Vol 10 No 3, March 2001
1009-1963/2001/10(03)/0186-03

Chinese Physics

© 2001 Chin. Phys. Soc.
and IOP Publishing Ltd

QUANTUM STATISTICS OF AN ATOM LASER IN THE PRESENCE OF A STRONG INPUT LIGHT

JING HUI(景 辉)[a], MIAO YUAN-XIU(苗元秀)[b], and HAN YI-ANG(韩毅昂)[c]
[a] *Theoretical Physics Division, Nankai Institute of Mathematics, Nankai University, Tianjin* 300071, *China*
[b] *Center for Advanced Study, Tsinghua University, Beijing* 100084, *China*
[c] *Department of Physics, Zhengzhou University, Zhengzhou* 450052, *China*

(Received 29 June 2000; revised manuscript received 30 November 2000)

Within the framework of quantum dynamical theory, we present a new method to control the quantum statistics of an atom laser by applying a powerful input light. Differing from the case in the rotating wave approximation, the non-classical properties can appear in the output atom laser beam with the evolution of time. By choosing a suitable phase of the input light, it is capable of realizing a steady and brighter output of coherent atom laser.

Keywords: atom laser, non-classical effect
PACC: 0365

I. INTRODUCTION

Recently, there has been much interest in creating an atom laser and exploring its novel properties.[1−3] In 1997, the MIT group first realized a pulsed atom laser;[4] subsequently Sun *et al.*[5] presented a wonderful theoretical analysis for the MIT output coupler of the trapped Bose–Einstein condensate with the rotating wave approximation, which showed that the initially coherent condensate remains in such a state while the output atomic pulse is also in a coherent state. In this paper, we will generalize the work of Sun *et al.* to the case without the rotating wave approximation, which corresponds to a powerful input light, and the virtual photon process can no longer be ignored.[6] In the resonant case, it is shown that the non-classical properties, such as sub-Poisson distribution and quadrature squeezing effect, can exist in the output atomic field. This indicates an interesting optical control on quantum statistics of the output matter wave.

II. MODEL AND SOLUTIONS

We begin by assuming that the atoms have two states, $|1\rangle$ and $|2\rangle$, with the initial condensation occurring in the trapped state $|1\rangle$. State $|2\rangle$, which has different trapping properties and is typically unconfined by the magnetic trap, is coupled to $|1\rangle$ by a coherent light tuned near the $|1\rangle \rightarrow |2\rangle$ transition with frequency ω_0. The Hamiltonian we consider here of a fully coupled photon–atom system can be written as ($\hbar = 1$)

$$H = \omega_0 b_2^\dagger b_2 + \omega_a a^\dagger a + g(ab_2^\dagger + a^\dagger b_2 + ab_2 + a^\dagger b_2^\dagger), \quad (1)$$

where $b_2^\dagger$, $a^\dagger$, b_2 and a are respectively the creation and annihilation operators for the atoms in the untrapped state $|2\rangle$ and the photons with frequency ω_a; g is the coupling constant. Note that the nonlinear atom–atom interaction has been ignored, which is reasonable for a dilute atomic system; and the Hamiltonian (1) is the effective one in the Bogoliubov approximation.[5]

The initial state of the system is supposed to be: $|\psi(0)\rangle = |\varphi\rangle_1 \otimes |\Phi(0)\rangle$ with $|\Phi(0)\rangle = |0\rangle_2 \otimes |\alpha\rangle$. Here $|\varphi\rangle_1$ is a Glauber coherent state of the operator b_1 characterizing the condensed atoms in the trapped state $|1\rangle$, namely, $b_1|\varphi\rangle = \sqrt{N_c}\mathrm{e}^{-\mathrm{i}\phi}|\varphi\rangle$; $|0\rangle_2$ represents that the initial untrapped state $|2\rangle$ is a vacuum state since there is no occupying atom in it; and the initial state of the input optical field is a coherent state $|\alpha\rangle$, i.e., $|\alpha\rangle = D(\alpha)|0\rangle_a$, where $D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$, and $\alpha = |\alpha|\mathrm{e}^{i\theta}$. Here we only need to study the component $|\Phi(0)\rangle = D(\alpha)|0\rangle$, $|0\rangle = |0\rangle_2 \otimes |0\rangle_a$, since the condensed component being initially in a coherent state $|\alpha\rangle_1$ remains in such a state in the Bogoliubov approximation.

