Vol 10 No 3, March 2001
1009-1963/2001/10(03)/0194-08

Chinese Physics

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LOW-LYING SPECTRA AND $E2$ TRANSITION RATES IN EVEN–EVEN Pd ISOTOPES IN THE INTERACTING BOSON MODEL*

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(Received 18 January 2000; revised manuscript received 3 September 2000)

The positive parity collective states in the even–even ^{100}Pd–^{116}Pd isotopes are studied in the framework of the interacting boson model. A schematic Hamiltonian capable of describing their spectra and transition is used. It is found that the spectra can be well described by a $U(5)$ plus $SU(3)$ perturbation interaction. As the valence neutron number changes, the structure of the isotopes shows regular patterns of change. In the meantime, the behaviour of 0_2^+ can also be partially described by the model.

Keywords: Spectra, electromagnetic transition, positive parity low-lying collective state, staggering phenomenon

PACC: 2160F, 2320

I. INTRODUCTION

The interacting boson model[1–3] is a very effective phenomenological model for describing low-lying collective properties of nuclei across an entire major shell. Previous model calculations[4–11] demonstrated that they are $U(5)$ to $O(6)$ transitional nuclei for Pd, Xe and Ba isotopes. It was believed until recently that many Ba, Pd and Xe isotopes are typical examples of the $O(6)$-like nuclei[4,8,12–16]. But it has recently been shown[17–19] that one can describe the low-lying structure of Ba and Xe isotopes by a transition from $U(5)$ to $SU(3)$. One may expect that Pd isotopes are also $U(5)$ to $SU(3)$ transitional nuclei. This is particularly convincing because the spectra of Pd and Xe isotopes are very similar.[16] It is the purpose of this study to see if a non-$O(6)$ Hamiltonian can describe its low-lying structure. Systematics is important in the study of the properties of nuclei.[20–23] We have analysed the systematics of the spectra and electromagnetic transitions of the even Pd isotopes in this work. It was found from this work that the Pd isotopes could be well described by the $U(5)$ to $SU(3)$ dynamical symmetry.

II. THE SCHEMATIC IBM HAMILTONIAN

The general IBM Hamiltonian contains seven terms. However, for our study, we take the following schematic Hamiltonian:

$$\hat{H} = \varepsilon_{\rm d}\hat{n}_{\rm d} + K\boldsymbol{Q}\cdot\boldsymbol{Q} + K_L\boldsymbol{L}\cdot\boldsymbol{L},$$

where

$$Q_u = (s^+\tilde{d} + d^+s)^2 + \chi(d^+\tilde{d})^2_\mu, L_q = \sqrt{10}(d^+\tilde{d})^{(1)}_q.$$

This Hamiltonian is capable of giving symmetries with the following parameters: $U(5)$: K=0, χ=0; $SU(3)$: ε_d=0, $\chi = -\sqrt{7}/2$; $O(6)$: ε_d=0, χ=0; $K_L\boldsymbol{L}\cdot\boldsymbol{L}$ term removes some of the degeneracy for different L values. Since there are only two parameters, K/ε_d and χ, that determine the phase transition, if K/ε_d=0, the Hamiltonian is vibrational; if this ratio is ∞, $\chi = -\sqrt{7}/2$, the Hamiltonian is rotational. If the ratio lies in between, the Hamiltonian, denoted by H^a, is in the transition between $U(5)$ and $SU(3)$; similarly, if χ=0, the Hamiltonian, denoted by H^b, is in the transition between $U(5)$ and $O(6)$. The parameters in the Hamiltonian can be determined by fitting to the experimental spectra. After the determination of the spectra, the wavefunction is determined. The electric and magnetic transition properties can then be obtained accordingly. For example, the $E2$ transition operator is

$$T(E2)^2_\mu = e_2[(s^+\tilde{d} + d^+s)^2_\mu + \chi(d^+\tilde{d})^2_\mu].$$

*Project supported in part by the National Natural Science Foundation of China, (Grant No. 19775026), Foundation for Outstanding Young University Teachers, Ministry of Education of China, and by the Fok Ying Tong Education Foundation.

