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MICRO-MOTION EFFECT OF A TRAPPED ULTRA-COLD ION IN A STANDING-WAVE LASER*

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In the absence of the requirements of the Lamb–Dicke limit and rotating wave approximation, we semi-classically investigate the dynamics of a trapped ultra-cold ion in the standing-wave laser, with the consideration of the time-dependent potential and pseudo-potential of the Paul trap. The specific calculations show that the larger the Lamb–Dicke parameter η and the Rabi frequency Ω, the greater the difference between the dynamics in the time-dependent potential and the pseudo-potential.

Keywords: ultra-cold ion, standing-wave laser, micro-motion
PACC: 4250, 3280

I. INTRODUCTION

The Paul trap[1] provides an almost ideal quantum system for ultra-cold ions, which can be modelled by the quantum optics theory and considered for applications in the field of frequency standard[2], precision spectroscopy[3] and quantum computation.[4] The general treatments[5–7] of such systems were made by using the Jaynes–Cummings model (JCM)[8] under the pseudo-potential approximation (PPA)[9], in which the effect of the micro-motion brought about by the time-dependent potential (TDP) of the Paul trap is neglected. However, the PPA is not always valid in the stability region of the Paul trap, since the effect of the micro-motion has been observed experimentally as additional resonance in the excitation spectra.[10,11] So it is necessary to carefully study the influence of the TDP on the quantum motion of the trapped ion. Previous work[12–14] focusing on this problem were semi-classical treatments, starting from the quantum Mathieu equation for the motion of the ion, and then mapping the motion of the ion into the internal dynamics of the ion in the rotating wave approximation (RWA). The numerical results[14] obtained by such treatments showed that the difference between the dynamics of the ion under the TDP and the pseudo-potential was small, although the difference would be enlarged with the increase of the Lamb–Dicke parameter. A recent study[15] has also shown that the RWA is not always valid in the interaction of the trapped ultra-cold ion with lasers. Therefore, it is natural to exclude the RWA from the above studies.

In this paper, we will compare the internal dynamics of the trapped ion interacting with a standing-wave laser in the absence of the RWA by considering respectively the explicit TDP and the pseudo-potential. It will be seen that only simple formulae need to be used in our work and the motion of the ion will be neither restricted to the Lamb–Dicke limit ($\eta \ll 1$), nor under weak excitation regime ($\Omega < \upsilon$).

II. MODEL AND CALCULATION

The description will be made in the frame rotating with the laser frequency. First of all, we consider the case under the TDP. The Hamiltonian of a trapped ultra-cold ion interacting with a standing-wave laser in the TDP can be described as

$$H = \frac{\hbar\Delta}{2}\sigma_z + \frac{p^2}{2m} + \frac{1}{2}mW(t)x^2 + \frac{\hbar}{2}\Omega\sigma_x \sin(kx+\phi), \quad (1)$$

where $\Delta = \omega_0 - \omega_L$, with ω_0 being the transition frequency of two levels of the ion, ω_L the frequency of the standing-wave laser. $W(t) = \frac{\upsilon^2}{4}(a - 2q\cos\upsilon t)$, with υ being the micro-motion frequency of the ion, a and q being the two dimensionless variables relative to the dc voltage U_0 and ac voltage V_0 applied to the trap electrodes respectively. It is generally defined that $a = -8eU_0/(m\upsilon^2 r_0^2)$ and $q = -4eV_0/(m\upsilon^2 r_0^2)$, with r_0 being the geometric parameter of the Paul

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trap and e being the ion's charge; $\frac{p^2}{2m}$ is the kinetic energy of the ion with mass m, in which m can be set to 1 for the convenience of theoretic calculation. Ω is the Rabi frequency, σ_i $(i = \pm, z)$ are Pauli operators, a^+ and a are creation and annihilation operators of the phonon respectively, which satisfy the commutation $[a, a^+] = 1$. Moreover, we denote the position of the ion in the standing wave by ϕ.