The Heisenberg equations about the operators b_2, a, $b_2^\dagger$ and $a^\dagger$ are

$$\mathrm{i}\frac{\partial}{\partial t}\begin{pmatrix} b(t) \\ a(t) \\ b^{\dagger}(t) \\ a^{\dagger}(t) \end{pmatrix} = \begin{pmatrix} \omega_0 & g & 0 & g \\ g & \omega_a & g & 0 \\ 0 & -g & -\omega_0 & -g \\ -g & 0 & -g & -\omega_a \end{pmatrix} \begin{pmatrix} b(t) \\ a(t) \\ b^{\dagger}(t) \\ a^{\dagger}(t) \end{pmatrix}, \tag{2}$$

where we have replaced b_2 by b. By applying the technique of diagonalizing the coefficient matrix, the above equations can be solved exactly. To avoid tedious calculations, we here consider the resonant case, namely, $\omega_a = \omega_0 = \omega$, and $g \ll \omega$, then in the first-order approximation of g/ω, one can obtain the following solutions:

$$\begin{aligned} b(t) &= \gamma_1 b(0) + \gamma_2 b^{\dagger}(0) + \gamma_3 a(0) + \gamma_4 a^{\dagger}(0), \\ a(t) &= \gamma_1 a(0) + \gamma_2 a^{\dagger}(0) + \gamma_3 b(0) + \gamma_4 b^{\dagger}(0), \end{aligned} \tag{3}$$

where

$$\begin{aligned} \gamma_1 &= \frac{1}{2}\left[\left(1+\frac{g}{\omega}\right)\mathrm{e}^{-\mathrm{i}\nu_1 t} + \left(1-\frac{g}{\omega}\right)\mathrm{e}^{-\mathrm{i}\nu_2 t}\right], \\ \gamma_2 &= \frac{g}{4\omega}(\mathrm{e}^{-\mathrm{i}\nu_1 t} - \mathrm{e}^{\mathrm{i}\nu_1 t} - \mathrm{e}^{-\mathrm{i}\nu_2 t} + \mathrm{e}^{\mathrm{i}\nu_2 t}), \\ \gamma_3 &= \frac{1}{2}\left[\left(1+\frac{g}{\omega}\right)\mathrm{e}^{-\mathrm{i}\nu_1 t} - \left(1-\frac{g}{\omega}\right)\mathrm{e}^{-\mathrm{i}\nu_2 t}\right], \\ \gamma_4 &= \frac{g}{4\omega}(\mathrm{e}^{-\mathrm{i}\nu_1 t} - \mathrm{e}^{\mathrm{i}\nu_1 t} + \mathrm{e}^{-\mathrm{i}\nu_2 t} - \mathrm{e}^{\mathrm{i}\nu_2 t}) \end{aligned} \tag{4}$$

and $\nu_1 = \omega + g$, $\nu_2 = \omega - g$.

Using these solutions, we can calculate the average numbers and the fluctuations of the out-state photons, as well as the output atoms. For the out-state photons, the final results are

$$\begin{aligned} < N_a(t) > &= \langle \Phi(0)|a^{\dagger}(t)a(t)|\Phi(0)\rangle \\ &= \frac{1}{2}[1+\cos(2gt)]|\alpha|^2 \end{aligned} \tag{5}$$

and

$$\begin{aligned} < \Delta N_a^2(t) > = &-\frac{g}{4\omega}|\alpha|^2[\sin(4gt) \\ &+ 2\sin(2gt)]\sin(2\theta) + < N_a(t) > . \end{aligned} \tag{6}$$

Similarly, the fluctuation of the output atoms is

$$\begin{aligned} < \Delta N_b^2(t) > = &-\frac{g}{4\omega}|\alpha|^2[\sin(4gt) \\ &- 2\sin(2gt)]\sin(2\theta) + < N_b(t) >, \end{aligned} \tag{7}$$

where $< N_b(t) > = [1-\cos(2gt)]|\alpha|^2/2$.

It is well known that, to determine the statistical properties of a quantum field, we can define the Mandel Q parameter as[6]:

$$Q(t) = \frac{< \Delta N^2(t) >}{< N(t) >} - 1 \begin{cases} < 0: & \text{Sub} - \text{Poisson distribution}, \\ = 0: & \text{Poisson distribution}, \\ > 0: & \text{Super} - \text{Poisson distribution}. \end{cases} \tag{8}$$

