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PHOTON ACCELERATION DRIVEN BY AN INTENSE LASER PULSE*

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Interaction of a laser field with a plasma wave is studied by metric optics. Analysis shows that the frequency upshifting of the laser pulse results from the plasma density gradient. A laser beam can be thought of as a packet of photons moving in a plasma and thus the laser frequency upshifting is equivalent to photon acceleration. Examination of the three-dimensional motion equations shows that a laser beam diffraction occurs in the presence of a radial variation of the plasma density. It is argued that the focusing mechanism originating from the plasma wave can curb laser diffraction so that photons may be trapped in the plasma wave and accelerated continuously.

Keywords: photon acceleration, frequency upshifting, plasma wave, optical metric
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I. INTRODUCTION

The interaction of an intense laser field with plasma has received much theoretical and experimental attention during the last decade. The plasma wave excited by an intense laser pulse can generate such an extremely high accelerating field of 100GV/m[1−4] that it may become the next generation of particle accelerator. It is further found that such a high gradient plasma wave can upshift the frequency of the laser pulse trailing the plasma wave.[5−7] The trailing laser pulse absorbs the energy from the plasma wave which is saved by a driving laser pulse in the plasma and then the energy is transformed into laser frequency upshifting, i.e., photons being accelerated.

In this paper, the interaction of the laser pulse with plasma is studied using metric optics.[8−10] The basic idea is illustrated as follows. The plasma wave produced owing to the ponderomotive force exerted on the plasma by the driving laser pulse is treated as a Riemannian geometric background. This means that the plasma electron density perturbation and corresponding distribution of the refractive index are described as some geometric quantities such as optical metric, connection coefficients and curvature tensors etc. Then, the trailing laser pulse propagating through the plasma is described as a photon packet travelling in the Riemannian geometry. Finally, the metric optics allows us to use a unified geometric formulation to examine the focusing and defocusing characteristics of the laser beam propagating through the plasma.

II. GENERATION OF THE PLASMA WAVE

Starting from the relativistic cold two-fluid plasma model, when a driving laser beam is travelling through a plasma, plasma ions are assumed to be stationary and plasma electrons obey the following equations:

$$\frac{\mathrm{d}P^\alpha}{\mathrm{d}\tau} = -\frac{e}{c}F^{\alpha\beta}U_\beta, \tag{1}$$

$$\partial_t n + \nabla\cdot(n\boldsymbol{u}) = 0, \tag{2}$$

$$\nabla\cdot\boldsymbol{E} = -4\pi e(n-n_0), \tag{3}$$

where P^α and U^β are the four-momentum and the four-velocity of the plasma electron, respectively, $F^{\alpha\beta}$ is the electromagnetic tensor, and n_0 and n are the plasma electron densities before and after the driving laser beam is injected into the plasma, respectively. Eq.(1) can be expanded as

$$\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = -\frac{e}{m\gamma}\left(\boldsymbol{E}+\frac{1}{c}\boldsymbol{u}\times\boldsymbol{B}-\frac{1}{c^2}\boldsymbol{u}\boldsymbol{u}\cdot\boldsymbol{E}\right), \tag{4}$$

where $\boldsymbol{u}$ is the electron velocity and $\gamma = 1\Big/\left(1-\dfrac{u^2}{c^2}\right)^{1/2}$ is the relativistic factor. Introducing the normalized vector potential of the laser field $\boldsymbol{a}_L = \dfrac{e}{mc^2}\boldsymbol{A}_L$, under the weak relativistic condition $|\boldsymbol{a}_L|^2 \ll 1$, Eqs.(1)–(4) can be expanded to one order in a_L

$$\partial_t \boldsymbol{u}^{(1)}/c = \partial_t \boldsymbol{a}_L, \tag{5}$$

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$$\partial_t n^{(1)} + n_0 \nabla \cdot \boldsymbol{u}^{(1)} = 0, \tag{6}$$

