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NONLINEAR LOCALIZED STRUCTURE IN DUSTY PLASMA*

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The nonlinear effect of ponderomotive force of high-frequency ion motion on the low-frequency motion of ions coupling with the dust density fluctuation is investigated. The nonlinear localized structure described by Zakharov equations and nonlinear Schrödinger equation in dusty plasma are obtained. Envelope solitons for high-frequency ion motion and the dust density cavitons are also obtained and discussed.

Keywords: nonlinear localized structure, dusty plasma, dust density cavitons
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I. INTRODUCTION

Recently, the physics of dusty plasma has attracted a great deal of research interest in laboratory and astrophysics plasmas.[1−4] Significant progress has been made in the studies of the collective phenomena such as low-frequency dust acoustic waves,[5−8] plasma crystal,[9] dust shielding and long-range correlation[10,11] and of their potential applications.

As is well known, the coupling between the collective motions of ions and dusts can play a significant role in the dust density fluctuations as well as in the possible formation of dust lattice in a dusty plasma.[12,13] In many cases, for example, when a laser field passes through the plasma, the ponderomotive effect of high-frequency field on the low-frequency mode is very important. This force usually arises from an oscillating electric field propagating in a dielectric medium, which results in a slow timescale effect, for example, self-focusing of laser field. A general treatment for the ponderomotive force in view of quasiparticles in the field has been done recently by Silva *et al.*[14] They also pointed out the importance of the problem associated with the dust wave excitation and instability that can be involved in the ponderomotive force, To our knowledge, the ponderomotive effect in dusty plasma has not been clearly understood. Therefore, our attention in the present work is focused on the study of the nonlinear dust localized structure in a dusty plasma.

II. FORMULATIONS AND RESULTS

For simplicity, we assume that the electrons are depleted and consider a dusty plasma only consisting of two components, i.e., ions and dusts. It is also assumed that the quasineutrality condition at the equilibrium state remains valid, so we have

$$n_{i0} = Z_{\rm d} n_{\rm d0},$$

where n_{i0} and $n_{\rm d0}$ are the unperturbed ion and dust number densities respectively, and $Z_{\rm d}$ is the number of charges residing on the dust grains, which is assumed to be constant.

The fluid equations for the ions are

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = \frac{e}{m_i} E - \frac{1}{m_i n_i} \frac{\partial p_i}{\partial x}, \tag{2}$$

$$\frac{p_i}{n_i^{\gamma}} = \text{const},$$

where $\gamma = 3$ is for high-frequency perturbations, and $\gamma = 1$ for low-frequency ones. We divide the ion fluctuations into high- and low-frequency components

$$n_i = n_{i0} + n_{ih} + n_{il},$$

$$u_i = u_{ih} + u_{il},$$

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where the subscripts h and l denote high- and low-frequency components, respectively, and n_{i0} is the uniform background ion density. In general, we have n_{ih}, $n_{il} \ll n_{i0}$ and $u_{il} \ll u_{ih}$.

For spatially-dependent electrostatic fluctuation, we can write

$$\frac{\partial E}{\partial t} + 4\pi J = 0, \tag{3}$$

where J is the longitudinal part of the current density. Equation (3) is readily obtained from Poisson's equation and the charge continuity equation. Linearizing Eq.(3) and taking the high-frequency component yields

$$\frac{\partial E_h}{\partial t} = -4\pi e(n_{i0}u_{ih} + n_{il}u_{ih}), \tag{4}$$

where we have kept zero- and first-order quantities while neglecting the second-order perturbation $n_{ih}u_{il}$ on the right-hand side of the equation. The high-frequency component of the ion momentum equation is

$$\frac{\partial u_{ih}}{\partial t} + u_{il}\frac{\partial u_{ih}}{\partial x} + u_{ih}\frac{\partial u_{il}}{\partial x} = \frac{e}{m_i}E_h - \frac{3v_i^2}{n_{i0}}\frac{\partial n_{ih}}{\partial x}, \tag{5}$$

where we have used $p_{ih} = 3m_i v_i^2 n_{ih}$. On the one hand, the low-frequency component in timescale is much slower than the high-frequency one, i.e. $\partial u_{il}/\partial t \approx 0$. On the other hand, the low-frequency mode we consider is of long wavelength, so that $\partial u_{il}/\partial x \approx 0$. Furthermore, we assume that the low-frequency ion velocity is much lower than the phase velocity of high-frequency response of ions, i.e. $u_{il} \ll v_{ph}$. (This assumption will be discussed and can be seen more clearly at the end of this paper.) Therefore we have

$$u_{il}\frac{\partial^2 u_{ih}}{\partial t \partial x} \ll \frac{\partial^2 u_{ih}}{\partial t^2}. \tag{6}$$