Microscopically, the transition operators can be derived from the shell model by the mapping procedure.[24–27] In practice, it is more convenient to treat them as free parameters. Here we adopt the consistent $Q \cdot Q$ formalism.[28] As is known, this convention is not an essential requirement of the model, and sometimes it is even necessary to use a different Q operator in $E2$ transition calculation to describe the low-lying $E2$ transitions,[29,30] for example, the structure for neutron-rich and neutron-deficient Sr nuclei[31–33], neutron-rich Cd nuclei[34], and neutron-deficient and neutron-rich Nd nuclei.[35,36] Noticeably, the reduction in the collectivity problem can be solved by using a different operator in the transition from that in the Hamiltonian.[37–39] However, in most cases, the consistent $Q \cdot Q$ formalism can give a good first description of the $E2$ transition properties. Since there are few experimental data available, we adopt the consistent $Q \cdot Q$ formalism in the first place. When there are more experimental data on the $E2$ transition in the future, one can refine the $E2$ transition operators to produce the details.

III. RESULTS AND DISCUSSION

In Table 1, we give the parameters of the Hamiltonian and of the $E2$ transition operator in each nucleus studied. In principle, all parameters can be varied independently in fitting the energy spectrum and the electromagnetic properties of any nucleus.

From Table 1, the values for ε_d, K and K_L are rather constant, with the exception of ^{104}Pd. The value of ε_d decreases with increasing mass number. Moreover, the largest change is observed for ^{106}Pd to ^{108}Pd. Within a geometrical language, the coexistence and coupling of slightly oblate and more strongly deformed prolate systems is one of the salient features of the Pd nuclei. Using these parameters, we calculate the energy levels and the values of reduced matrix elements of quadrupole transition, i.e. $B(E2)$ value(ratios), for each nucleus. The comparisons between calculated and experimental energy levels[40–46] for ^{100}Pd–^{116}Pd are shown in Figs.1–9, respectively. In general, the agreement is quite good, especially for the ground-state band levels with $J^\pi \leq 8^+$. However, there are still some discrepancies.

In general, Pd isotopes exhibit staggering in the gamma band. In ^{114}Pd and ^{116}Pd, the staggering phenomenon in calculation is stronger than that in experiment. The agreement between the calculated and experimental data may be improved by the use of cubic terms[47] in IBM-1 and the quadrupole interaction between like nucleons[48–51] in IBM-2. In addition, we find the calculated energies of the 2_2^+ and 0_2^+ are smaller than those of the experimental 2_2^+ and 0_2^+ states, especially in the 0_2^+ states. As a consequence, we suspect that these states are intruder states.[52] This deviation may be solved in the interacting proton–neutron boson model. However, no conclusion can be drawn from the energies alone, since it is very likely that both intruder and collective 0_2^+ states will occur in the same energy region. It must be stressed that in a number of cases, more decisive conclusions can be drawn from electromagnetic properties, which will be discussed later.

Table 1. Parameters of energy levels and $B(E2)$ operator for Pd isotopes.

