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MULTISTAGE ADAPTIVE HIGHER-ORDER NONLINEAR FINITE IMPULSE RESPONSE FILTERS FOR CHAOTIC TIME SERIES PREDICTIONS*

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A multistage adaptive higher-order nonlinear finite impulse response (MAHONFIR) filter is proposed to predict chaotic time series. Using this approach, we may readily derive the decoupled parallel algorithm for the adaptation of the coefficients of the MAHONFIR filter, to guarantee a more rapid convergence of the adaptive weights to their optimal values. Numerical simulation results show that the MAHONFIR filters proposed here illustrate a very good performance for making an adaptive prediction of chaotic time series.

Keywords: chaotic time series, adaptive higher-order nonlinear finite impulse response filters
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I. INTRODUCTION

Recently, nonlinear adaptive filters such as Volterra-type filters and higher-order nonlinear finite impulse response (FIR) filters have successfully been used in nonlinear and low-dimensional chaotic time series predictions.[1−8] Adaptive higher-order nonlinear finite impulse response (AHONFIR) filters based on the sparse higher-order Volterra expansions, which the filter's weights are updated by direct application of the minimum mean square error criterion, have high predictive ability, good fitting accuracy compared with predictive performance of the second-order Volterra filters and neural networks.[4−6] However, because a large number of higher-order nonlinear terms are often required, it is usually very difficult to guarantee more rapid convergence of the adaptive weights to their optimal values.

In this paper, a multistage adaptive higher-order nonlinear finite impulse response (MAHONFIR) filter is made to predict chaotic time series based on the sparse higher-order Volterra expansions and multichannel linear factorization implementation. In contrast to previously developed filters, the MAHONFIR filters have a computationally efficient and modular structure, which show a high predictive ability, good fitting accuracy and require less computer resources, compared with the previous direct-embedded linear structure of AHONFIR filters.

This paper is organized as follows. In section II we review the higher-order nonlinear FIR filters for modelling of chaotic series predictions. In section III, the structure of the MAHONFIR filters is formulated and is shown to be equivalent to that implemented as multi-channel linear moving average filters. We present some simulations and results in section IV. Conclusions are given in section V.

II. PRELIMINARIES

Suppose a scalar time series $\{x(t)(t = 1, 2, ..., N)\}$ is a measurement on the chaotic dynamical system in the state space. If the embedding dimension m and the time delay τ have been chosen for the phase reconstruction, a reconstructed vector can be written as follows:

$$\boldsymbol{x}(t) = (x(t), x(t+\tau), ..., x(t+(m-1)\tau)). \qquad (1)$$

By Taken's embedding theorem or its extensions, there exists a function $F(\cdot)$ such that

$$x(t+T) = F(\boldsymbol{x}(t)) \qquad t = 1, 2, ..., N, \qquad (2)$$

where the integer T is the look-ahead time. Let the reconstructed attractor be denoted by S, then $F : S \to S$. Thus it is necessary to fit F only in the domain S by using a prediction model. A model for time series is obtained by finding a functional form $\hat{F}$ for the training set pairs, $(\boldsymbol{x}(t), d(t))(t = 1, ..., N)$, where $d(t) = x(t+T)$ is the output of the model, and $\boldsymbol{x}(t)$ is the input of the model. $\hat{F}$ relates $\boldsymbol{x}(t)$ to $d(t)$:

$$x(t+T) = \hat{F}(\boldsymbol{x}(t)) + \varepsilon(t), \qquad (3)$$

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where $\varepsilon(t)$ represents the noise or fitting error.

A wide class of discrete time nonlinear systems can be represented by the Volterra filter's model.[11] Such systems with finite memory 'm' can be approximated by a truncated homogeneous and causal pth-order Volterra series as follows:

$$\begin{aligned} & x(n+T) = F(\boldsymbol{x}(n)) \\ =& h_0 + \sum_{i=0}^{m-1} h_1(i)x(n-i) \\ & + \sum_{i_1=0}^{m-1}\sum_{i_2=0}^{m-1} h_2(i_1,i_2)x(n-i_1)x(n-i_2) + \cdots \\ & + \sum_{i_1=0}^{m-1}\sum_{i_2=0}^{m-1}\cdots\sum_{i_p=0}^{m-1} h_p(i_1,i_2,\cdots,i_p) \\ & \cdot x(n-i_1)x(n-i_2)\cdots x(n-i_p) + \cdots, \end{aligned} \tag{4}$$

