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LEVEL DENSITY AND FINITE-TEMPERATURE SPECIFIC HEAT OF NUCLEUS ^{104}Pd UNDER MICROSCOPIC IBM*

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By using the microscopic sdgIBM-$F_{\max}$ approach, the procedure of canonical ensemble average and the saddle point approximation, the thermodynamics of the nucleus is established under microscopic IBM. Calculations of spectrum, level density and finite-temperature specific heat for the nucleus ^{104}Pd are carried out. The calculated values are coincident with the experimental data reported recently. The results predict that the shape phase transition in the ground-state band appears at about $T \approx 0.230$ MeV and the phase transition of the thermal excitation mode takes place at $T \approx 0.630$ MeV for the nucleus ^{104}Pd.

Keywords: spectrum, level density, finite-temperature specific heat, finite-temperature phase transition, ^{104}Pd

PACC: 2100, 2700

I. INTRODUCTION

In recent years, considerable effort has been made to try to develop microscopic methods for the calculation of relatively accurate values of level density as a function of excitation energy. Among these, presently of interest are the so-called shell model Monte Carlo (SMMC) method[1] and the static path approximation (SPA) plus random path approximation (RPA), i.e., SPA+RPA approach. The former may be applied even at very low temperatures although the detail of the spectrum cannot be described. The latter can be used for moderately low and high temperatures ($T > 0.2$ MeV), and the calculation takes less time, yet it cannot offer much detail of the nuclear spectrum. Recently, the level density has been extracted for medium mass nucleus ^{104}Pd through the measurement of proton production in the reaction ^{93}Nb (^{12}C, p) ^{104}Pd.[2] The experimental data are available for the excitation energy (E^*) ranging from 7–25 MeV, or equivalently $T \approx 0.7$–1.5 MeV. It provides a possibility to compare the applicability in various approaches. In Ref.[3], the authors calculated level density as a function of excitation energy E^* for nucleus ^{104}Pd, using the SPA+RPA model with a quadrupole–quadrupole interaction model Hamiltonian

$$H = H_0 + \frac{1}{2}\chi \sum_{\mu=-2}^{2} (Q_\mu)^2.$$

The calculated energy values are in close agreement with those reported in Ref.[2].

Based on the experience of describing heavy and middle mass nuclei in microscopic IBM,[4–6] thermodynamics is established by using the microscopic sdgIBM-$F_{\max}$ approach,[4] the procedure of canonical ensemble average[7] and the saddle point approximation.[8] The level density of the nucleus ^{104}Pd is calculated. The physical interpretation of model parameters is obvious. The finite-temperature specific heat is also calculated. It is predicted theoretically that the shape phase transition in the ground-state band appears at about $T \approx 0.230$ MeV and the phase transition of the thermal excitation mode takes place at $T \approx 0.630$ MeV for nucleus ^{104}Pd. In this approach, the calculation is relatively simple and more information is obtained.

II. THEORETICAL APPROACH

In order to obtain more information about the spectrum, the microscopic sdgIBM-$F_{\max}$ approach was used and the g-boson number $N_g \leq 2$ was taken.

According to the canonical ensemble theory, as

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long as the nuclear spectrum at zero temperature has been obtained, the partition function can be calculated; and other thermodynamic functions may be calculated in terms of thermodynamic relations, then the thermodynamics of the nucleus may be established.[7] Specific heat has the following form:

$$C_{\rm N}(\beta) = \beta^2(< E^2 > - < E >^2), \qquad (1)$$

with

$$\begin{aligned} < E > =& \Sigma_{j,\nu(j)}(2j+1)E_{j,\nu(j)} \\ & \cdot \exp(-\beta E_{j,\nu(j)})/Z(\beta), \\ < E^2 > =& \Sigma_{j,\nu(j)}(2j+1)E^2_{j,\nu(j)} \\ & \cdot \exp(-\beta E_{j,\nu(j)})/Z(\beta), \\ Z(\beta) =& \Sigma_{j,\nu(j)}(2j+1) \\ & \cdot \exp(-\beta E_{j,\nu(j)}), \end{aligned}$$

where $\beta = 1/T, \nu(j)$ denotes the state in each irreducible representation with angular momentum j.

