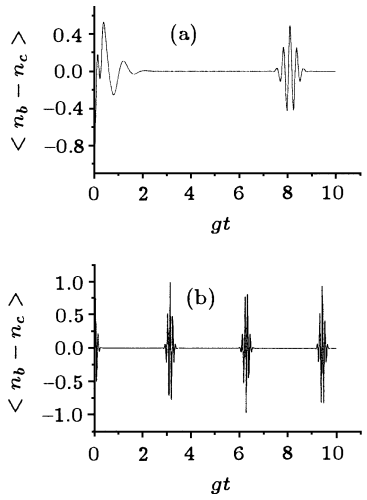


Fig.1. Comparision of atom population inversion between the generalized JC model and the original JC model with RWA, where $N = 30$: (a) the result of the generalized JC model ; (b) the result of the original JC model.

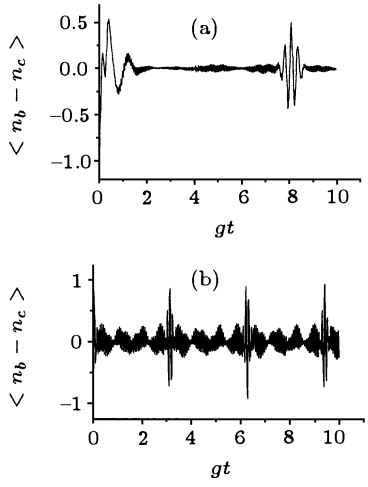


Fig.2. The temporal evolution of atomic population inversion without RWA when $N = 30$, g/ω_0=0.04: (a) the result of the generalized JC model; (b) the result of the original JC model.

the maximum amplitude of atomic inversion decreases. We can infer that the behaviour of 'collapse and revival' is related with angular momenta of the two atomic levels, because the sum of 'm' is related to the value of J_b and J_c. We believe the conclusion is

reasonable. The atomic transition between different J_b and J_c should result in some difference, but in the original JC model the collapse and revival has nothing to do with the two different levels of the atom if only the atom may be treated as a two-level atom.

We investigate the effect of the generalized JC model without RWA. Figures 2(a) and 2(b) are the curves of atom population inversion without RWA corresponding to the generalized and the original, respectively. They both show quantum chaos but the chaos of the generalized JC model are much weaker than that of the original JC model with the same index number g/ω_0, which is regarded as the important index number for deciding whether it is necessary to take RWA or not. Consequently, when we use the generalized JC model, the condition for utilizing RWA is not so strict. This means that even when the original JC model with RWA becomes invalid, we can still use the generalized JC model with RWA. In Fig.3, we change the value of g/ω_0 to 0.08. With the increase of index number g/ω_0, the quantum chaos of both 3(a) and 3(b) become stronger. Nevertheless, the quantum chaos of the original JC model increases faster than that of the generalized JC model. In other words, with the increase of g/ω_0 the quantum chaos of the generalized JC model increases more slowly than that of the original JC model. We conclude that the degeneracy of angular momentum can restrain the phenomenon of quantum chaos, i.e. we can employ RWA reassurance in a larger range.

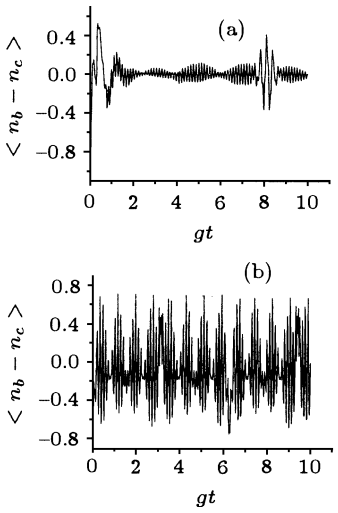


Fig.3. The same as Fig.2, but with $g/\omega_0 = 0.08$.

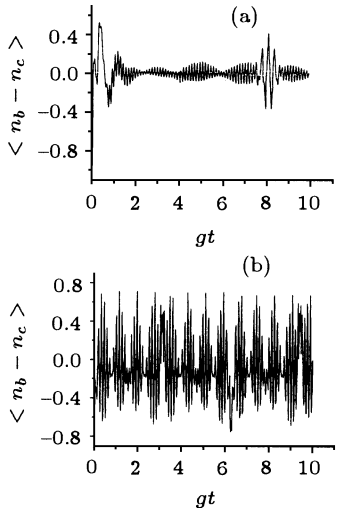


Fig.4. The temporal evolution of power spectrum where (a) and (b) correspond to Figs.3(a) and 3(b), respectively.

Figures 4 (a) and 4(b) show the power spectra of $< n_b - n_c >$,

$$P(\omega) = \left| \frac{2}{t_{\max}} \int_0^{t_{\max}} < n_b - n_c > \mathrm{e}^{-\mathrm{i}\omega t} \mathrm{d}t \right|, \qquad (14)$$

corresponding to Figs.3(a) and 3(b), respectively. The power spectrum is often employed as the diagnostics of quantum chaos. In Fig.3 we could not observe the typical signature of chaos, the broadband and continuous component, and we cannot make diagnostics that the system has no quantum chaos. Fukuo[13] believed that chaotic properties inherent in the non-RWA JC model, which are usually suppressed by the linearity of the Schrödinger equation, are recovered due to the breakdown of time reversibility under the continuous quantum nondemolition measurement (CQND). When the spectrum calculated by using classical Maxwell–Bloch equations exhibits chaos, the unmeasured non-RWA JC model does not show chaos. Because our system here is a 'closed' one, it is not surprising that we do not find the typical characteristics of chaos. Figure 4 shows that the temporal evolution has no period and quasiperiod. We judge they are quantum chaos. We find that the quantum chaos of the generalized JC model is stronger than that of the original one. The amplitude of the spectrum in the original JC model is greater than that of the generalized JC model. This justifies the conclusion of Figs.3 and 2.

IV. CONCLUSIONS

Considering that the two-photon process and atomic levels degenerate in the projections of the angular momenta, we establish the quantum theory of

degenerate two-photon JC model with and without RWA. Comparing the atomic population inversion of the generalized JC model with that of the original JC model, we found the following. (1) With RWA, the revival period of the generalized JC model becomes longer and the maximum amplitude of atomic inversion decreases. We conclude that the 'collapse and revival' is related to the angular momenta of the two atomic levels. (2) Without RWA, the quantum chaos of the generalized JC model are much weaker than that of the original JC model with the same index number g/ω_0. The degeneracy of angular momentum can restrain the phenomenon of quantum chaos.

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