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PARISI–SOURLAS CONFINEMENT MECHANISM OF QUANTUM CHROMODYNAMICS

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Using Parisi and Sourlas dimensional reduction, four-dimensional quantum chromodynamics is reduced to a two-dimensional principal chiral model by suitable superspace embedding. The frame T^a cannot be regarded as a fixed one and the frame connection field $\varpi_\mu(x)$ becomes a dynamical gauge field in two dimensions, giving rise to a confining potential. As a result of the original $SU(3)$ Yang–Mills field obtains another $SU(3)$ local symmetry and turns into $SU(3)\times SU(3)$ local symmetry—one group element as $\exp(\mathrm{i}\delta\varphi^a T^a)$ with fixed frame T^a, the other gauging the frame T^a.

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It is widely believed that the strong interaction is described by a non-Abelian $SU(3)$ gauge theory of quarks and gluons[1,2] which is called quantum chromodynamics (QCD) with the Lagrangian density as

$$\mathcal{L}=-\frac{1}{2e^2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}+\sum_{i=1}^{N_f}\bar{\psi}_i\left(\mathrm{i}\gamma^\mu D_\mu-M_i\right)\psi_i, \quad (1)$$

where $F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+e\epsilon_{abc}A^a_\mu A^a_\nu$ is an $SU(3)$ gauge field, and ψ_i are quark fields with the index i labelling flavours with $N_f=3$. Varied phenomena of QCD appear, such as colour confinement, dynamical chiral-symmetry breaking, asymptotic freedom and quantum anomalies. In particular, confinement is the most outstanding feature in nonperturbative QCD. An isolated quark or an isolated anti-quark has never been observed in experiments. Nowadays they are considered to be confined in the hadrons. This is the hypothesis of quark confinement.

To explain confinement one should admit in the QCD vacuum the nonperturbative component and split the total gluonic vector potential A_μ as

$$\tilde{A}_\mu=A_\mu+\varpi_\mu,$$

where ϖ_μ is the nonperturbative and A_μ the perturbative part. As for ϖ_μ, it can be: a) a superposition of classical solutions like instantons, monopoles, etc; b) purely quantum (but nonperturbative). In this picture, ϖ_μ may be a Parisi–Sourlas field in lower dimensions.

It is very popular that the nonperturbative contributions ϖ_μ come only from magnetic monopole which exactly determines the low-energy effective Abelian gauge theory. These examples show that the quark confinement is caused by the condensation of magnetic monopoles.[3–9] Instead of paying attention to monopoles or instantons, we are more interested in the Parisi–Sourlas mechanism of confinement. Based on this mechanism we will show the area law of the Wilson loop.

In this paper, we give a reasonable generalization to the Faddeev–Popov method. The Lorentz type gauge fixing term is written as

$$\mathcal{L}_{gf}=-\mathrm{Tr}\left(\nu\partial_\mu A^\Omega_\mu+\frac{\alpha}{2}\nu^2+\bar{\mathcal{C}}\partial_\mu D_\mu\left[(A^\Omega_\mu)\right]\mathcal{C}\right), \quad (2)$$

where the gauge field A^Ω_μ is gauge dependent $A^{\Omega=g}_\mu=g^{-1}A_\mu g+\dfrac{1}{\mathrm{i}e}g^{-1}\partial_\mu g$. Because the above gauge fixing term $\mathcal{L}_{gf}$ is gauge mode $g(x)$ dependent, we do not isolate out the integral $\int \mathrm{D}\Omega$ but keep the nonphysical gauge modes $g(x)$ as a dynamical field at first. After considering the physical effects of gauge modes $g(x)$, we absorb $g(x)$ into gauge field $A_\mu(x)$ through a gauge transformation $\Omega(x)^{-1}$ at last.