where $h_p(i_1,i_2,\cdots,i_p)$ are the pth-order Volterra kernels, $x(n+T)$ is the output and $x(n),\cdots,x(n-m+1)$ are the input. The order of the nonlinearity 'p' and the memory length of the filter 'm' are finite. Note that there are $O(m^p)$ coefficients in this polynomial expansion. One big disadvantage of the Volterra system model, Eq.(4), is that the complexity of implementing filters using this model can be very large, even for moderate values of m and p.[2,9] Consequently, most of the practical applications of the systems employing Volterra series expansion are the second-order Volterra filter. This second-order Volterra filter with adaptive algorithms has been successfully used to predict some low-dimensional chaotic time series.[1,3] There exist some limitations for predicting high-dimensional time series because the power of approximating higher-order nonlinear functions is very limited.[4−6] Therefore, a higher-order Volterra series expansion should be used to predict chaotic time series. But one of the major drawbacks of the higher-order Volterra filters is the large number of parameters associated with such a structure. However, its non-parsimonious representations motivate the study of other structures.[9,10]

Recently, in order to make an adaptive prediction of chaotic time series, an adaptive higher-order nonlinear FIR filtering prediction model has been proposed, which is constructed by a sparse pth-order Volterra series expansion based on fourth-order cumulants analysis results of chaotic time series.[4−6] This adaptive higher-order nonlinear FIR filtering prediction model can be described as follows:

$$\begin{aligned} x(n+T) = F(\boldsymbol{x}(n)) &= h_0 + \sum_{i=0}^{m-1} h_1(0,i)x(n-i) \\ &+ \sum_{i=0}^{m-1} h_2(0,i)x^2(n-i) + \sum_{i=1}^{m-1} h_2(1,i)x(n)x(n-i) \\ &+ \sum_{i=0}^{m-1} h_3(0,i)x^3(n-i) + \sum_{i=1}^{m-1} h_3(1,i)x^2(n)x(n-i) \\ &+ \sum_{i=1}^{m-1} h_3(2,i)x(n)x^2(n-i) + \cdots \\ &+ \sum_{i=0}^{m-1} h_p(0,i)x^p(n-i) \\ &+ \sum_{i=1}^{m-1} h_p(1,i)x^{p-1}(n)x(n-i) \\ &+ \sum_{i=1}^{m-1} h_p(2,i)x(n)x^{p-1}(n-i), \end{aligned} \tag{5}$$

where the total number of this pth-order nonlinear filter's coefficients is $M = 3m + P(3m-2), p > 2$, its complexity is smaller than that of the pth-order Volterra series expansions. We will refer to the sparse higher-order Volterra series expansion in Eq.(5) as the higher-order nonlinear finite impulse response (HONFIR) filters.[12]

Adaptive Volterra filters can be developed using linear techniques by embedding the nonlinear problem into a linear multivariate problem, in which each Volterra kernel coefficient becomes the coefficient of a separate variable. The corresponding HONFIR filter in Eq.(5) can be embedded as follows:

$$x(n+T) = H^{\mathrm{T}}(n)U(n), \tag{6}$$

where

$$\begin{aligned} U(n) =& [1, x(n), \cdots, x(n-m+1), x^2(n), \cdots, \\ & x^2(n-m+1), x(n)x(n-1), \cdots, \\ & x(n)x(n-m+1), x^3(n), \cdots, \\ & x(n)x^{p-1}(n-1), \cdots, x(n)x^{p-1}(n-m+1)]^{\mathrm{T}}, \end{aligned}$$

$$\begin{aligned} H(n) =& [h_0, h_1(1), \cdots, h_1(m-1), h_2(0,0), \cdots, \\ & h_2(0,m-1), h_2(1,1), \cdots, h_2(1,m-1), \\ & h_3(0,0), \cdots, h_p(0,0), \cdots, \\ & h_p(0,m-1), h_p(1,1), \cdots, \\ & h_p(2,1), \cdots, h_p(2,m-1)]^{\mathrm{T}}. \end{aligned}$$

This HONFIR filter can thus be realized by a linear, multivariable filter with the augmented coefficient vector '$H(n)$'. Any of the well-known adaptive linear

filtering algorithms, in principle, can be used to develop the adaptive HONFIR filters. Unfortunately, this approach is seldom efficient. In the following, we develop a multistage adaptive higher-order nonlinear FIR filter and show that it is more easy to guarantee more rapid convergence of the adaptive weights to their optimal values.