It is well known that the state density is the inverse Laplace transformation of $Z(\beta)$. According to the saddle point approximation,[8] the level density $\rho(E)$ is approximately

$$\rho(E) = [2\pi\beta^{-2}C_{\rm N}(\beta)]^{-1/2}\exp(S(E))/(2 < j > +1), \qquad (2)$$

with

$$\beta^{-2}C_{\rm N}(\beta) = -{\rm d}E/{\rm d}\beta, \qquad (3)$$

$$S(E) = \beta E + \log(Z(\beta)). \qquad (4)$$

Ⅲ. CALCULATED RESULTS AND DISCUSSIONS

The starting point of the microscopic sdgIBM-$F_{\rm max}$ approach includes shell model configurations, effective interactions between valence nucleons. For the nucleus ^{104}Pd, the valence nucleon configurations used in the calculation are $(2{\rm p}_{3/2}, 1{\rm f}_{5/2}, 2{\rm p}_{1/2}, 1{\rm g}_{9/2})^{\overline{4}}$ $(2{\rm d}_{5/2}, 1{\rm g}_{7/2}, 3{\rm s}_{1/2}, 2{\rm d}_{3/2}, 1{\rm h}_{11/2})^8$, where $\overline{4}$ indicates four proton-holes. Single-particle levels, given in Table 1, are taken from Ref.[4]. The nucleon–nucleon effective interactions are pairing force, quadrupole pairing force, hexadecapole force and quadrupole–quadrupole force between identical nucleons, (denoted respectively by $g_0^{(\sigma)}$, $G_2^{(\sigma)}$, $G_4^{(\sigma)}$ and $K^{(\sigma)}$,σ=n, p). The neutron–proton interaction $K^{(\rm np)}$ is of the quadrupole–quadrupole type. The strength parameters, given in Table 2, are also taken from Ref.[4], but slightly changed to obtain a close agreement with the observed levels of higher spin states.

Table 1. Single-particle levels[4] (MeV).

$n\ l\ j$	$2{\rm d}_{5/2}$	$1{\rm g}_{7/2}$	$3{\rm s}_{1/2}$	$2{\rm d}_{3/2}$	$1{\rm h}_{11/2}$	$n\ l\ j$	$2{\rm p}_{3/2}$	$1{\rm f}_{5/2}$	$2{\rm p}_{1/2}$	$1{\rm g}_{9/2}$
Neutron	0.00	0.90	1.90	2.30	3.30	Proton	0.00	0.78	1.11	3.80

Table 2. Effective interacting parameters (MeV).

$g_0^{(\rm n)}$	$G_2^{(\rm n)}$	$G_4^{(\rm n)}$	$K^{(\rm n)}$	$g_0^{(\rm p)}$	$G_2^{(\rm p)}$	$G_4^{(\rm n)}$	$K^{(\rm p)}$	$K^{(\rm np)}$
0.0615	0.0815	0.0000	0.0090	0.0575	0.0765	0.0000	0.0090	0.0120

The calculated spectrum, given in Fig.1, shows that the theoretical levels are in close agreement with the data reported recently.[9–11] We note that the experimental level spacing between the states with I=10 and I=12 is narrow, and our calculated result does not reproduce the spacing. The reason is that a pair of neutrons or protons aligns along the rotating axis for the state with I=12, at the time the first band crossing is observed. In the boson–fermion model, this phenomenon is described by breaking a boson into a pair of unpaired nucleons. In our present approach, however, the broken boson is not taken into account, thus the energy spacing increases continuously.♯ The specific heat curves $C_{\rm N}(\beta)$ and $C_{\rm g}(\beta)$ at finite temperatures are plotted in Fig.2. In the light of thermodynamics,[7] the appearance of the peak of the specific heat $C_{\rm g}$ curve indicates that the shape phase transition in the ground-state band takes place at $T \approx$0.230 MeV, and the whole shape phase transition at a finite-temperature takes place at $T \approx$0.630 MeV for nucleus ^{104}Pd. This is coincident with the abrupt change of level density which appears experimentally at $T \approx$0.70 MeV.[3]

♯The authors wish to thank the referee for pointing this out to us.

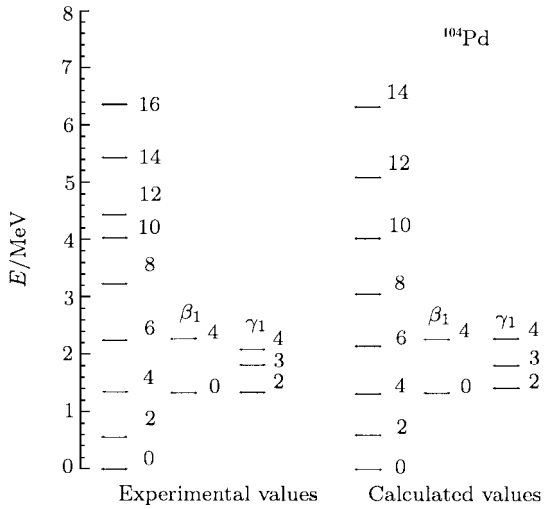


Fig.1. Comparison of the calculated levels in ^{104}Pd with the experimental ones.[9]