By BRST δ_B and anti-BRST $\bar{\delta}_\mathrm{B}$, [10–16] $\mathcal{L}_{gf}$ turns into a very compact form

$$\begin{aligned}\mathcal{L}_{gf}=&-\mathrm{Tr}\left(\nu\partial_\mu A^\Omega_\mu+\frac{\alpha}{2}\nu^2+\bar{\mathcal{C}}\partial_\mu D_\mu\left[(A^\Omega_\mu)\right]\mathcal{C}\right)\\ =&\mathrm{i}\delta_\mathrm{B}\bar{\delta}_\mathrm{B}\mathrm{Tr}\left[\frac{1}{2}(A^\Omega_\mu)^2+\frac{\mathrm{i}\alpha}{2}\mathcal{C}\bar{\mathcal{C}}\right], \end{aligned}\quad (3)$$

where BRST δ_B is

$$\begin{aligned}&\delta_\mathrm{B}A^\Omega_\mu(x)=D_\mu\mathcal{C}(x),\ \delta_\mathrm{B}\nu(x)=0,\\ &\delta_\mathrm{B}\mathcal{C}(x)=0,\ \delta_\mathrm{B}\overline{\mathcal{C}}(x)=\mathrm{i}\nu(x);\end{aligned}\quad (4)$$

and anti-BRST $\bar{\delta}_{\rm B}$ is

$$\bar{\delta}_{\rm B}A_\mu^\Omega(x)=D_\mu\overline{\mathcal{C}}(x),\ \bar{\delta}_{\rm B}\bar{\nu}(x)=0,$$
$$\bar{\delta}_{\rm B}\mathcal{C}(x)=\mathrm{i}\bar{\nu}(x),\ \bar{\delta}_{\rm B}\overline{\mathcal{C}}(x)=0 \tag{5}$$

with the Lagrangian multiplier field ν. In a (4+2)-dimensional superspace, $\left[\frac{1}{2}(A_\mu^\Omega)^2+\frac{\mathrm{i}a}{2}\mathcal{C}\bar{\mathcal{C}}\right]$ can be a contravariance supervector which transforms like the supercoordinate under Osp$(4\mid 2)$.[10−16]

The (4+2)-dimensional superspace $\mathcal{M}$ with coordinates is written as

$$X^M=(x^\mu,\theta,\bar{\theta})\in\mathcal{M},\quad x\in R^4,$$

where x^μ denotes the coordinates of the four-dimensional Euclidean space, θ and $\bar{\theta}$ are anti-Hermitian Grassmann numbers. The inner product of two vectors is defined by introducing the superspace (covariant) metric tensor η_{MN} with components

$$\eta_{\mu\nu}=\delta_{\mu\nu},\quad \eta_{\theta\bar{\theta}}=-\eta_{\bar{\theta}\theta}=-2/\gamma,\quad \text{others}=0.$$

The connection one-form (superspace vector potential) $\mathcal{A}(X)$ and its curvature (superspace field strength) $\mathcal{F}(X)$ in the superspace are defined as:

$$\begin{aligned}\mathcal{A}(X)=&\mathcal{A}_\mu(x,\theta,\bar{\theta})\mathrm{d}x^\mu+\mathcal{A}_\theta(x,\theta,\bar{\theta})\mathrm{d}\theta\\&+\mathcal{A}_{\bar{\theta}}(x,\theta,\bar{\theta})\mathrm{d}\bar{\theta},\\\mathcal{F}(X)=&\tilde{\mathrm{d}}\mathcal{A}(X)+\frac{1}{2}[\mathcal{A}(X),\mathcal{A}(X)]\\=&-\frac{1}{2}\mathcal{F}_{NM}(X)\mathrm{d}X^M\mathrm{d}X^N.\end{aligned}\tag{6}$$

The horizon condition[11] being

$$\mathcal{F}_{M\theta}(X)=\mathcal{F}_{M\bar{\theta}}(X)=0, \tag{7}$$

it turns out that the derivatives in the direction of θ and $\bar{\theta}$ are proportional to the BRST and the anti-BRST transformations,

$$\frac{\partial}{\partial\theta}=s\delta_{\rm B},\quad \frac{\partial}{\partial\bar{\theta}}=s\bar{\delta}_{\rm B}, \tag{8}$$