III. MULTISTAGE ADAPTIVE HIGHER-ORDER NONLINEAR FIR FILTERS

A. Structure

The structure of a multistage adaptive higher-order nonlinear FIR filter is depicted in Fig.1. Each stage of the filter consists of the HONFIR filter nonlinear homogeneous order components. Define the kth stage filter input vectors $x_k(n) = G_k(n), k = 1, \cdots, p$, which contains all the homogeneous kth-order components. i.e.,

$$x_k(n) = \begin{cases} \{x(n-i)\}_{i=0,\cdots,m-1} & k = 1, \\ \{x^2(n-i), \\ x(n)x(n-i-1)\}_{i=0,\cdots,m-1} & k = 2, \\ \{x^k(n-i), x^{k-1}(n)x(n-i-1), \\ x(n)x^{k-1}(n-i-1)\}_{i=0,\cdots,m} & k > 2. \end{cases} \tag{7}$$

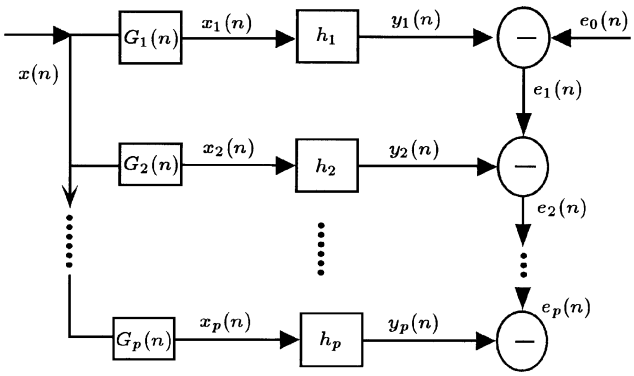


Fig.1. Structure of the multistage adaptive higher-order nonlinear FIR filter.

We further arrange the corresponding weights into a vector $h_k = \{h_k(0,i), h_k(1,i), h_k(2,i)\}_{i=0,\cdots,m-1}$. With the notations defined above, the output of the filter Eq.(5) may be written as

$$x(n+T) = \sum_{k=1}^{p} y_k(n) = \sum_{ik=1}^{p} h_k^{\mathrm{T}} x_k(n), \tag{8}$$

where

$$y_k(n) = h_k^{\mathrm{T}} x_k(n).$$

The resulting error of the kth stage filter output $y_k(n)$, compared with $e_{k-1}(n)$, is

$$e_k(n) = e_{k-1}(n) - h_k^{\mathrm{T}} x_k(n), \text{and } e_0(n) = x(n+T), \tag{9}$$

where $e_0(n)$ is the observation value at time $n+T$.

Accordingly, the multistage structure offers a modular, computationally efficient and unified treatment of HONFIR filters. Appropriate selection of the linear filtering building blocks allows us to rapidly switch between system types. In the multistage implementation, the flexibility and modularity of the multistage HONFIR filter make its implementation quite attractive.

B. Adaptive algorithms

Based on the multistage adaptive HONFIR filter's construction of Fig.1, as is the case with linear filters,[13] optimal weights h_k of each stage filter may be determined by minimizing the mean square error $E(h_k)$, i.e.,

$$E(h_k) = E\{|e_{k-1}(n) - h_k^{\mathrm{T}} x_k(n)|^2\}. \tag{10}$$

Thus, for the objective function, it can be easily shown that a normalized LMS (NLMS) adaptation algorithm[11,12] for each stage is given by

$$e_k(n) = e_{k-1}(n) - h_k(n) x_k(n), \tag{11}$$

$$h_k(n+1) = h_k(n) + \frac{\mu_k}{1 + \| x_k(n) \|^2} x_k(n) e_k(n). \tag{12}$$

The normalized factor contains higher-order power of the input signal and, therefore, often has very large statistical variations associated with $x_k(n)$. In order to mitigate the ill effects, such as large dynamics, on the update equation, the normalization factor $\| x_k(n) \|^2$ may be replaced by a smoothed version obtained by using a single-pole low pass filter[11] as follows:

$$\sigma_k^2(n) = \beta \sigma_k^2(n-1) + (1-\beta) \| x_k(n) \|^2, \tag{13}$$

where β is a constant between 0 and 1 and is usually very close to 0. So we slightly modify the recursion of Eq.(12) as follows:

$$h_k(n+1) = h_k(n) + \frac{\mu_k}{1 + \sigma_k^2(n)} x_k(n) e_k(n). \tag{14}$$

IV. SIMULATION AND RESULTS

To illustrate the effectiveness of the MAHONFIR filter used for making predictions, the Mackey–Glass