By taking a time derivative of Eq.(5) and substituting Eq.(4) into it, we get

$$\frac{\partial^2 u_{ih}}{\partial t^2} + \frac{3v_i^2}{n_{i0}}\frac{\partial^2 n_{ih}}{\partial t \partial x} = -\omega_{\mathrm{p}i}^2 u_{ih} - \frac{4\pi e^2}{m_i}n_{il}u_{ih}. \tag{7}$$

In deriving this equation the inequality (6) has been used. The high-frequency component of the linearized continuity equation of ions reads

$$\frac{\partial n_{ih}}{\partial t} + n_{i0}\frac{\partial u_{ih}}{\partial x} = 0. \tag{8}$$

Substituting it into Eq.(7) gives

$$\frac{\partial^2 u_{ih}}{\partial t^2} + \omega_{\mathrm{p}i}^2 u_{ih} - 3v_i^2\frac{\partial^2 u_{ih}}{\partial x^2} + \frac{4\pi e^2}{m_i}n_{il}u_{ih} = 0. \tag{9}$$

To obtain an equation for the low-frequency fluctuations, we must take into account both the ion and the dust responses. We can neglect ion inertia for the low-frequency ion motion. By using the isothermal equation of state, the low-frequency component of the momentum equation of ions becomes

$$\frac{e}{m_i}E_l - \frac{\partial}{\partial x}|u_{ih}|^2 - \frac{v_i^2}{n_{i0}}\frac{\partial n_{il}}{\partial x} = 0. \tag{10}$$

Obviously this equation indicates that the force acting on the ions due to the low-frequency electrostatic field is balanced by the ion pressure and the ponderomotive force from high-frequency field.

For one-dimensional low-frequency dust motions, we have the following equations:

$$\frac{\partial n_{\mathrm{d}}}{\partial t} + \frac{\partial}{\partial x}(n_{\mathrm{d}}u_{\mathrm{d}}) = 0, \tag{11}$$

$$\frac{\partial u_{\mathrm{d}}}{\partial t} + u_{\mathrm{d}}\frac{\partial u_{\mathrm{d}}}{\partial x} = -\frac{eZ_{\mathrm{d}}}{m_{\mathrm{d}}}E - \frac{1}{m_{\mathrm{d}}n_{\mathrm{d}}}\frac{\partial p_{\mathrm{d}}}{\partial x}, \tag{12}$$

where n_{d}, u_{d} and m_{d} refer to the dust grain density, dust fluid velocity and dust grain mass, respectively. Here the dust charge variable is $Q_{\mathrm{d}} = eZ_{\mathrm{d}}$, where Z_{d} is the charge number of dust grains measured in units of electron charge. The density and velocity of dusts are assumed to be

$$n_{\mathrm{d}} = n_{\mathrm{d}0} + n_{\mathrm{d}l},$$

$$u_{\mathrm{d}} = u_{\mathrm{d}l},$$

then we have the linearized continuity and momentum equations for the dusts

$$\frac{\partial n_{\mathrm{d}l}}{\partial t} + n_{\mathrm{d}0}\frac{\partial u_{\mathrm{d}l}}{\partial x} = 0, \tag{13}$$

$$\frac{\partial u_{\mathrm{d}l}}{\partial t} = -\frac{eZ_{\mathrm{d}}}{m_{\mathrm{d}}}E_l - \frac{3v_{\mathrm{d}}^2}{n_{\mathrm{d}0}}\frac{\partial n_{\mathrm{d}l}}{\partial x}. \tag{14}$$