$$\nabla^2 \phi^{(1)} = 4\pi e n^{(1)}. \tag{7}$$

where ϕ is the scalar potential of the electromagnetic field. The solutions of the first-order equations are $\boldsymbol{u}^{(1)} = c\boldsymbol{a}_L, n^{(1)} = \phi^{(1)} = 0$. To second order in a_L, Eqs.(1)–(3) give with the gauge condition $\nabla \cdot \boldsymbol{A} = 0$

$$\partial_t \boldsymbol{u}^{(2)} = -\frac{c^2}{2}\nabla a_L^2 + \frac{e}{mc^2}\nabla\phi^{(2)}, \tag{8}$$

$$\partial_t n^{(2)} + n_0 \nabla \cdot \boldsymbol{u}^{(2)} = 0, \tag{9}$$

$$\nabla^2 \phi(2) = 4\pi e n^{(2)}. \tag{10}$$

Differentiating Eq.(9) with respect to time and substituting Eqs.(8) and (10) into Eq.(9), we have

$$(\partial_t^2 + \omega_{p0}^2)\frac{n^{(2)}}{n_0} = \frac{c^2}{2}\nabla^2 a_L^2, \tag{11}$$

where $\omega_{p0}^2 = \dfrac{4\pi e^2 n_0}{m}$ is the plasma frequency before the laser beam is injected into the plasma and $-\dfrac{mc^2}{2}\nabla a_L^2$ is the ponderomotive force exerted on the plasma by the incident laser field. The solution of Eq.(11) is derived as

$$\frac{n^{(2)}(r,z,t)}{n_0} = \frac{c^2}{2\omega_{p0}}\int_{t_0}^{t} \mathrm{d}t' \sin\omega_p(t-t')\nabla^2 a_L^2. \tag{12}$$

It is convenient to perform a transformation of the speed of light frame $\xi = z - ct$, $\tau = t$ and then the solution (12) reads

$$\frac{n^{(2)}(r,\xi)}{n_0} = \frac{1}{2k_{p0}}\int_{\xi_0}^{\xi} \mathrm{d}\xi' \sin k_p(\xi-\xi')\nabla^2 a_L^2, \tag{13}$$

where $k_{p0} = \omega_{p0}/c$. To be specific, we consider a driving laser pulse in the following profile:

$$a_L(r,\xi) = \begin{cases} a_{L0}\exp(-r^2/r_L^2)\sin(\pi\xi/l_L) & 0 \le \xi \le l_L \\ 0 & \xi < 0, \end{cases} \tag{14}$$

where l_L and r_L are the pulse length and the effective radius of the pulse section, respectively. Inserting Eq.(14) into solution (13), the density perturbation is derived

$$\frac{n^{(2)}(r,\xi)}{n_0} = \begin{cases} \dfrac{a_{L0}^2\pi^2}{4\pi^2 - k_{p0}^2 l_L^2}\left\{\cos k_{p0}(\xi - l_L) - \cos\left(\dfrac{2\pi}{l_L}\xi\right) + \dfrac{8}{k_{p0}^2 r_L^2}\left(1 - \dfrac{2r^2}{r_L^2}\right)\left[\cos k_{p0}(\xi - l_L)\right.\right. \\ \left.\left. -1 - \dfrac{k_{p0}^2 l_L^2}{4\pi^2}\left(\cos\left(\dfrac{2\pi}{l_L}\xi\right) - 1\right)\right]\right\}\exp\left(-\dfrac{2r^2}{r_L^2}\right), & 0 \le \xi \le l_L, \\ \dfrac{a_{L0}^2\pi^2}{4\pi^2 - k_{p0}^2 l_L^2}\left[1 + \dfrac{8}{k_{p0}^2 r_L^2}\left(1 - \dfrac{2r^2}{r_L^2}\right)\right]\left[\cos k_{p0}(\xi - l_L) - \cos\left(\dfrac{2\pi}{l_L}\xi\right)\right]\exp\left(-\dfrac{2r^2}{r_L^2}\right), & \xi < 0. \end{cases} \tag{15}$$