From Eqs.(5), (6) and (7), we can derive an interesting result for the Q parameters of the out-state optical field and the output atomic field:

$$\begin{pmatrix} Q_a(t) \\ Q_b(t) \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \sin(2gt)\sin(2\theta), \tag{9}$$

which yields $Q_a(0) = Q_b(0) = 0$, as it should be. For $\sin(2\theta) > 0$ (or $n\pi < \theta < n\pi + \pi/2$, n is an integer), when the evolution of time t_0 satisfies $\sin(2gt_0) > 0$, we have $Q_a(t) < 0$, $Q_b(t) > 0$. This means that the optical field is in a sub-Poisson distribution and that the atomic field is in a super-Poisson distribution; but when $\sin(2gt_0) < 0$, the situation will be quite different: the atomic field has a sub-Poisson distribution. This periodically oscillating behaviour may be expected to be observed in the laboratory. Besides, we can see that the phase θ of the initial optical field has a great impact on the statistical properties of the atom laser. For the case of $\theta = n\pi/2$, the outcoupled atomic pulse will always remain in the coherent state. This suggests a method to realize a steady coherent atom laser through removing the non-classical disturbance.

Now we proceed to calculate the quadrature squeezing of the output atomic field. According to Ref.[6], the field quadratures X_{1a}, X_{1b}, X_{2a} and X_{2b} are defined as

$$\begin{aligned} X_{1a} &= \frac{1}{2}(a + a^{\dagger}), \quad X_{2a} = \frac{1}{2\mathrm{i}}(a - a^{\dagger}), \\ X_{1b} &= \frac{1}{2}(b + b^{\dagger}), \quad X_{2b} = \frac{1}{2\mathrm{i}}(b - b^{\dagger}). \end{aligned} \tag{10}$$

Following Bruzek *et al.*,[7] we introduce the squeezed coefficients

$$S_i = \frac{< (\Delta X_i)^2 > -1/2|}{|< [X_1, X_2] > |/2} \quad (i = 1, 2), \tag{11}$$

then we obtain

$$\begin{aligned} S_{1b}(t) =& 2 < N_b(t) > + 2\mathrm{Re} < b^2(t) > \\ &- (\mathrm{Re} < b(t) >)^2, \\ S_{2b}(t) =& 2 < N_b(t) > - 2\mathrm{Re} < b^2(t) > \\ &- (\mathrm{Im} < b(t) >)^2, \end{aligned} \tag{12}$$

where

$$< b^2(t) >= (\gamma_1\gamma_2+\gamma_3\gamma_4)+\sigma^2; \quad \sigma = \gamma_3\alpha+\gamma_4\alpha^*. \quad (13)$$

It is interesting that a non-zero term exists in the case of $|\alpha| = 0$, i.e.

$$\begin{pmatrix} S_{1b}(t) \\ S_{2b}(t) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \frac{g}{\omega} \sin(2\omega t)\sin(2gt). \quad (14)$$

This means that the squeezing effect occurs even in the vacuum stage. Obviously, for the initial state of the atomic field, Eq.(14) yields $S_{ib}(0) = 0$, which means that there is no squeezing, as it should be. When the evolution of time satisfies: $n\pi < \omega t < n\pi + \pi/2$, we have: $S_{1b}(t) > 0, \quad S_{2b}(t) < 0$. This means that the quadrature component X_{2b} is squeezed. However, when the evolution of time satisfies: $n\pi + \pi/2 < \omega t < n\pi + \pi$, but $n\pi < gt < n\pi + \pi/2$, the squeezing effect will transfer to X_{1b} component.

Ⅲ. CONCLUSION

We have probed the impact of the strong optical field on the properties of the output atom laser. In the resonant case, it is shown that non-classical properties, such as the sub-Poisson distribution and the quadrature squeezing effect, may appear in the output atomic field. This differs from our previous work using a squeezing input light.[8] Our investigations also shed new light on the problem of realizing a steady and brighter coherent atom laser beam.

References

[1] Bloch I, Hänsch T W and Esslinger T 1999 *Phys. Rev. Lett.* **82** 3008

[2] Steck H, Naraschevski M and Wallis H 1998 *Phys. Rev. Lett.* **80** 1

[3] Trippenbach M, Band Y, Edwards M, Doery M and Julienne P S 1999 e-print cond-mat/9906033

[4] Mewes M O, Andrews M R, Kurn D M, Durfee D S, Townsend C G and Ketterle W 1997 *Phys. Rev. Lett.* **78** 582

[5] Sun C P, Zhan H, Miao Y X and Li J 1998 *Commun. Theor. Phys.* **29** 161

[6] Walls D F and Milburn G J 1994 *Quantum Optics* (Berlin: Springer-Verlag)

[7] Buzek V, Barranco A V and Knight P L 1992 *Phys. Rev.* A **45** 6570

[8] Jing H, Chen J L and Ge M L 2000 *Phys. Rev.* A (at press)