	ε_d/MeV		K/MeV		K_L/MeV		e_2/eb	
Nucleus	H^a	H^b	H^a	H^b	H^a	H^b	H^a	H^b
^{100}Pd	0.6796	0.5980	−0.0084	−0.0079	0.0063	0.0093		
^{102}Pd	0.6288	0.5650	−0.0084	−0.0079	0.0050	0.0093	0.1234	0.1284
^{104}Pd	0.6288	0.6150	−0.0054	−0.0079	0.0050	0.0093	0.1258	0.1258
^{106}Pd	0.5888	0.5880	−0.0074	−0.0079	0.0098	0.0740	0.1224	0.1262
^{108}Pd	0.4888	0.4950	−0.0074	−0.0089	0.0118	0.0093	0.1172	0.1208
^{110}Pd	0.4588	0.4450	−0.0074	−0.0079	0.0108	0.0093	0.1106	0.1189
^{112}Pd	0.4548	0.4350	−0.0074	−0.0079	0.0108	0.0099	0.0856	0.0934
^{114}Pd	0.4448	0.4050	−0.0084	−0.0128	0.0108	0.0123	0.0648	0.0659
^{116}Pd	0.4446	0.4050	−0.0084	−0.0128	0.0098	0.0123	0.0951	0.0950

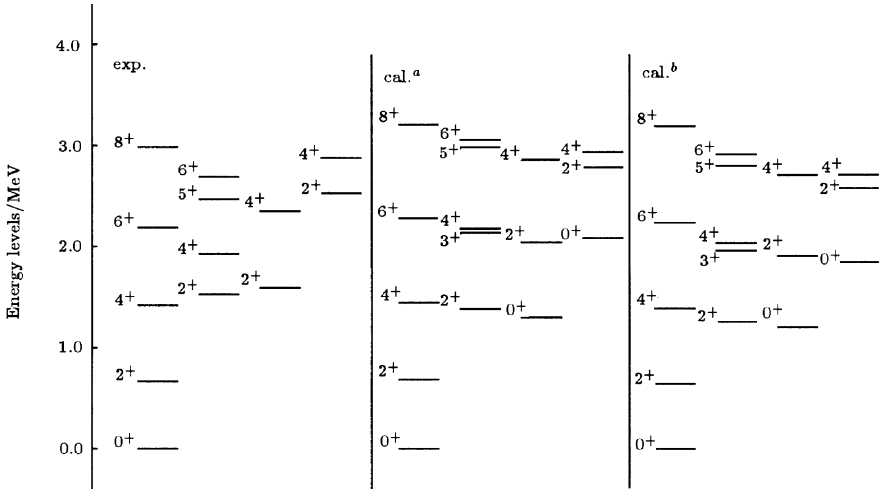


Fig.1. Spectrum for ^{100}Pd.

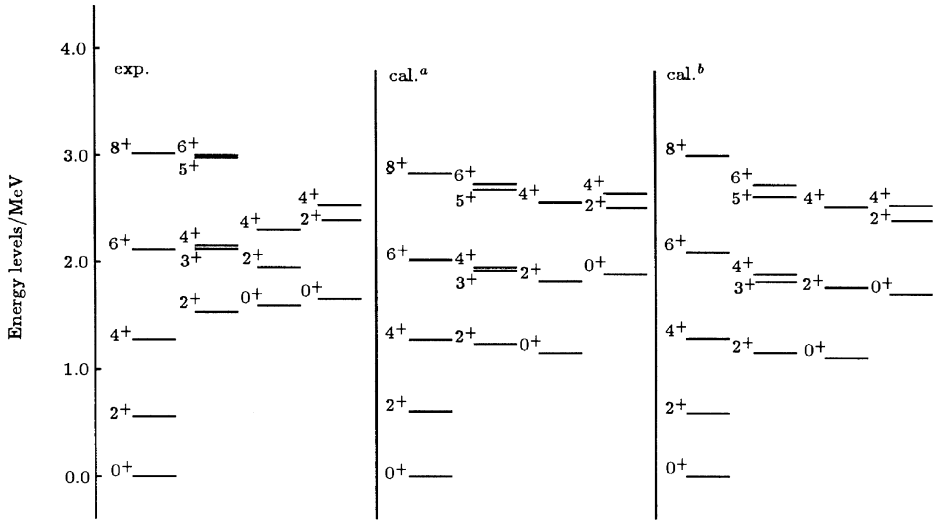


Fig.2. Spectrum for ^{102}Pd.