As an ultra-cold ion is nearly perturbation-free in the trap, we may suppose that υ is much greater than the atomic decay rate (strong confinement limit), so that one can neglect the effect of the atom decay. Using the Heisenberg equation $\dot{F} = \frac{i}{\hbar}[H, F]$, undamped Bloch–Maxwell equations can be derived directly from Eq.(1)

$$\dot{\sigma}_x = -\frac{\Delta}{\upsilon}\sigma_y, \tag{2a}$$

$$\dot{\sigma}_y = \frac{\Delta}{\upsilon}\sigma_x - \frac{\Omega}{\upsilon}\sigma_z \sin(\tilde{x} + \phi), \tag{2b}$$

$$\dot{\sigma}_z = \frac{\Omega}{\upsilon}\sigma_y \sin(\tilde{x} + \phi), \tag{2c}$$

$$\begin{aligned}\ddot{\tilde{x}} &= -\frac{W(t)}{\upsilon^2}\tilde{x} - \frac{\eta^2\upsilon_0\Omega}{\upsilon^2}\sigma_x \cos(\tilde{x} + \phi) \\ &= -\frac{1}{4}(a - 2q\cos\tau)\tilde{x} \\ &\quad - \frac{1}{2}\frac{\eta^2\sigma_x\Omega}{\upsilon^2}\sqrt{a + \frac{q^2}{2}}\cos(\tilde{x} + \phi),\end{aligned} \tag{2d}$$

where $k = \eta\sqrt{2\upsilon_0}$ has been used in the PPA, $\upsilon_0^2 \approx \frac{\upsilon^2}{4}\left(a + \frac{q^2}{2}\right)$, for $a \ll 1$, $q < 0.4$, and the external motion of the ion is treated classically by setting $\tilde{x} = kx = \eta(a^+ + a)$, η is the Lamb–Dicke parameter, and the dot denotes the time derivation $\frac{\partial}{\partial\tau}$ with $\tau = \upsilon t$.

For comparison, we write the Hamiltonian of a trapped ultra-cold ion in the pseudo-potential of the trap experiencing a standing wave

$$H = \frac{\hbar\Delta}{2}\sigma_z + \hbar\upsilon_0 a^+ a + \frac{\hbar\Omega\sigma_x}{2}\sin(kx + \phi), \tag{3}$$

where the parameters are the same as those defined in Eq.(1). With similar deduction, we can have another set of undamped Bloch–Maxwell equations

$$\dot{\sigma}_x = -\frac{\Delta}{\upsilon}\sigma_y, \tag{4a}$$

$$\dot{\sigma}_y = \frac{\Delta}{\upsilon}\sigma_x - \frac{\Omega}{\upsilon}\sigma_z \sin(\tilde{x} + \phi), \tag{4b}$$

$$\dot{\sigma}_z = \frac{\Omega}{\upsilon}\sigma_y \sin(\tilde{x} + \phi), \tag{4c}$$

$$\begin{aligned}\ddot{\tilde{x}} &= -\frac{\upsilon_0^2}{\upsilon^2}\tilde{x} - \frac{\eta^2\upsilon_0\Omega}{\upsilon^2}\sigma_x \cos(\tilde{x} + \phi) \\ &= -\frac{1}{4}\left(a + \frac{q^2}{2}\right)\tilde{x} - \frac{1}{2}\frac{\eta^2\sigma_x\Omega}{\upsilon^2}\sqrt{a + \frac{q^2}{2}}\cos(\tilde{x} + \phi).\end{aligned} \tag{4d}$$

Obviously, in our semi-classical treatment of the Heisenberg representation, we can readily calculate $< \sigma_i >$ $(i = x, y, z)$ simply by substituting σ_i with $< \sigma_i >$ in Eqs.(2) and (4) respectively, where $< \sigma_i >$ is the expectation value of the corresponding Pauli operator σ_i with respect to the internal states of the ion. In what follows, we will focus on the study of the observable variable $< \sigma_z(\tau) >$, which is the expectation value of the population inversion of the ion, experimentally corresponding to the variation of the fluorescence intensity with time. The ion is supposed to be prepared in the internal excited state $|e\rangle$, that is to say, $< \sigma_x(0) >=< \sigma_y(0) >= 0$ and $< \sigma_z(0) >= 1$. Moreover, we chose $a = 0$, $q = 0.3$, $\phi = \frac{\pi}{4}$, $x(0) = 0$ and $\dot{x}(0) = \eta$. Equations (2) and (4) will be integrated forward in time with the fourth-order Runge–Kutta procedure by adjusting the parameters η, $\frac{\Omega}{\upsilon}$, $\frac{\Delta}{\upsilon}$. The results shown in Figs.1–4 illustrate the cases under the TDP and pseudo-potential. From the time evolution of the population inversion, we know that the results obtained by the above two treatments are almost the same in the case of small values of η and Ω in a limited period of time, this is in agreement with the treatment in present ion trap experiments,[4] that averages the micro-motion by the PPA in the case of small values of η and Ω. As the values of the parameters in Fig.1 are taken from the data of actual experiment,[4] our results further confirm the validity of the treatment in Ref.[4] in the case of neglecting the RWA. However, with the increase of η and Ω, the differences become more and more apparent. Note that the large value of Ω has not been achieved in the experiments with ultra-cold ions because it corresponds to the high intensity of the laser beam which will drive the ions out of the trap. However, a heuristic proposal[16] for quantum computation with hot trapped ions, which requires a large value of Ω, has stimulated experimenters to pay much effort to increase Ω. In the hot ion quantum computation model, the computing process is nothing to do with the motional states of the ion, which much simplifies the prerequisite of the quantum computation, i.e. cooling the ion to the ultra-cold state.[17]