where s is the superspace embedding factor to be determined. Requiring $\mathcal{A}_\mu(X)\mid_{\theta=0,\bar{\theta}=0}=\mathcal{A}_\mu(x,0,0)=A_\mu(x)$, 'the superspace embedding relation' is found as

$$\mathcal{A}_\theta(x)=s\mathcal{C}(x),\ \mathcal{A}_{\bar{\theta}}(x)=s\bar{\mathcal{C}}(x), \tag{9}$$

$$\begin{aligned}\eta_{NM}\mathcal{A}^M(X)\mathcal{A}^N(X)=&[\mathcal{A}^\mu(X)]^\Omega[\mathcal{A}_\mu(X)]^\Omega\\&+\mathrm{i}\alpha\mathcal{C}(X)\bar{\mathcal{C}}(X).\end{aligned}\tag{10}$$

From this, the relation between s and γ in superspace is obtained as $\left(\frac{4s^2}{\gamma}\right)=\mathrm{i}a$.

Then the action is reduced to a two-dimensional sigma model as

$$\begin{aligned}S_{gf}=&\frac{\mathrm{i}}{2s^2}\int\mathrm{d}x^4\int\mathrm{d}\theta\mathrm{d}\bar{\theta}\mathrm{Tr}(\eta_{NM}\mathcal{A}^M(X)\mathcal{A}^N(X))\\=&\frac{1}{2\lambda}\int\mathrm{d}x^2\mathrm{Tr}\big[\left(g^{-1}\partial_\mu g+\mathrm{i}eg^{-1}A_\mu g\right)^2\\&+\mathrm{i}\alpha\mathcal{C}(x)\bar{\mathcal{C}}(x)\big],\end{aligned}\tag{11}$$

where $\lambda=\dfrac{s^4e^2}{\pi\alpha}$.

The superspace embedding factor $s\,(p)$ is a key parameter to bridge the original Yang–Mills field A_μ and the gauge modes $g\,(x)$ in the principal chiral model, but it is still unknown. Because the coupling constant λ for the principal chiral model is dependent on e^2, α and s, we can solve for s from the known $\beta\,(p/\Lambda)$ function of e^2, α and λ.[17]

For four-dimensional QCD, the $\beta\left(e^2\right)$ function of e^2 is known as

$$\begin{aligned}\beta\left(e^2\right)=&-b_0^ee^4,\\b_0^e=&\frac{11c_v}{48\pi^2}-\frac{N_f}{24\pi^2}=\frac{9}{16\pi^2},\end{aligned}\tag{12}$$

where $c_v=3$ is the quadratic Casimir operator in adjoin representation.

The $\beta\,(\lambda)$ function of λ is given by

$$\beta\,(\lambda)=\Lambda\frac{\partial\lambda}{\partial\Lambda}=-b_0^\lambda\lambda^2 \tag{13}$$

with $b_0^\lambda=\dfrac{c_v}{8\pi}$. Up to one loop at high-energy limit $p^2\to\infty$, the renormalization behaviours of the known parameters are

$$\begin{aligned}\left(p^2/\Lambda^2\right)=&\exp\left[-\frac{1}{b_0^\lambda\lambda\,(p\to\infty)}\right],\\\left(p^2/\Lambda^2\right)=&\exp\left[-\frac{1}{b_0^ee^2\,(p\to\infty)}\right],\end{aligned}\tag{14}$$

where Λ is an energy cut-off. Then we can leave for the present the coupling constant $\lambda=\dfrac{b_0^ee^2}{b_0^\lambda}$ which is independent of gauge parameter α. And the solution of s of one loop is

$$s=\left(\frac{\pi\alpha\lambda}{e^2}\right)^{1/4}\mid_{p\to\infty}=\left(\frac{\pi\alpha b_0^e}{b_0^\lambda}\right)^{1/4}. \tag{15}$$

To higher loops, $s\to s\,(p)$ is rather involved and dependent on $\dfrac{p}{\Lambda}$.