In the momentum equation we have used the adiabatic state equation for dusts. Now we take the time derivative of Eq.(13), $\partial/\partial x$ of Eq.(14), and after eliminating the common term $\partial^2 u_{\mathrm{d}l}/\partial t \partial x$ one obtains

$$\frac{\partial^2 n_{\mathrm{d}l}}{\partial t^2} - 3v_{\mathrm{d}}^2\frac{\partial^2 n_{\mathrm{d}l}}{\partial x^2} - n_{\mathrm{d}0}\frac{eZ_{\mathrm{d}}}{m_{\mathrm{d}}}\frac{\partial E_l}{\partial x} = 0. \tag{15}$$

Substituting E_l of Eq.(10) into Eq.(15) and bearing in mind that $n_{i0} = Z_{\mathrm{d}}n_{\mathrm{d}0}$ and $n_{il} = Z_{\mathrm{d}}n_{\mathrm{d}l}$, we obtain

$$\frac{\partial^2 n_{\mathrm{d}l}}{\partial t^2} - c_{\mathrm{d}}^2\frac{\partial^2 n_{\mathrm{d}l}}{\partial x^2} - \omega_{\mathrm{pd}}^2\frac{m_i}{4\pi e^2 Z_{\mathrm{d}}}\frac{\partial^2}{\partial x^2}|u_{ih}|^2 = 0, \tag{16}$$

where

$$c_{\mathrm{d}} = \sqrt{\frac{Z_{\mathrm{d}}T_i + 3T_{\mathrm{d}}}{m_{\mathrm{d}}}}$$

is the dust acoustic speed and

$$\omega_{\rm pd} = \sqrt{\frac{4\pi n_{\rm d0} e^2 Z_{\rm d}^2}{m_{\rm d}}}$$

is the dust plasma frequency.

On the other hand, with assumption $u_{ih} \sim \widetilde{u}_{ih}\exp(-{\rm i}\omega_{{\rm p}i}t) + {\rm c.c.}$, Eqs.(9) and (16) become

$$2{\rm i}\omega_{{\rm p}i}\frac{\partial \widetilde{u}_{ih}}{\partial t} + 3v_i^2\frac{\partial^2 \widetilde{u}_{ih}}{\partial x^2} - \frac{4\pi e^2 Z_{\rm d}}{m_i} n_{\rm dl}\widetilde{u}_{ih} = 0,$$

$$\frac{\partial^2 n_{\rm dl}}{\partial t^2} - c_{\rm d}^2\frac{\partial^2 n_{\rm dl}}{\partial x^2} - \omega_{\rm pd}^2\frac{m_i}{4\pi e^2 Z_{\rm d}}\frac{\partial^2}{\partial x^2}|\widetilde{u}_{ih}|^2 = 0. \quad (17)$$

Now let us introduce the normalized variables $t' = \omega_{{\rm p}i}t$, $x' = x/\lambda_i$, $u'_{ih} = \widetilde{u}_{ih}/v_i$ and $n'_{\rm dl} = n_{\rm dl}/n_{\rm d0}$. We have the following coupled equations (the primes of the normalized variables have been omitted)

$$2{\rm i}\frac{\partial u_{ih}}{\partial t} + 3\frac{\partial^2 u_{ih}}{\partial x^2} - n_{\rm dl}u_{ih} = 0, \quad (18)$$

$$\frac{\partial^2 n_{\rm dl}}{\partial t^2} - \frac{\alpha Z_{\rm d}}{\mu_{{\rm d}i}}\frac{\partial^2 n_{\rm dl}}{\partial x^2} - \frac{Z_{\rm d}}{\mu_{{\rm d}i}}\frac{\partial^2}{\partial x^2}|u_{ih}|^2 = 0, \quad (19)$$

where

$$\alpha = 1 + \frac{3T_{\rm d}}{Z_{\rm d}T_i}, \mu_{{\rm d}i} = \frac{m_{\rm d}}{m_i}.$$

Usually the condition $\partial^2 n_{\rm dl}/\partial t^2 \ll 1$ is readily satisfied, so that

$$n_{\rm dl} = -|u_{ih}|^2/\alpha,$$

then we obtain

$${\rm i}\frac{\partial u}{\partial t} + \frac{3}{2}\frac{\partial^2 u}{\partial x^2} + \frac{1}{2\alpha}|u|^2 u = 0.$$