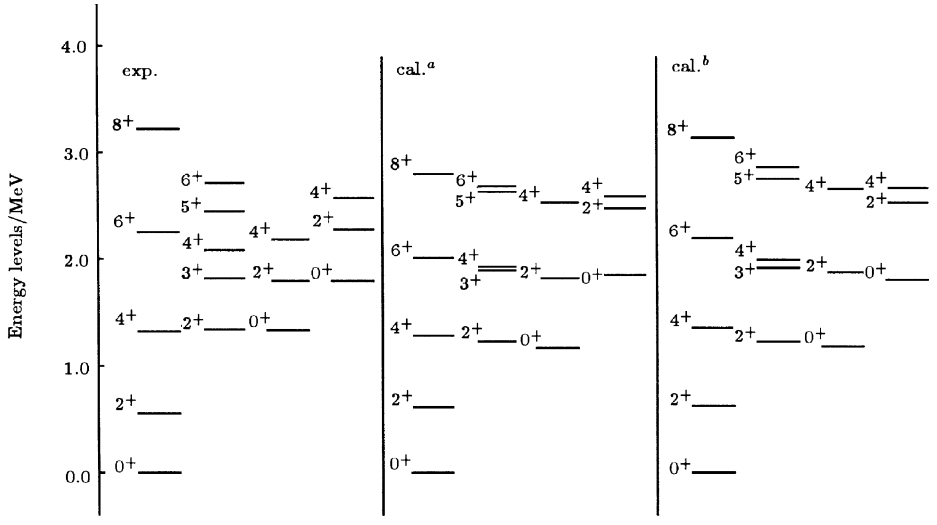


Fig.3. Spectrum for ^{104}Pd.

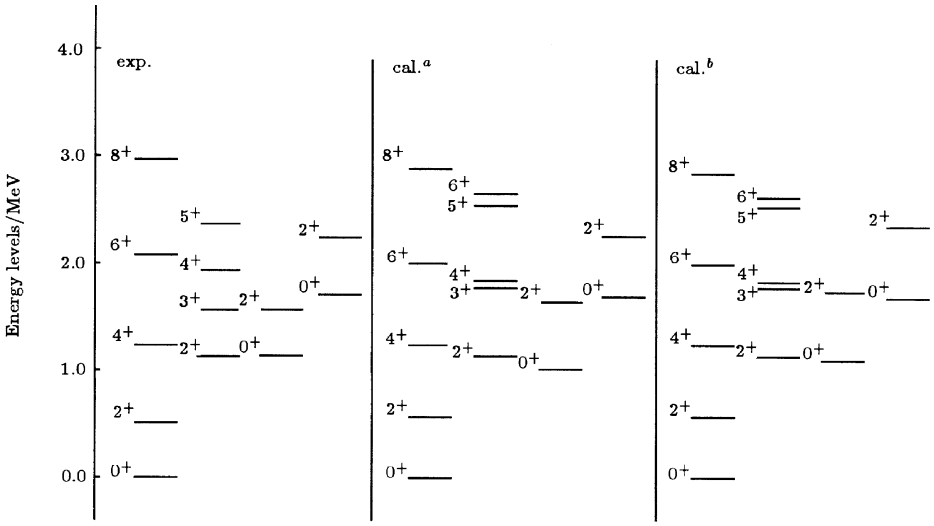


Fig.4. Spectrum for ^{106}Pd.