When $l_L = \lambda_{p0} = \dfrac{2\pi}{k_{p0}}$, Eq.(15) is reduced to

$$\frac{n^{(2)}(r,\xi)}{n_0} = -\frac{\pi}{4}a_{L0}^2\left[1 + \frac{8}{k_{p0}^2 r_L^2}\left(1 - \frac{2r^2}{r_L^2}\right)\right] \cdot \exp\left(-\frac{2r^2}{r_L^2}\right)\sin(k_{p0}\xi). \tag{15'}$$

Since $k_{p0}^2 r_L^2 \gg 1$ under the general laboratory condition (e.g. taking $n_0 \sim 10^{17}\mathrm{cm}^{-3}$ and $r_L \sim 1\mathrm{mm}$, $k_{p0}^2 r_L^2 \sim -3.53 \times 10^3$), the maximum density perturbation is expressed as

$$\max\left\{\frac{n^{(2)}}{n_0}\right\} = \frac{\pi}{4}a_{L0}^2. \tag{16}$$

One can estimate the value $\max\{n^{(2)}/n_0\} = 57.3\%$, by using the identity $a_{L0} = \dfrac{e}{mc^2}A_{L0} = 8.54 \times 10^{-10}\lambda\,[\mu\mathrm{m}]\, I_0^{\frac{1}{2}}[\mathrm{W/cm}^2]$, where the laser intensity $I_0 = 10^{18}\mathrm{W/cm}^2$ and $\lambda = 1\mu\mathrm{m}$.

III. MOTION EQUATIONS OF PHOTONS

Let us examine a laser beam trailing the plasma wave, which can be described as a packet of photons travelling in the plasma. In general relativity, the interaction of photons with a gravitational field is interpreted as photon motion along the null geodesic in a manifold which is characterized by the ambient mass-energy. In metric optics, by analogy with general relativity, the interaction of laser pulses with a plasma can be illustrated as photon motion along the null geodesic in a spacetime that is specified by the electromagnetic response of the ambient medium.

A. Null geodesic equations

The photon motion obeys the null geodesic equations as follows[11]:

$$\frac{\mathrm{d}k^\mu}{\mathrm{d}\sigma} + \Gamma^\mu_{\nu\lambda}k^\nu k^\lambda = 0, \tag{17}$$

where σ is a parameter associated with the proper time, the wave four-vector is

$$k^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\sigma} = \left(\frac{\omega}{c}, \boldsymbol{k}\right),$$

the connection coefficients are

$$\Gamma^\mu_{\nu\lambda} \equiv \frac{1}{2} g^{\mu\rho}(\partial_\nu g_{\rho\lambda} + \partial_\lambda g_{\nu\rho} - \partial_\rho g_{\nu\lambda}), \qquad (18)$$

and the optical metric is[12]

$$g_{\mu\nu} = \eta_{\mu\nu} + \left(1 - \frac{1}{\epsilon\mu}\right) u_\mu u_\nu, \qquad (19)$$

where $\eta_{\mu\nu}$ is Minkowski metric (the metric for flat spacetime), u^μ is four-velocity of the medium with respect to the laboratory, ϵ and μ are the dielectric constant and the magnetic permeability, respectively. In the case of a static non-magnetic isotropic medium, the optical metric reads

$$g_{\mu\nu} = \begin{bmatrix} -\frac{1}{\epsilon} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \qquad (19)'$$