delay differential equation, Kawakami map and a four-dimensional nonlinear dynamical system are chosen as examples of hyper-chaotic time series. Each example uses corresponding 2100 values after omitting initial transient points. The generated time series are divided into two parts: the first 50 values are the training data set used to build the model, and the final 1000 points are the test data set to the prediction ability of the result. The normalized root-mean-square error (RMSE) is used to evaluate the prediction accuracy, where

$$\mathrm{RMSE} = < (x(n+T) - \hat{x}(n+T))^2 >^{1/2} / < (x(n+T)^2 >^{1/2} . \quad (15)$$

To clearly show the predictive results of the MAHONFIR filter used for making one step and multi-step future predictions of chaotic series, only one is plotted for each four points compared with actual values. Numerical experiments are given as follows.

Example 1 Mackey–Glass differential equation, with Δ=17, and initial conditions $x(t)$=0.9, $0 \le t \le 17, D_2 = 2.1$, for λ_1=0.0086, λ_2=0.001, λ_3=−0.0395, λ_4=−0.0504,

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = -0.1x(t) + \frac{0.2x(t-\Delta)}{1+x(t-\Delta)^{10}}.$$

The Mackey–Glass series prediction results are given in Fig.2.

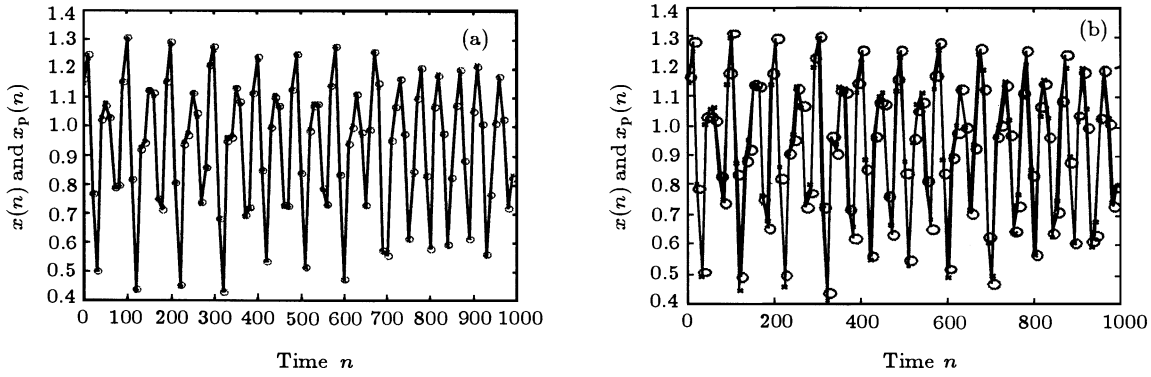


Fig.2. Prediction results of the Mackey–Glass series by the MAHONFIR filter: (a) T=1 and RMSE=3.7121 × 10^{-4}; (b) T=4 and RMSE=0.0013 (prediction values $x_p(t)$—○, true values $x(t)$—×, nonlinear order p=3, input size m=4).

Example 2 A four-dimensional nonlinear dynamical system with integral step 0.01. After all transients have been diminished, 2100 x-component values from these points are taken as our experimental data

$$\begin{aligned} \dot{x} &= ax - y, & a &= 0.56, & \lambda_1 &= 0.106, \\ \dot{y} &= x - yz^2, & b_1 &= 1.0, b_2 = 1.0, & \lambda_2 &= 0.023, \\ \dot{z} &= -b_1 y - b_2 z - b_3 w, & b_3 &= 6.0, & \lambda_3 &= -3.07 \times 10^{-4}, \\ \dot{w} &= z + cw, & c &= 0,80, & \lambda_4 &= -0.630, \end{aligned} \qquad D_2 = 3.205.$$

The prediction results of this four-dimensional dynamical system x-component series are given in Fig.3.