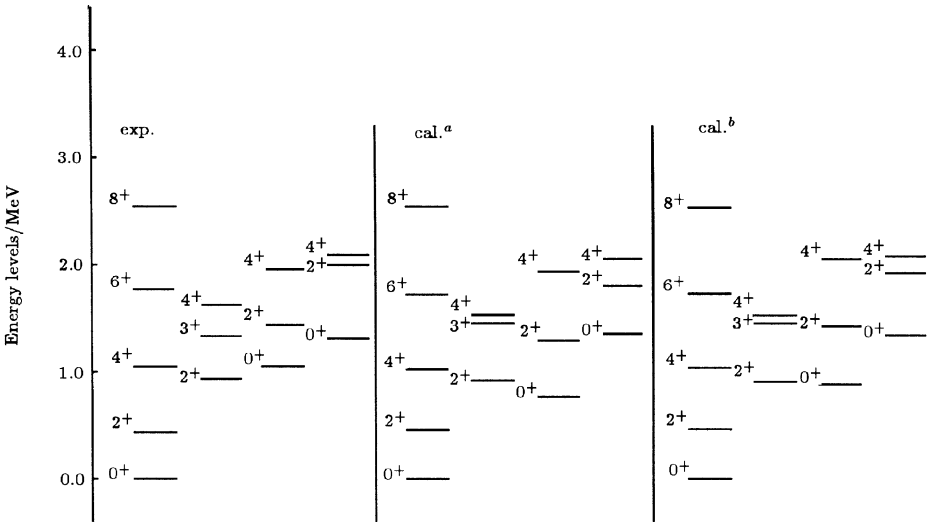


Fig.5. Spectrum for ^{108}Pd.

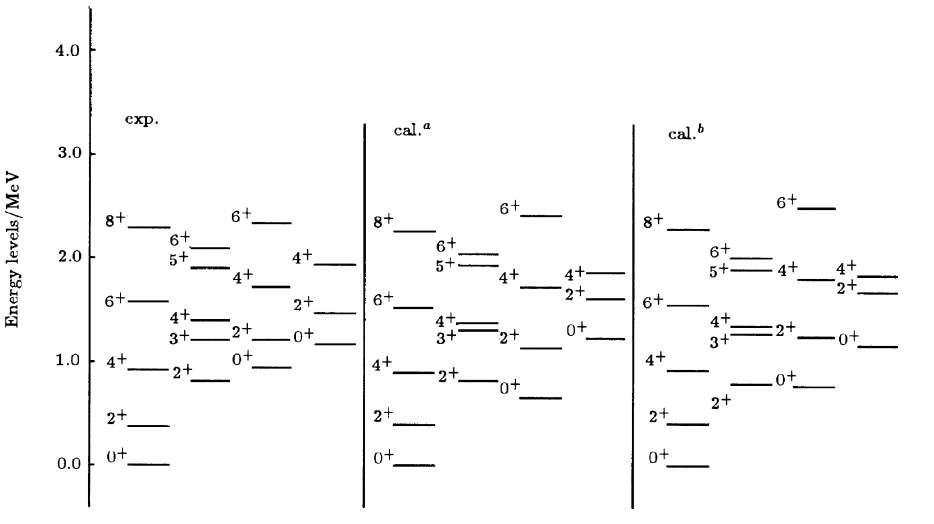


Fig.6. Spectrum for ^{110}Pd.