Since $\frac{\omega_p^2}{\omega^2} \ll 1$ in the case of a general experiment, one can write $1/\epsilon = 1/\left(1 - \frac{\omega_p^2}{\omega^2}\right) \simeq 1 + \frac{\omega_p^2}{\omega^2} = 1 + \frac{\omega_{p0}^2}{\omega^2}\left(1 + \frac{\delta n}{n_0}\right) = 1 + h(1+\Delta)$, where $h = \frac{\omega_{p0}^2}{\omega^2}$, $\Delta = \frac{\delta n}{n_0}$. Under transforming coordinates into the cylindrical light speed frame: $x^\mu = (t, x, y, z) \to \bar{x}^\alpha = (\tau, r, \theta, \xi)$, the optical metric is transformed as

$$\bar{g}_{\alpha\beta}(r,\theta,\xi,\tau) = \frac{\partial x^\mu}{\partial \bar{x}^\alpha}\frac{\partial x^\nu}{\partial \bar{x}^\beta} g_{\mu\nu} = \begin{bmatrix} -h(1+\Delta) & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}. \qquad (20)$$

The corresponding contravariant optical metric gives

$$\bar{g}^{\alpha\beta}(r,\theta,\xi,\tau) = \begin{bmatrix} -\frac{1}{1+h(1+\Delta)} & 0 & 0 & \frac{1}{1+h(1+\Delta)} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & r^{-2} & 0 \\ \frac{1}{1+h(1+\Delta)} & 0 & 0 & \frac{h(1+\Delta)}{1+h(1+\Delta)} \end{bmatrix}. \qquad (21)$$

Using Eqs.(20) and (21), all the nonzero connection coefficients are given as

$$\Gamma^0_{00} = -\frac{1}{2}\bar{g}^{03}\bar{g}_{00,3} = \frac{1}{2}\frac{h}{1+h(1+\Delta)}\partial_\xi \Delta,$$

$$\Gamma^0_{01} = \Gamma^0_{10} = \frac{1}{2}\bar{g}^{00}\bar{g}_{00,1} = \frac{1}{2}\frac{h}{1+h(1+\Delta)}\partial_r \Delta,$$

$$\Gamma^0_{03} = \Gamma^0_{30} = \frac{1}{2}\bar{g}^{00}\bar{g}_{00,3} = \frac{1}{2}\frac{h}{1+h(1+\Delta)}\partial_\xi \Delta,$$

$$\Gamma^1_{00} = -\frac{1}{2}\bar{g}^{11}\bar{g}_{00,1} = \frac{1}{2}h\partial_r \Delta,$$

$$\Gamma^1_{22} = \frac{1}{2}\bar{g}^{11}\bar{g}_{22,1} = -r, \qquad (22)$$

$$\Gamma^2_{21} = \Gamma^2_{12} = \frac{1}{2}\bar{g}^{22}\bar{g}_{22,1} = \frac{1}{r},$$

$$\Gamma^3_{00} = -\frac{1}{2}\bar{g}^{33}\bar{g}_{00,3} = \frac{1}{2}\frac{h(1+\Delta)}{1+h(1+\Delta)}h\partial_\xi \Delta,$$

$$\Gamma^3_{03} = \Gamma^3_{30} = \frac{1}{2}\bar{g}^{30}\bar{g}_{00,3} = -\frac{1}{2}\frac{h}{1+h(1+\Delta)}\partial_\xi \Delta,$$

$$\Gamma^3_{01} = \Gamma^3_{10} = \frac{1}{2}\bar{g}^{30}\bar{g}_{00,1} = -\frac{1}{2}\frac{h}{1+h(1+\Delta)}\partial_r \Delta,$$

where the notation "," denotes the partial derivative with respect to a coordinate component. The geodesic equations in the light speed frame are then rewritten as

$$\frac{\mathrm{d}\bar{k}^\alpha}{\mathrm{d}\sigma} + \Gamma^\alpha_{\beta\gamma}\bar{k}^\beta\bar{k}^\gamma = 0, \qquad (23)$$

where $\bar{k}^\alpha = (\partial \bar{x}^\alpha/\partial x^\mu)k^\mu$.