Moreover, the technical difficulty for increasing Ω may be overcome with some experimental skill[18] in the near future. However, the results in our paper remind us that the model in Ref.[16] is probably unsuitable for the quantum computation because large errors will appear in the final readout of the quantum computation, depending on the exact detection of $< \sigma_z(\tau) >$. Therefore, a more hopeful quantum computation model with hot ions is expected to either include the effect of the micro-motion or keep the ion under the condition of $\Omega < \upsilon$.[19] Furthermore, specific calculation showed that the variation of detuning Δ is not essential to the complicated variation of the population inversion, although the difference of results obtained under the PPA and TDP is greater in the case of large detunings.

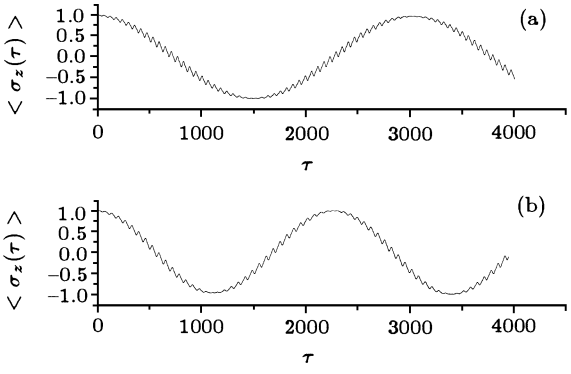


Fig.1. (a) Time evolution of population inversion $< \sigma_z(\tau) >$ under the pseudo-potential, with $\eta = 0.2$, $\frac{\Delta}{\upsilon} = 0.0$ and $\frac{\Omega}{\upsilon} = 0.01$. (b) Time evolution of population inversion $< \sigma_z(\tau) >$ under the time-dependent potential with the same parameters as in (a).

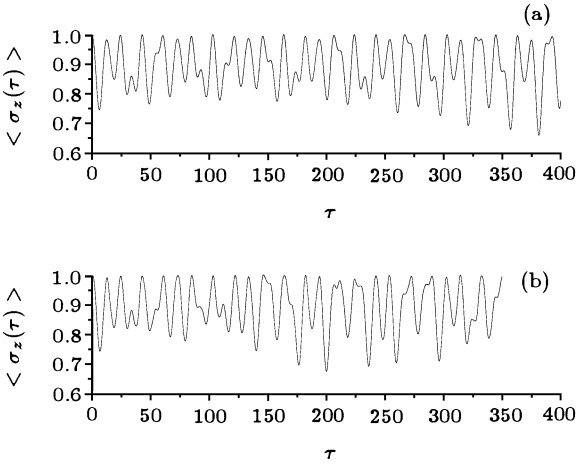


Fig.2. Same as Fig.1, but for $\eta = 0.2$, $\frac{\Delta}{\upsilon} = 0.5$ and $\frac{\Omega}{\upsilon} = 0.2$.

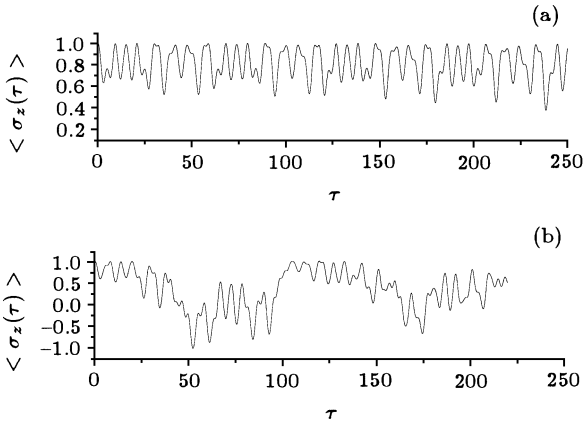


Fig.3. Same as Fig.1, but for $\eta = 0.5$, $\frac{\Delta}{\upsilon} = 1.0$ and $\frac{\Omega}{\upsilon} = 0.5$.