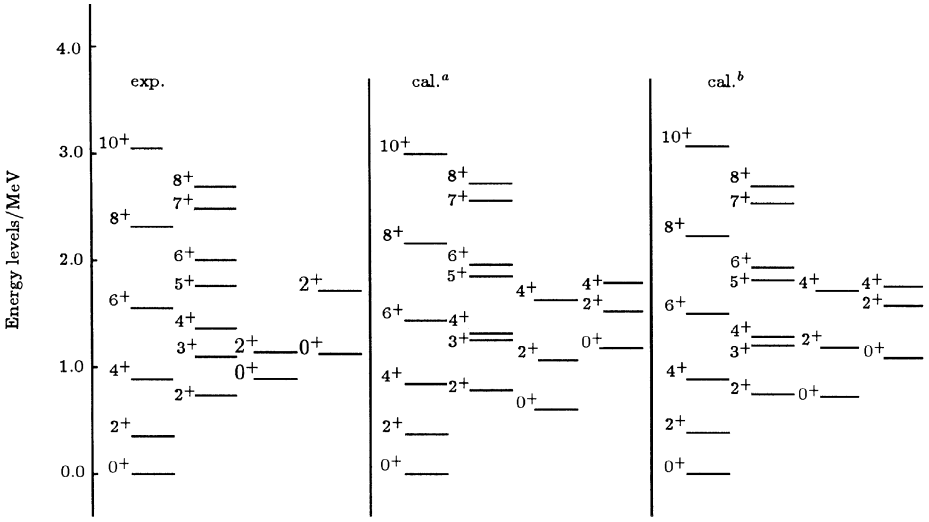


Fig.7. Spectrum for ^{112}Pd.

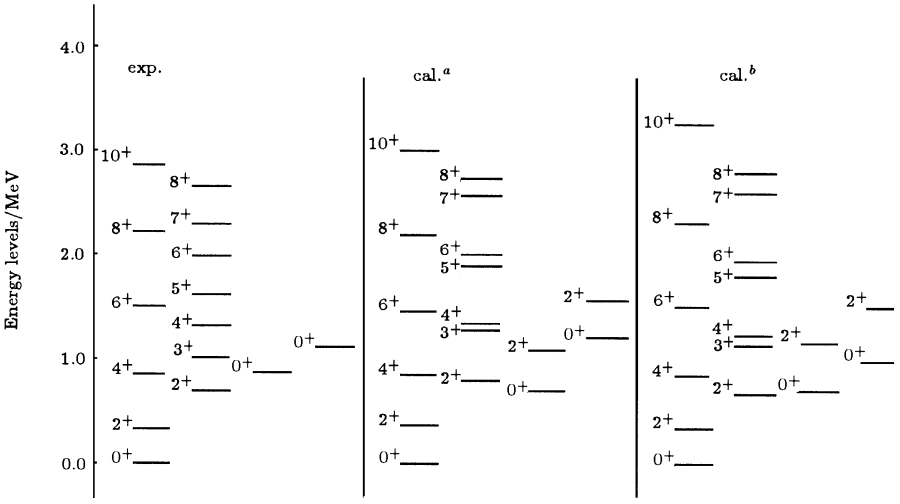


Fig.8. Spectrum for ^{114}Pd.

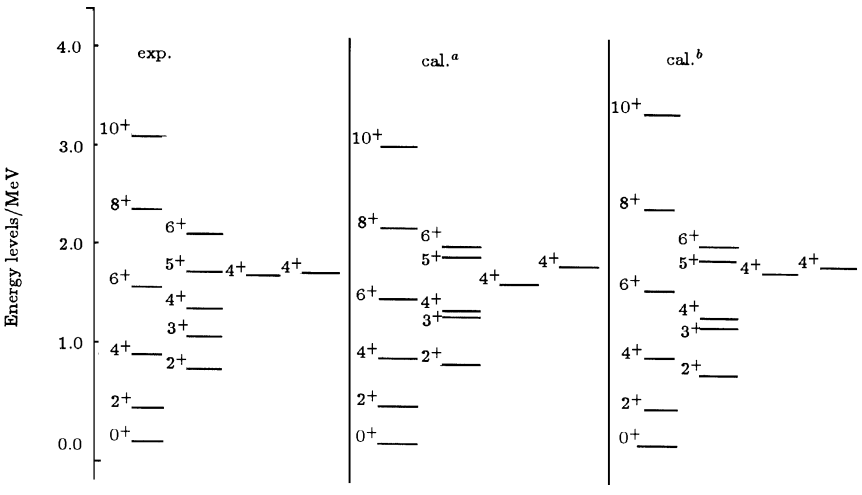


Fig.9. Spectrum for ^{116}Pd.