B. Solutions for the one-dimensional motion equations

In the one-dimensional (1D) case $\Delta = \Delta(\xi)$, the motion equation (23) can be expanded as

$$\frac{\mathrm{d}\bar{k}^0}{\mathrm{d}\sigma} + \Gamma^0_{00}\bar{k}^0\bar{k}^0 + 2\Gamma^0_{30}\bar{k}^3\bar{k}^0 = 0, \qquad (23a)$$

$$\frac{\mathrm{d}\bar{k}^1}{\mathrm{d}\sigma} + \Gamma^1_{22}\bar{k}^2\bar{k}^2 = 0\,, \tag{23b}$$

$$\frac{\mathrm{d}\bar{k}^2}{\mathrm{d}\sigma} + 2\Gamma^2_{21}\bar{k}^2\bar{k}^1 = 0\,, \tag{23c}$$

$$\frac{\mathrm{d}\bar{k}^3}{\mathrm{d}\sigma} + \Gamma^3_{00}\bar{k}^0\bar{k}^0 + 2\Gamma^3_{30}\bar{k}^3\bar{k}^0 = 0\,. \tag{23d}$$

Under the initial conditions $\bar{k}^1|_{z,t=0} = \bar{k}^2|_{z,t=0} = 0$, Eqs.(23b) and (23c) give the solutions $\bar{k}^1 = \bar{k}^2 = 0$. Using $\dfrac{\mathrm{d}}{\mathrm{d}\sigma} = (\mathrm{d}\bar{x}^0/\mathrm{d}\sigma)\dfrac{\mathrm{d}}{\mathrm{d}\bar{x}^0}$ and $\bar{k}^0 = k^0$, Eq.(23a) gives

$$\frac{\mathrm{d}k^0}{\mathrm{d}\bar{x}^0} + \left(\Gamma^0_{00} + 2\Gamma^0_{30}\frac{\mathrm{d}\bar{x}^3}{\mathrm{d}\bar{x}^0}\right)k^0 = 0\,. \tag{23a}'$$

Noting that the tracks of photons are just the null geodesics, i.e.,

$$\bar{g}_{\alpha\beta}\mathrm{d}\bar{x}^\alpha\mathrm{d}\bar{x}^\beta = 0 \tag{24}$$

one can obtain $(\mathrm{d}\bar{x}^3/\mathrm{d}\bar{x}^0) = -1 + \sqrt{1+h(1+\Delta)}$. And then Eq.(23a)′ is solved by

$$k^0 = \frac{\omega_0}{c}\exp(-\Phi\tau), \tag{25}$$

where $\Phi = \left[\left(\sqrt{1+h(1+\Delta)} - \dfrac{1}{2}\right)/(1+h(1+\Delta))\right] \cdot hc\partial_\xi\Delta$. To one order in h the solution (25) is rewritten as

$$k^0 = \frac{\omega_0}{c}\exp\left(-\frac{1}{2}hc\tau\partial_\xi\Delta\right). \tag{25}'$$

One can therefore obtain the relative frequency upshifting

$$\frac{\delta\omega}{\omega_0} = \frac{\omega-\omega_0}{\omega_0} \simeq -\frac{1}{2}hc\tau\partial_\xi\Delta. \tag{26}$$

From $\bar{k}^3 = (\mathrm{d}\bar{x}^3/\mathrm{d}\bar{x}^0)k^0$ and $\bar{k}^3 = k^3 - k^0$, one has

$$k_z = \frac{\omega_0}{c}\sqrt{1+h(1+\Delta)}\exp(-\Phi\tau). \tag{27}$$

To one order in h the solution (27) is reduced to

$$k_z = k_0\left(1 - \frac{1}{2}hc\tau\partial_\xi\Delta\right), \tag{27}'$$

where $k_0 = \omega_0/c\sqrt{1+h(1+\Delta)}$. Noting the fact that $1/\sqrt{1+h(1+\Delta)} = \epsilon^{1/2}$ is just the refractive index of the medium, k_0 can be interpreted as the wavenumber in the absence of the plasma wave, where ω_0/c is the wavenumber in free space. One then has the relative variation of the wavenumber

$$\frac{\delta k_z}{k_0} = \frac{k_z - k_0}{k_0} = \frac{1}{2}hc\tau\partial_\xi\Delta. \tag{28}$$