Because QCD is asymptotic free, it can be reduced to the corresponding $SU(3)_R \times SU(3)_L$ principal chiral model.[21] Hence the gauge fixing term in this case is

$$S_{gf} = \frac{1}{2\lambda}\int \mathrm{d}x^2 \mathrm{Tr}\big[\left(g^{-1}\partial_\mu g + eg^{-1}A_\mu g\right)^2 + \mathrm{i}\alpha\mathcal{C}(x)\bar{\mathcal{C}}(x)\big], \tag{16}$$

where

$$\lambda = \frac{b_0^e e^2}{b_0^\lambda} = \frac{3}{2\pi}e^2.$$

The two-dimensional $SU(3)_R \times SU(3)_L$ principal chiral model has only one phase—a disorder phase without long range order,

$$< g >|_{2D} = 0. \tag{17}$$

Also, Goldstone excitons are massive. Up to one loop, the mass gap of the principal chiral model in two dimensions has been obtained by large N approximation as

$$m^2 = \Lambda^2 \exp\left(-\frac{8\pi}{3\lambda}\right) = \Lambda^2 \exp\left(-\frac{1}{b_0^e e^2}\right), \tag{18}$$

where Λ is a cutoff.

It turns out that the spectrum of the $SU(3)_R \times SU(3)_L$ principal chiral model contains two kinds of massive particles.[22] They form multiplets belonging to the diagonal of the $SU(3) \times SU(3)$ algebra, namely, the vector representation and all antisymmetric tensors according to the Dynkin diagram. The spectrum of masses is

$$m_1 = m,\ m_2 = 3\sqrt{2}m, \tag{19}$$

where $1, 2$ is the rank of a fundamental representation.

The large N limit is highly nontrivial. At small fields, an inverse logarithmic singularity was found in the ground-state energy at the mass gap which indicates that as $N \to \infty$ the spectrum of the theory contains extended objects rather than point-like particles. For $N \to \infty$, the string type field from gauge modes $g(x)$ in QCD may be described by the AdS/CFT correspondence in terms of the corresponding limit of the string background.[23,24]

We have considered a Yang–Mills theory with fixed frame T^a. However, in the disordered phase, frame fluctuations are induced by quantum fluctuations of gauge modes and cannot be regarded as a fixed frame T^a. We introduce frame field $\boldsymbol{\phi}(x) = U^{-1}(x)T^a U(x)$ $(U(x) \in SU(3)_R \times SU(3)_L/SU(3))$ to indicate the colour-direction variable. $\boldsymbol{\phi}(x)$ is a composite field of the gauge modes $g(x)$, and is an elementary degree of freedom in the disordered phase.

Without a fixed frame, the gauge field $A_\mu(x) = A_\mu^a(x)T^a$ changes its path-integral form. Accordingly, the gauge field $\tilde{A}_\mu^\phi(x)$ without fixed frame turns into $A_\mu^\phi(x) + \frac{1}{e}\varpi_\mu$, where

$$A_\mu^\phi(x) = 2\mathrm{Tr}\{A_\mu(x)\boldsymbol{\phi}(x)\}\cdot\boldsymbol{\phi}(x) = \boldsymbol{A}_\mu^\phi \cdot \boldsymbol{\phi}(x), \tag{20}$$

A_μ^ϕ is the image of gauge field $A_\mu(x)$ projected into the original $SU(3)$ gauge manifold without a fixed frame. ϖ_μ is a new $U(3)$ gauge field in two dimensions denoted as

$$\varpi_\mu(x) = \mathrm{i}U^{-1}(x)\partial_\mu U(x). \tag{21}$$

The gauge transformation is $g(x) = \exp\left[\mathrm{i}\boldsymbol{\phi}\cdot\boldsymbol{\varphi}(x)\right]$ for given frame $\boldsymbol{\phi}$. And the $\boldsymbol{\phi}$-direction covariant derivative operator $\hat{D}_\mu$ of the gauge theory without a fixed frame is performed with

$$\hat{D}_\mu^\phi = \hat{\partial}_\mu + \mathrm{i}eA_\mu^\phi + \mathrm{i}\varpi_\mu. \tag{22}$$