In addition, comparisons of the experimental spectra of ^{100}Pd–^{116}Pd with those from the H^a and the H^b calculations, are made. Although both the H^a and H^b calculations give nearly identical spectra for the low-lying levels, the H^a calculation is better. Meanwhile, the spectra were calculated by using a scheme[23] in which the Hamiltonian is $\hat{\boldsymbol{H}} = \boldsymbol{KQ}\cdot\boldsymbol{Q} + \boldsymbol{K}_L\boldsymbol{L}\cdot\boldsymbol{L}$, and $Q_u = s^+\tilde{d} + d^+s)^2 + \chi(d^+\tilde{d})^2_\mu$. If $|\chi| \to \infty$ the Hamiltonian is vibrational, and if $\chi = \pm\sqrt{7}/2$, the Hamiltonian is rotational. That is to say, if $|\chi| > \sqrt{7}/2$, the Hamiltonian is in the transition between $U(5)$ and $SU(3)$. It is found that the quality of agreement between the calculated and experimental data is also good for $|\chi| > \sqrt{7}/2$. However, the transition properties favour the $U(5)$ to $SU(3)$ transition, i.e. $\chi = -\sqrt{7}/2$. The transition properties with $\chi = -\sqrt{7}/2$ and χ=0 are given in Table 2, where the ratios $R = E(2_2^+)/E(2_1^+), R_{4/2} = E(4_1^+)/E(2_1^+), R_{6/2} = E(6_1^+)/E(2_1^+)$ and $R^* = E(3_1^+)/[E(2_1^+) + E(2_2^+)]$ for all nine nuclei are shown. It is found that for Pd isotopes, with the exception of ^{108}Pd, the agreement between H^a calculations and experimental data is better also. We analysed each value in the H^a calculation by systematics. It is obvious that the R-value reflects the vibrational character; the R^* values for all nine nuclei are near 1.00, the model value of a triaxial rotor. The $R_{4/2}$ value increases from a value of 2.11 for ^{100}Pd to 2.28 for ^{116}Pd. The $R_{6/2}$ value increases to 3.96 in ^{114}Pd. This value lies between those for a good vibrator ($R_{6/2}$=3) and a good rotor ($R_{6/2}$=7) and is close to 4.5 for an $O(6)$ nucleus. Sometimes it is confusing that the ratios are suggesting the $O(6)$ limit. Indeed, most of the energy ratios in Table 2 are not in disagreement with the vibrator to gamma-soft rotor transition. However, our Hamiltonian is the $U(5)$ to $SU(3)$ one, reflecting the transition from $U(5)$ to $SU(3)$. This similarity between the $O(6)$ and $U(5)$ to $SU(3)$ has been discussed[18] recently.

Table 2. Comparison of the experimental[35–41] values (ratios) for $^{100-116}$Pd with the H_a calculation, and with the H^b calculation.