To one order in h the wavenumber shifting of the laser pulse is of the same form as the frequency shifting of the laser pulse. Substituting Eq.(15)′ into Eq.(26), one can estimate a maximum value of the frequency upshifting

$$\max\left\{\frac{\delta\omega}{\omega_0}\right\} = \frac{\pi}{8}a^2_{L0}h\tau\omega_{p0}\,. \tag{29}$$

Setting $\lambda = 1\mu$m, $a_{L0} = 0.5$, $n_0 = 10^{19}\mathrm{cm}^{-3}$, and $\tau = 10^{-12}$s, one has

$$\max\left\{\frac{\delta\omega}{\omega_0}\right\} = 15.7\%.$$

C. Solutions for the three-dimensional motion equations

In the three-dimensional (3D) case $\Delta = \Delta(r,\xi)$, the motion equations read

$$\frac{\mathrm{d}\bar{k}^0}{\mathrm{d}\sigma} + \Gamma^0_{00}\bar{k}^0\bar{k}^0 + 2\Gamma^0_{10}\bar{k}^1\bar{k}^0 + 2\Gamma^0_{30}\bar{k}^3\bar{k}^0 = 0, \tag{30a}$$

$$\frac{\mathrm{d}\bar{k}^1}{\mathrm{d}\sigma} + \Gamma^1_{00}\bar{k}^0\bar{k}^0 + \Gamma^1_{22}\bar{k}^2\bar{k}^2 = 0, \tag{30b}$$

$$\frac{\mathrm{d}\bar{k}^2}{\mathrm{d}\sigma} + 2\Gamma^2_{21}\bar{k}^2\bar{k}^1 = 0, \tag{30c}$$

$$\frac{\mathrm{d}\bar{k}^3}{\mathrm{d}\sigma} + \Gamma^3_{00}\bar{k}^0\bar{k}^0 + 2\Gamma^3_{10}\bar{k}^1\bar{k}^0 + 2\Gamma^3_{30}\bar{k}^3\bar{k}^0 = 0. \tag{30d}$$

Under the initial condition $\bar{k}^2|_{z,t=0} = 0$, Eq.(30c) is solved by $\bar{k}^2 = 0$. Equations (30a) and (30b) can read

$$\frac{\mathrm{d}\bar{k}^0}{\mathrm{d}\bar{x}^0} + \left(\Gamma^0_{00} + 2\Gamma^0_{10}\frac{\mathrm{d}\bar{x}^1}{\mathrm{d}\bar{x}^0} + 2\Gamma^0_{30}\frac{\mathrm{d}\bar{x}^3}{\mathrm{d}\bar{x}^0}\right)\bar{k}^0 = 0, \tag{30a}'$$

$$\frac{\mathrm{d}\bar{k}^1}{\mathrm{d}\bar{x}^0} + \Gamma^1_{00}\bar{k}^0 = 0. \tag{30b}'$$

In the 3D case, Eq.(24) becomes

$$\left(\frac{\mathrm{d}\bar{x}^3}{\mathrm{d}\bar{x}^0}\right)^2 + 2\frac{\mathrm{d}\bar{x}^3}{\mathrm{d}\bar{x}^0} + \left(\frac{\mathrm{d}\bar{x}^1}{\mathrm{d}\bar{x}^0}\right)^2 + \bar{g}_{00} = 0. \tag{24}'$$

By solving this, one obtains

$$\frac{\mathrm{d}\bar{x}^3}{\mathrm{d}\bar{x}^0} = -1 + \sqrt{1+h(1+\Delta) - \left(\frac{\mathrm{d}\bar{x}^1}{\mathrm{d}\bar{x}^0}\right)^2}. \tag{31}$$