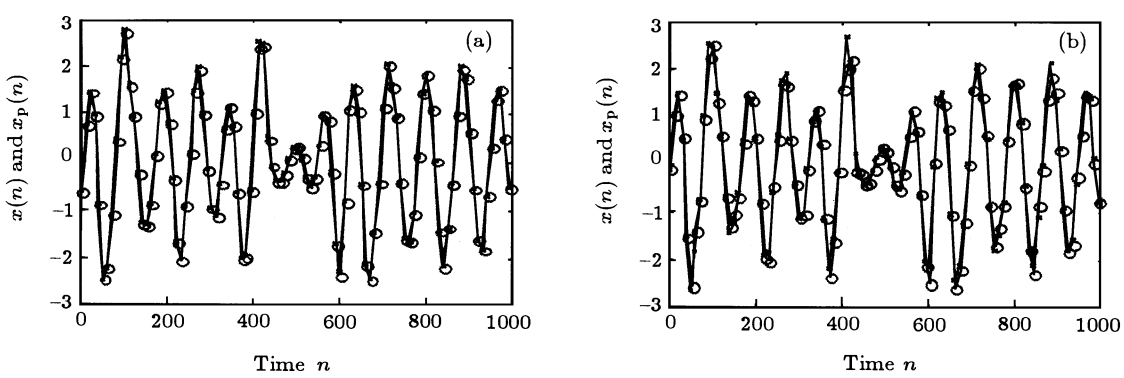


Fig.3. Prediction results of the four-dimensional nonlinear dynamical system x-component series by the MAHONFIR filter: (a) T=1 and RMSE=2.9365 × 10^{-4}; (b) T=4 and RMSE=0.0017 (prediction values $x_p(t)$—○, true values $x(t)$—×, nonlinear order p=3, input size m=4).

Example 3 Kawakami map

$$\begin{aligned} x_{n+1} &= -ax_n + y_n, \\ y_{n+1} &= x_n^2 - b, \end{aligned} \qquad a = 0.1, \quad b = 1.6, \quad \lambda_1 = 0.231, \quad \lambda_2 = 0.144.$$

The prediction results of the Kawakami map x-component series are depicted in Fig.4.

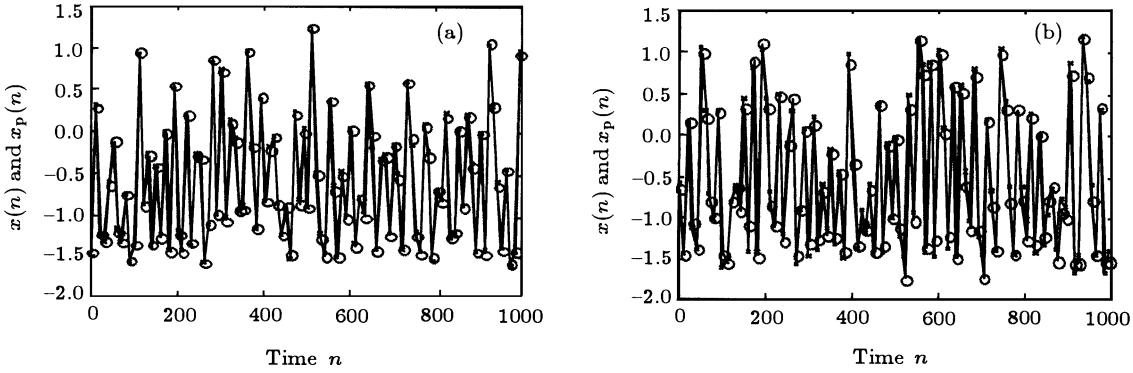


Fig.4. Prediction results of the Kawakami map x-component series by the MAHONFIR filter: (a) T=1 and RMSE=0.0013; (b) T=4 and RMSE=0.0130 (prediction values $x_p(t)$—∘, true values $x(t)$—×, nonlinear order p=3, input size m=4).

From these experimental results listed above, we can see clearly that the predicted values for the case of one step agree well with the true values; the predicted results for four-step are not much better than those for one step. Obviously, the MAHONFIR filter proposed here is capable of capturing the underlying hyper-chaotic dynamics of the system based on remarkably less data points. As we expected, the multi-step predictions by the MAHONFIR filter are very successful. This is due to the inherent adaptation and the stronger power of the approximating nonlinear functions of this MAHONFIR filter.

V. CONCLUSION

We have proposed a new multistage adaptive HONFIR filter to predict chaotic time series. This multistage adaptive HONFIR model provides high predictive ability, good fitting accuracy and requires less computer resources, compared with the previous direct embedded linear structure of adaptive HONFIR filters. In short, the multi-channel structure has three main advantages: (a) modularity; (b) computational efficiency; and (c) local adaptation. Since it consists of adaptive linear building blocks, much of our knowledge of implementing linear filters is useful. It is interesting to note that adaptive filtering methods can provide quite useful and efficient alternatives in the study of chaotic dynamics.

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