Furthermore, by changing the temporal and spatial scale as

$$t \to t/2\alpha,\ x \to x/\sqrt{3\alpha}$$

we obtain the standard nonlinear Schrödinger equation (NLS)

$${\rm i}\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} + |u|^2 u = 0. \quad (20)$$

A standard envelope soliton solution has the form

$$u = \sqrt{2}\eta\sec[\eta(x - v_g t)]\exp\left[{\rm i}\frac{v_g}{2}x - {\rm i}\left(\frac{v_g^2}{4} - \eta^2\right)t\right], \quad (21)$$

and the dust density fluctuation has the following form of cavitons

$$n_{\rm dl} = -\frac{2\eta^2}{\alpha}\sec^2[\eta(x - v_g t)].$$

In fact, Eqs.(18) and (19) are the famous Zakharov equations, which also have an analytical solution that is similar in its form to the solution of Eq.(21) except for the altered amplitude (see, for example, Ref.[15]).

Now let us check the validity of the assumption Eq.(6), i.e. dimensionless $u_{il} \ll u_{ih} \sim \sqrt{2}\,\eta$, since the dust density is positive-definite, one has $n_{\rm dl} \geq -1$, i.e. $\eta < \sqrt{\alpha/2}$, which leads to the conclusion $u_{il} \ll \sqrt{\alpha} \sim O(1)$. On the other hand, the dimensionless phase velocity is $v_{ph} = (2\sqrt{\alpha}/\sqrt{3})/(v_g/2) \sim \sqrt{\alpha}/\eta \sim O(1)$, then $u_{il} \ll v_{ph} \sim O(1)$ is indeed satisfied self-consistently for the dusty plasma system.

Ⅲ. DISCUSSIONS AND SUMMARY

It is interesting to compare our result with that of Ref.[12]. It is noticed that in Ref.[12] the authors also obtained a nonlinear Schrödinger equation; however, they started from an equation for the recombination of electrons and ions on the surface of dust grains. Their results come from the nonlinear effect of long wavelength instability driven by the recombination processes. In this limit, the modification of the real part of the frequency due to collisions is assumed important. Instead, in our model with the assumption of depletive electrons we have shown that it is the ponderomotive force, a nonlinear effect of high-frequency field acting on the low-frequency mode, that plays a significant role on the formation of caviton structure.

We here stress that our present results would be helpful in understanding some impressive features of the instabilities of dust number density modes in a recent experiment by Samsonov and Goree at the Max Planck Institute,[4] especially the discovery of the so-called great void mode, i.e. dust-free region. We can give a brief illustration and discussion for the possibility of void in our present study. Usually the frequency of the applied voltage is near to the high frequency of the system, ω_i for the excitation of ions motion. We then approximately obtain the critical voltage with which the ions acquire the velocity of the magnitude of thermal speed

$$V_{\rm c}[V] = \frac{LT_i[{\rm eV}]}{\lambda_i}, \quad (22)$$

where L is the distance between the electrodes, T_i the ions plasma temperatures and λ_i the ions plasma Debye radii. It is therefore easy to get $u'_{ih} \sim v_i$ at the initial stage so that $\sqrt{2}\,\eta \sim 1$ when the voltage $V > V_{\rm c}$ is applied. With a given ion temperature T_i, it is clear that the lower the ion density number, the smaller

the V_c. On the other hand, for smaller T_d and larger Z_d, one has $\alpha \sim 1$. Thus the caviton profile of dust density fluctuation would be highly negative with the dipped amplitude $2\eta^2/\alpha$. This amplitude is of the order of n_{d0} in the dimension expression. This indicates a void can form for dust density in some of the cases mentioned above. For example, for $L = 1$cm, $\lambda_i = 0.01$cm, $T_i = 0.5$eV, and $T_d = 0.5$eV, $Z_d = 3000$, the minimum magnitude of the applied voltage for introducing a void is $V_c = 50$V. In our case the model is one-dimensional. For a practical situation where the problem is high-dimensional, as is well known, an NLS can collapse.[16,17] This phenomenon is probably related to the filamentary structures of dust density in Ref.[4], much similar to the filamentation effect due to nonlinear ponderomotive force in intense laser–plasma interactions. Certainly much attention is expected to be paid to this interesting problem in future studies.

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