Considering that the phase velocity of the laser in a plasma is faster than that in a vacuum, i.e.,

$$\frac{\mathrm{d}\bar{x}^3}{\mathrm{d}\bar{x}^0} = \frac{\mathrm{d}}{c\mathrm{d}t}(z-ct) = \frac{1}{c}(v_p - c) > 1,$$

where v_p is the phase velocity in a plasma, it is shown that $\left|\dfrac{\mathrm{d}\bar{x}^3}{\mathrm{d}\bar{x}^0}\right| \ll 1$ and $\left|\dfrac{\mathrm{d}\bar{x}^1}{\mathrm{d}\bar{x}^0}\right| \ll 1$. Neglecting high-order terms in h the solution of Eq.(30a)′ is written as

$$k^0 = \frac{\omega_0}{c}\exp(-\Phi\tau), \tag{32}$$

where $\Phi = -hc\partial_\xi \Delta/2[1+h(1+\Delta)]$. To one order in h the relative frequency upshifting presents

$$\frac{\delta\omega}{\omega_0} = -\frac{1}{2}hc\tau\partial_\xi\Delta\,. \tag{33}$$

This is the same result as that in the 1D case. Using solution (32), Eq.(30b)′ is solved as

$$\bar{k}^1 = k_r = \frac{\Gamma^1_{00}}{\Gamma^0_{00}}\frac{\omega_0}{c}[\exp(-c\Gamma^0_{00}\tau)-1]. \tag{34}$$

To one order in h, solution (34) is rewritten as

$$k_r = -\frac{1}{2}hc\tau(\partial_r\Delta)\frac{\omega_0}{c}. \tag{34}'$$

Using $\bar{k}^3 = \bar{k}^0\dfrac{\mathrm{d}\bar{x}^3}{\mathrm{d}\bar{x}^0} = k^3 - k^0$ and Eq.(31), one obtains

$$k^3 = k_z = \sqrt{(\bar{k}^0)^2[1+h(1+\Delta)] - (\bar{k}^1)^2}. \tag{35}$$

To one order in h, solution (35) is reduced as

$$k_z = k_0\left(1 - \frac{1}{2}hc\tau\partial_\xi\Delta\right), \tag{35}'$$

which is the same form as the solution (27)′ in the 1D case. In the 3D case, the transverse component k_r of the wavevector is produced owing to the presence of radial gradient of the electron density that results from the quiver motion of the plasma electrons. The transverse component is responsible for the laser beam diffraction, which severely limits the accelerating distance of the trailing laser in the plasma. It is therefore necessary to examine the focusing and defocusing characteristics of the trailing laser pulse in the plasma.

IV. FOCUSING ANALYSIS FOR ACCELERATING PHOTONS

Using the focusing theorem in general relativity, the focusing equation for a photon beam presents[13,14]

$$\frac{\mathrm{d}^2\mathcal{A}^{1/2}}{\mathrm{d}\sigma^2} + \left(|\Sigma|^2 + \frac{1}{2}R_{\alpha\beta}k^\alpha k^\beta\right)\mathcal{A}^{1/2} = 0, \tag{36}$$

where $\mathcal{A}$ is the cross-sectional area of the laser which is filled with photons,

$$|\Sigma|^2 \equiv \frac{1}{2}k_{\alpha;\beta}k^{\alpha;\beta} - \frac{1}{4}(k^\alpha_{;\alpha})^2, \tag{37}$$

where the notation ";" is the covariant derivative, and the Ricci curvature tensor

$$R_{\alpha\beta} = \partial_\rho\Gamma^\rho_{\alpha\beta} - \partial_\beta\Gamma^\rho_{\alpha\rho} + \Gamma^\rho_{\sigma\rho}\Gamma^\sigma_{\alpha\beta} - \Gamma^\rho_{\sigma\beta}\Gamma^\sigma_{\alpha\rho}. \tag{38}$$

Inserting Eq.(22) into Eqs.(37) and (38), to one order in h, after a tedious calculation one can obtain

$$|\Sigma|^2 + \frac{1}{2}R_{\alpha\beta}\bar{k}^\alpha\bar{k}^\beta \simeq \frac{h}{4}\left[\frac{1}{r}\partial_r(r\partial_r\Delta)\right](\bar{k}^0)^2. \tag{39}$$