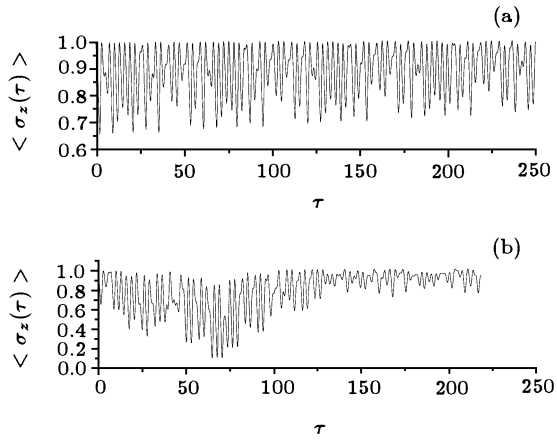


Fig.4. Same as Fig.1, but for $\eta = 0.5$, $\frac{\Delta}{\upsilon} = 2.0$ and $\frac{\Omega}{\upsilon} = 1.0$.

III. DISCUSSION AND CONCLUSION

In the previous work[20] within the purely quantum mechanical framework, the complicated variations of the population inversion were also studied by using nonlinear JCM under the RWA and PPA. The nonlinearity in those works was caused by the high-order terms of $\eta(a^+ + a)$. We also noted that a semi-classical treatment of many identical two-level atoms interacting with a single mode electromagnetic field has been carried out in the absence of the RWA.[21] The complicated behaviour was also found in the evolution of population inversion when the coupling between the atoms and the field became large enough, and this nonlinearity was attributed to the counter-rotating terms $a^+\sigma_+ + a\sigma_-$. As no approximation was made in our work, the complicated variation in the population inversions obviously resulted from the high-order terms of $\eta(a^+ + a)$ and the counter-rotating

terms. Compared with the results in Ref.[14], we know that the large difference between the dynamics under TDP and pseudo-potential is mainly produced by the counter-rotating terms.

With the above results, we have made some discussions on the recent proposal for quantum computation with hot trapped ions, in which we assumed that the hot ions still satisfied the Schrödinger equation, whereas their motional state need not be in ultra-cold state $|0\rangle$ or $|1\rangle$. However, as our semi-classical method treats the motional states of the ion classically, we have no way to distinguish the ultra-cold ion from the hot ion in the Schrödinger equation. Nevertheless, by means of our method, we find that the requirement for large value of Ω in Ref.[16] is inappropriate for the precise read-out of the final data in the quantum computation.

In summary, we have studied semi-classically the dynamics of a trapped ultra-cold ion in the standing wave of a laser, with particular emphasis on the influence of micro-motion, in which the motion of the ion is described classically and its internal degree is treated quantum mechanically. By means of the familiar Bloch–Maxwell equation, we investigated the internal dynamics of the trapped ion in a wide range of parameters. Compared with previous studies, the advantage of our treatment is that not only can we use simple formulas in calculation, but we also have the exact description in the absence of RWA, and thus we present the case of an actual configuration of trapped ions. Moreover, the dc and ac voltages applied to the trap electrodes can also influence the internal dynamics of ions, whereas their changes can be resorted to the variation of υ, for simplicity of the theoretical studies. The adjustments of $\frac{\Delta}{\upsilon}$ and $\frac{\Omega}{\upsilon}$ also reflect, to a certain extent, the influence of υ on the dynamics of the system. Furthermore, as $< \sigma_z(\tau) >$ can be detected with high precision in the current ion trap experiment by the quantum jump technique, the theoretical results presented in this paper could be tested in principle in experiments. As the ion trap configuration is supposed to be undamped in our model, the decoherence due to the spontaneous emission is not discussed here. In fact, if the decoherence is considered, the large value of Ω corresponds to the larger possibility of decoherence, unless the detuning is increased correspondingly. We believe the large differences between $< \sigma_z(\tau) >$ under the explicit time-dependent potential and that under the pseudo-potential, as shown in our study, will be useful in the experimental exploration for quantum properties of the trapped ion beyond the weak excitation regime.

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