The original $SU(3)$ gauge field strength turns into

$$\begin{aligned}\tilde{F}_{\mu\nu}^\phi(x) \equiv& \frac{1}{\mathrm{i}e}\left([\hat{D}_\mu^\phi, \hat{D}_\nu^\phi] - [\hat{\partial}_\mu, \hat{\partial}_\nu]\right)\\ =& \partial_\mu\tilde{A}_\nu^\phi(x) - \partial_\nu\tilde{A}_\mu^\phi(x)\\ &+ \mathrm{i}e[\tilde{A}_\mu^\phi(x), \tilde{A}_\nu^\phi(x)].\end{aligned} \tag{23}$$

From this, the gauge field strength is defined as

$$F_{\mu\nu}(A_\mu^\phi) = \partial_\mu A_\nu^\phi(x) - \partial_\nu A_\mu^\phi(x) + \mathrm{i}e[A_\mu^\phi(x), A_\nu^\phi(x)]. \tag{24}$$

The original $SU(3)$ Yang–Mills field obtains another $SU(3)$ local symmetry and turns into $SU(3)\times SU(3)$ gauge theory—one group element as $\exp(\mathrm{i}\delta\varphi^a\phi^a)$ with fixed frame ϕ^a, another group gauging the frame ϕ^a.

Because the frame may fluctuate, we replace the fixed frame T^a by a frame field $\boldsymbol{\phi}$ and $\exp\left[\mathrm{i}T^a\varphi^a(x)\right]$ by $\exp\left[\mathrm{i}\boldsymbol{\phi}\cdot\boldsymbol{\varphi}(x)\right]$. Gauge modes

$$g = g_0 \exp\left[\mathrm{i}\boldsymbol{\phi}(x)\cdot\boldsymbol{\varphi}(x)\right] \tag{25}$$

are divided into two parts:[19] one is the slowly varying vector g_0, the other the fast one $\boldsymbol{\varphi}(x)$. Using g_0 and $\boldsymbol{\varphi}(x)$, we can rewrite the action

$$S_{gf} = \frac{1}{2\lambda}\int \mathrm{d}x^2 \mathrm{Tr}\big[\left(g^{-1}\partial_\mu g + \mathrm{i}eg^{-1}A_\mu^\phi g\right)^2 + \mathrm{i}\alpha\mathcal{C}(x)\bar{\mathcal{C}}(x)\big] \tag{26}$$

as

$$S_{gf} = S\left(g_0, A_\mu^\phi\right) - \frac{1}{2\lambda}\int \mathrm{d}x^2 \mathrm{Tr}\{[\partial_\mu(\boldsymbol{\phi}\cdot\boldsymbol{\varphi})]^2 - \left(g_0^{-1}\partial_\mu g_0 + \mathrm{i}e g_0^{-1} A_\mu^\phi g_0\right) \cdot [\boldsymbol{\phi}\cdot\boldsymbol{\varphi}, \partial_\mu(\boldsymbol{\phi}\cdot\boldsymbol{\varphi})]\}. \quad (27)$$

In the disordered phase, Goldstone modes $\boldsymbol{\varphi}(x)$ have a mass gap m^2, and the relevant terms of Goldstone modes are:

$$\frac{1}{2\lambda}\int \mathrm{d}x^2 \left[\mid \boldsymbol{\phi}\cdot(\partial_\mu + 2\varpi_\mu)\boldsymbol{\varphi} \mid^2 + m^2\boldsymbol{\varphi}^2\right]. \quad (28)$$

Hence the Goldstone modes of the principal chiral model are bosons with charge two in the presence of an gauge field ϖ_μ.

We obtain the expression of the expansion of the renormalization two-point function $\varpi_\mu\varpi_\nu$ at one-loop order as:

$$4c_2\int \frac{\mathrm{d}^2k}{(2\pi)^2}\left[\frac{(k_\mu+2p_\mu)(k_\nu+2p_\nu)}{(k^2+m^2)((k+p)^2+m^2)} - 2\delta_{\mu\nu}\frac{1}{k^2+m^2}\right], \quad (29)$$

where $c_2 = \dfrac{N^2-1}{2N} = \dfrac{4}{3}$ is the quadratic Casimir operator in fundamental representation. At low energy limit $p^2 \to 0$, we simplify the above integral to the following one

