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# EARTHQUAKE SCALING PARADOX*

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Two measures of earthquakes, the seismic moment and the broadband radiated energy, show completely different scaling relations. For shallow earthquakes worldwide from January 1987 to December 1998, the frequency distribution of the seismic moment shows a clear kink between moderate and large earthquakes, as revealed by previous works. But the frequency distribution of the broadband radiated energy shows a single power law, a classical Gutenberg–Richter relation. This inconsistency raises a paradox in the self-organized criticality model of earthquakes.

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The scaling of the size of earthquakes, as represented by the frequency–size distribution, is one of the most important observational bases of the physics of earthquakes. Traditionally, this relation has been represented by the Gutenberg–Richter law,[1] that the number $N$ of earthquakes with magnitude $M$ depends on $M$ by

$$\log N = a - bM,$$

where $a$ and $b$ are constants, with $b$ near to 1.0. Taking the empirical relations between the magnitude $M$ and the seismic moment $M_0$ and radiated energy $E$,[2] one has

$$N \propto M_0^{-B_0}$$

and

$$N \propto E^{-B},$$

with $B_0$ and $B$ being scaling constants. When the concept of self-organized criticality (SOC)[3] was used to understand the phenomenology of seismicity, these relations were regarded as the manifestations of the SOC of earthquakes.[4,5] On the other hand, however, because of the limitation in observation, the empirical relations between the magnitude and the seismic moment and radiated energy have significant uncertainties, and cannot be used directly in the study of the frequency–moment or frequency–energy distributions.[6] In recent years, the development of broadband digital seismological observations has made it possible to measure the seismic moment and radiated energy directly. This development has led to some important questions in the study of the physics of earthquakes.

In principle, the broadband radiated energy and seismic moment have different physical significances. The broadband radiated energy, derived from the velocity power spectra of ground motion, is a measure of seismic potential for damage, while the seismic moment, derived from the low-frequency asymptote of displacement spectra, is more physically related to the final displacement of an earthquake.[7,8] Starting from recent years, the seismic moment has been widely used in the study of seismic hazards and the physics of earthquakes, but broadband radiated energy has been less used. One of the reasons for this situation was that the broadband radiated energy was not regarded as a better measure of an earthquake than the seismic moment. But this impression is wrong, because broadband radiated energy is not a better measure of an earthquake than seismic moment; rather it is another measure of an earthquake other than the seismic moment.

At present, routine measurements of centroid moment tensors (CMTs) for earthquakes worldwide over M5.5 are conducted by Harvard University and the National Earthquake Information Center (NEIC) of USGS using mantle waves and long-period body waves, respectively. The results are regularly published as research letters in *Phys. Earth Planet. Inter.* with electronic catalogues available. Broadband radiated energy measurements are conducted by the

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NEIC, with the results published firstly in *Bull. Seismol. Soc. Am.* and then in *Seismol. Res. Lett.*

We use the seismic moment measured by Harvard and the broadband radiated energy measured by NEIC to compare the frequency–energy and frequency–moment distributions for global seismicity. The catalogues contain the results for shallow earthquakes that occurred from January 1987 to December 1998. For the frequency–moment distribution it is found that there is a clear kink between moderate and large earthquakes,[9] different from the classical Gutenberg–Richter law.[1] Accordingly one of the aims of this study is to see what happens in the energy catalogue, and whether there are similar results in the frequency–energy distribution.

In the study of the frequency–size distribution, the analysis with under-sampled data was usually an obstacle preventing us from recognizing the detailed changes in the power law distribution. This difficulty can be overcome by applying rank-ordering statistics[10,11] in the analysis. Rank-ordering analysis orders by rank the energies or moments of earthquakes by placing the largest in the first rank, the next largest in the second rank and so on, and then plots the energy or moment versus their rank order. The statistics of cumulative distributions as used in the past are dominated by the most numerous small events, which limit the resolution of the analysis of large events, while rank-ordering statistics provide a perspective on the rare, largest elements of a population. By numerical experiments, Sornette *et al.*[12] showed that, for the case of only a few tens of samples with a single power law distribution, rank-ordering analysis can show the power law distribution clearly by a straight line on the double log rank-ordering plot. For the case of a transition of power laws between small and large events, rank-ordering analysis can distinguish the two regimes very clearly. However, the crossover between the two distributions is ill defined from the apparent crossover on the plot, and the estimate of the crossover, either by least-squares fit or by visual inspection, has at best a poor accuracy.

Figures 1 and 2 show the rank-ordered broadband radiated energy and scalar seismic moment versus their ranks. To avoid the incompleteness of the data set which usually happens to small events, we use only the first 100 earthquakes on the rank-ordering plot. From the figure it can be seen that, unlike the distribution of the seismic moment which shows a clear deviation from the Gutenberg–Richter law, i.e. a transition of moment distribution that falls off at a faster rate for large events than for smaller events, the distribution of broadband radiated energy for worldwide shallow earthquakes shows a single power law. Since we use the same set of earthquakes in the analysis, such a difference is convincing and cannot simply be explained by the lack of data samples. We used subsets of events with different focal mechanisms and subsets of events within special regions and obtained similar results.[13]

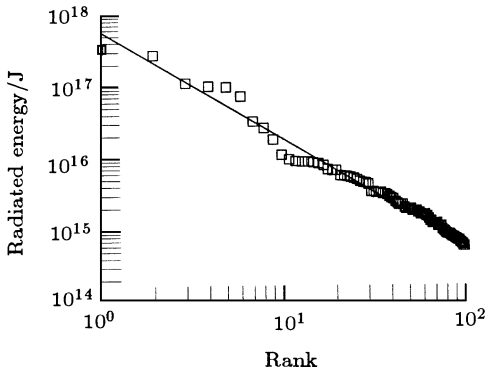



**Fig.1.** Log–log plot of the rank-ordered radiated energy versus its rank.

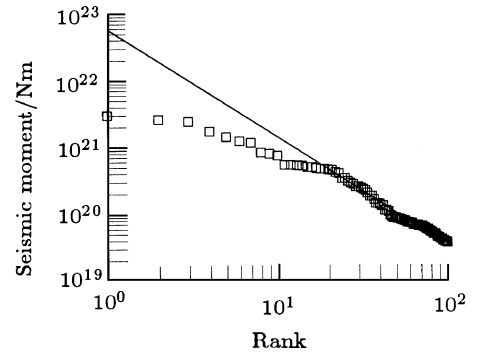



**Fig.2.** Log–log plot of the rank-ordered seismic moment versus its rank.

It is proposed that the frequency–size distribution is the 'footprint' of the state of evolution of a dynamic system, defining self-organized sub-critical, critical and super-critical states.[14] The comparison between the frequency–moment and the frequency–energy distributions raises an earthquake scaling paradox.

Seen in the perspective of seismic moment, global seismicity seems to be in a self-organized sub-critical state. On the other hand, however, seen in the perspective of broadband radiated energy, global seismicity seems to be in a self-organized critical state.

It seems that the models of the self-organization of global seismicity have to accommodate these two seemingly conflicting phenomena, which implies that some changes are needed in the present SOC models of earthquakes.

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