Nucleus		R	$R_{4/2}$	$R_{6/2}$	R^*	$\frac{B(E2,2_2^+\to 2_1^+)}{B(E2,2_2^+\to 0_1^+)}$	$\frac{B(E2,2_2^+\to 2_1^+)}{B(E2,2_1^+\to 0_1^+)}$	$\frac{B(E2,4_1^+\to 2_1^+)}{B(E2,2_2^+\to 2_1^+)}$
^{100}Pd	Exp.	2.29	2.13	3.29				
	Cal.^{a}	2.03	2.11	3.35				
	Cal.^{b}	1.96	2.16	3.49				
^{102}Pd	Exp.	2.76	2.29	3.79	1.02	3.57	0.46	3.39
	Cal.^{a}	2.04	2.11	3.34	1.04	3.55	0.29	7.80
	Cal.^{b}	1.96	2.18	3.54	1.04	4.00	1.55	1.00
^{104}Pd	Exp.	2.41	2.38	4.05	0.91	18.20	0.60	2.28
	Cal.^{a}	2.01	2.09	3.28	1.03	19.30	0.69	3.00
	Cal.^{b}	1.95	2.17	3.51	1.03	17.97	1.61	1.00
^{106}Pd	Exp.	2.20	2.40	4.06	0.95	37.8	0.98	1.61
	Cal.^{a}	2.01	2.18	3.35	1.04	38.90	0.71	3.00
	Cal.^{b}	1.98	2.16	3.48	1.04	38.80	1.65	1.00
^{108}Pd	Exp.	2.15	2.42	4.08	0.98	96.00	1.54	1.17
	Cal.^{a}	2.02	2.25	3.78	1.06	95.30	0.69	3.10
	Cal.^{b}	1.98	2.25	3.74	1.06	96.66	1.65	1.00
^{110}Pd	Exp.	2.18	2.46	4.25	1.11	69.60	0.98	1.74
	Cal.^{a}	2.08	2.28	3.85	1.08	69.80	0.56	3.95
	Cal.^{b}	1.96	2.28	3.82	1.06	67.25	2.54	1.00
^{112}Pd	Exp.	2.11	2.53	4.45	1.01	50.00		
	Cal.^{a}	2.13	2.30	3.92	1.08	49.90		
	Cal.^{b}	1.95	2.30	3.92	1.07	50.25		
^{114}Pd	Exp.	2.08	2.56	4.51	0.99	33.00		
	Cal.^{a}	2.15	2.31	3.96	1.10	32.30		
	Cal.^{b}	1.97	2.48	4.39	1.12	34.61		
^{116}Pd	Exp.	2.17	2.58	4.58	0.99	31.00		
	Cal.^{a}	2.13	2.28	3.87	1.09	31.00		
	Cal.^{b}	1.96	2.43	4.29	1.11	30.93		

Besides the energy levels, we are also interested in the electromagnetic properties of Pd isotopes, particularly in the electric $E2$ transition. Table 2 gives the comparison between calculated and experimental $B(E2)$ values(ratios) in the eight nuclei. Results obtained in the present work are in good agreement with those of experiments. $B(E2)$ reflects a transition from vibrational nuclei to deformed ones. In the lighter even Pd isotopes, the ratio of the $E2$ transition rates from the 2_2^+ state to the 2_1^+ or the 0_2^+ state is small, it increases with increasing mass number until that of ^{108}Pd where a maximum occurs. It then decreases and reaches a minimum in ^{112}Pd at N=66, exactly in the middle of the N=50–82 shell. Similar results are found for other ratios.

Moreover, to determine if $U(5)$ or $O(6)$ limit is taken can be achieved from the behaviour of low-lying 0^+ states.[4] In the $U(5)$ limit, the Δn_d=1 selection rule allows only a decay from the 0_2^+ state to 2_1^+ state. However, in the $O(6)$ limit, the $\Delta\sigma$=0 and $\Delta\tau$=1 selection rule allows only a decay from the 0_2^+ state to 2_2^+ state. From ^{104}Pd to ^{110}Pd, there exists the decay from 0_2^+ state to 2_1^+ state, but for all Pd isotopes, the decay from 0_2^+ state to 2_2^+ state is not observed. This fact fortifies the 0^+ state to contain intruder configurations. These states can be understood in terms of excitation of pairs of protons over subshell or shell gaps, a description, which is equivalent to the shape coexistence between states of different deformation.

IV. CONCLUSION

We have given, in a schematic way, a detailed study of the energy levels and $E2$ transitions in Pd isotopes in the framework of the interacting boson model. Our theoretical calculations incorporating the recent experimental data in this work indicate that Pd isotopes are also of the $U(5)$ to $SU(3)$ transitional nuclei. More data in future experiments will be important for verifying our conclusion.

ACKNOWLEDGMENTS

We are greatly indebted to Prof. G. L. Long for his continuing interest in this work and his helpful discussions. We would like to thank Prof. D. Ruan for his suggestions. Pak Jae-Yon is grateful to the China Education Ministry for financial support.

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