$$\frac{1}{\bar{e}_\varpi^2}\left(p_\mu p_\nu - \delta_{\mu\nu}p^2\right), \quad (30)$$

where the induced coupling constant is

$$\bar{e}_\varpi^2 = \frac{9\pi m^2}{4}. \quad (31)$$

For the reason of $SU(3)$ local symmetry, the corresponding action of the kinetic term is

$$S_{\mathrm{ind}}(\varpi_\mu) = \int \mathrm{d}^2x \left[\frac{1}{2\bar{e}_\varpi^2}\mathrm{Tr}\left(\partial_\mu\varpi_\nu - \partial_\nu\varpi_\mu + [\varpi_\mu, \varpi_\nu]\right)^2\right]. \quad (32)$$

Correspondingly, in the QCD vacuum, the total gluonic vector potential A_μ split into two components:

$$\tilde{A}_\mu = A_\mu^\phi + \frac{1}{e}\varpi_\mu. \quad (33)$$

One is the perturbative part A_μ^ϕ, the other the nonperturbative ϖ_μ. ϖ_μ is not a superposition of classical solutions like instantons, monopoles, etc, but induced purely by quantum fluctuations. In this picture, ϖ_μ is just a Parisi–Sourlas field in two dimensions which maintains the stochastic picture of the vacuum. In the propagator of the vector field $\varpi_\mu(x)$, a massless pole appears. Hence the frame connection field $\varpi_\mu(x)$ becomes a dynamical gauge field in two dimensions, giving rise to a confining potential.

Feynman diagrams for quark–anti-quark scattering by exchanging one gluon and one ϖ_μ show the interaction between them (massive or massless). It is not difficult to obtain the potential between quark and anti-quark (massive or massless) in four dimensions as

$$V(r) = -\frac{c_2}{4\pi}\frac{e^2}{r} - \frac{c_2}{2}e_\omega^2 r, \quad (34)$$

where

$$\bar{e}_\varpi^2 = \frac{3\pi m^2}{c_2} = \frac{9\pi\Lambda^2}{4}\exp\left[-\frac{1}{b_0^e e^2}\right]. \quad (35)$$

This linear potential is equivalent to a string tension

$$\sigma = \frac{3\pi\Lambda^2}{2}\exp\left[-\frac{1}{b_0^e e^2}\right]. \quad (36)$$

This result shows that in the weak coupling region the string tension obeys the scaling law suggested from the result of the renormalization group based on the loop calculations.[25]

Remember the fact that quantum fluctuations of ϖ_μ field lies in two dimensional superspace while all other fields in four dimensions. A dimensional constraining condition must be noted that for all Feynman diagrams the internal lines in two-dimensional superspace cannot connect to the four-dimensional internal lines. This condition is very important, for it ensures that the frame connection field ϖ_μ does not affect perturbation behaviour of QCD.

Let us give a summary.

This paper is based on the Parisi–Sourlas confinement mechanism and the concept of frame fluctuation. If the frame $\phi^a = U^{-1}(x)T^aU(x)$ is not fixed in QCD vacuum, the gauge field $A_\mu(x)$ turns into $A_\mu^\phi(x) + \dfrac{1}{e}\varpi_\mu$. The nonperturbative part ϖ is a kind of Parisi–Sourlas field which is induced by quantum fluctuations. As a result of the original $SU(3)$ Yang–Mills field obtains another $SU(3)$ local symmetry and turns into $SU(3)\times SU(3)$ local symmetry—one group element as $\exp(\mathrm{i}\delta\varphi^a T^a)$ with fixed frame T^a, the other group gauging the frame T^a.

Using the Parisi–Sourlas dimensional reduction, four-dimensional QCD is reduced to the two-dimensional principal chiral model by suitable superspace embedding. Because Goldstone modes are massive for this model, the kinetic term of ϖ_μ appears and the frame T^a cannot be regarded as a fixed one. In the propagator of the vector field $\varpi_\mu(x)$, a massless pole appears in two dimensions, giving rise to a confining potential.

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