Substituting the plasma density perturbation Eq.(15)′ into the above equation one has

$$\begin{aligned}&|\Sigma|^2 + \frac{1}{2}R_{\alpha\beta}\bar{k}^\alpha\bar{k}^\beta\\ =&\frac{2\pi a^2_{L0}}{r^2_L}\left\{\left(1-\frac{2r^2}{r^2_L}\right) + \frac{16}{k^2_p r^2_L}\left[1-\frac{2r^2}{r^2_L}\left(2-\frac{r^2}{r^2_L}\right)\right]\right\}\\ &\cdot\exp\left(-\frac{2r^2}{r^2_L}\right)\sin(k_p\xi)(\bar{k}^0)^2.\end{aligned} \tag{40}$$

Since $k^2_p r^2_L \gg 1$, Eq.(40) is reduced as

$$\begin{aligned}&|\Sigma|^2 + \frac{1}{2}R_{\alpha\beta}\bar{k}^\alpha\bar{k}^\beta\\ \simeq&\frac{2\pi a^2_{L0}}{r^2_L}\left(1-\frac{2r^2}{r^2_L}\right)\exp\left(-\frac{2r^2}{r^2_L}\right)\sin(k_p\xi)(\bar{k}^0)^2.\end{aligned} \tag{40}'$$

From Eqs.(36)and (40)′, obviously, focusing requires the trailing laser beam to be located in the region $r < r_L/\sqrt{2}$ and the phase of the plasma wave $\sin(k_p\xi) > 0$, or in the region $r > r_L/\sqrt{2}$ and the phase of the plasma wave $\sin(k_p\xi) < 0$. It is also shown that when the pulse is positioned at $r = r_L/\sqrt{2}$ it may be in the state of trapping, i.e., the state in which the laser beam could travel, in a constant section, for a long distance.

V. DISCUSSION AND CONCLUSION

In conclusion, it is found that photons are accelerated when they are positioned in the phase of the plasma wave such that $\partial_\xi\Delta < 0$ (accelerating phase). Analysis shows that photons will be trapped in the plasma wave and accelerated continuously if the density perturbation is propagating at the speed near that of light in the plasma. Since there is a radial component of the plasma density gradient, photons may be scattered transversely, i.e., the laser beam may be diffracted. Further study indicates, however, that if the laser pulse profile is properly specified, the focusing mechanism of the plasma wave is capable of limiting photon radial motion and keeping the packet from spreading. Furthermore, from Eqs.(33) and (40)′, to phase the laser pulse at a position on the plasma wave such that the plasma wave may both speed and focus the photons, one must confine the trailing pulse length in the scale $\leq \lambda_p/4$.

Another limitation on the photon accelerating length is the phase detuning distance L_t[15]. The phase detuning distance is the maximum distance in which

photons can be trapped in the accelerating phase and continuously gain the energy from the plasma wave:

$$L_t \equiv 2\lambda_p \Big/ \left(1 - \frac{v_g^2}{c^2}\right) = 2\lambda_p / \frac{\omega_p^2}{\omega^2}, \qquad (41)$$

where v_g and ω are the group velocity and frequency of the driving laser field, respectively. From Eqs.(15)′ and (26), one can estimate the maximum upshifting of the laser pulse as

$$\max\left\{\frac{\delta\omega}{\omega}\right\} = \frac{\pi}{8} a_{L0}^2 h L_t k_{p0}. \qquad (42)$$

Setting the frequency of the driving pulse equal to one of the trailing pulse, the normalized vector potential of the driving pulse $a_{L0} = 0.5$; and using Eq.(16), one has

$$\max\left\{\frac{\delta\omega}{\omega}\right\} = \frac{\pi^2}{2}\frac{a_{L0}^2}{1+\Delta} = 1.03.$$

Obviously, this is a very remarkable upshifting effect.

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