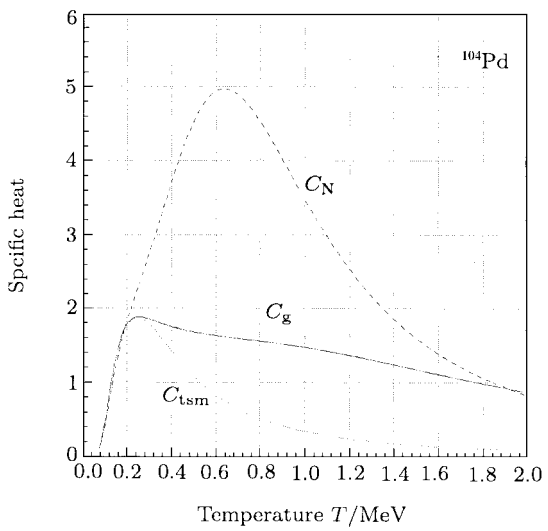


Fig.2. Finite-temperature specific heat curve of ^{104}Pd.

To study how the density ρ depends on the exited energy E^*, when the energy-level density is calculated according to Eqs.(2)–(4), we transform E into $E^* = N_b E + E_c$, and the obtained result is shown in Fig.3. One can see that the calculated level density curve $\log[\rho(E^*)]$ is close to the values reported recently, particularly in the range of low excitation energies. In the range of higher excitation energies, the calculated level density is slightly small, since the states with negative-parity cannot be calculated by the sdgIBM-F_{max} approach. It is worth emphasizing that the variables in the above transformation have an apparent physical interpretation. E^* is the energy absorbed by the whole nucleus, E is that by the excited single boson, E_c is that by unexcited bosons and inertia core; and N_b is the average number of excited bosons. For ^{104}Pd, E_c=5.70 MeV, N_b=3.68. Owing to the restriction of the saddle point method, it is impossible to calculate the level density at low temperatures ($T \approx$0.230 MeV), and as a result the prediction of the above shape phase transition in the ground-state band is verified by the result of an abrupt change of level density.

On the other hand, according to the two-state model (TSM) theory,[12,13] the Schottky peak of specific heat in the ground-state band is described by

$$C_{tsm} = N(E/T)^2 \frac{\exp(E/T)}{(1+\exp(E/T))^2}.$$

Here N, E and T are, respectively, the particle number, the energy gap between the two low-lying states and the temperature in this system. There are two states in C_{tsm}=0.0. One of them is a high-order state corresponding to T=0 MeV; the other is the thermal random state corresponding to the high-temperature state. Fitting the Schottky peak, we obtain N=3.78 and E=0.595 MeV. They are in close agreement with $< N_b >$=3.60, E=0.591 MeV in the sdgIBM-F_{max} approach, and N_b=3.68 in the above calculation of $[E^* - \mathrm{Log}\rho(E)]$. The temperature in the thermal random state (C_{tsm}=0.01) is near 6.20 MeV; it is close to the predicted value $T \approx$6.058 MeV in Ref.[14].

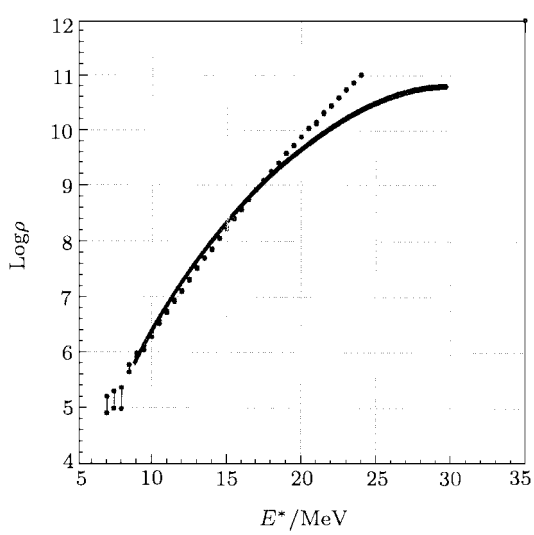


Fig.3. Comparison of experimental values $(\cdots)$[3] with the calculated values (—) of level density for ^{104}Pd.

IV. CONCLUSION

In summary, starting from the microscopic sdgIBM-F_{max} approach, the procedure of the canonical ensemble average and the saddle point approximation, the thermodynamics of the nucleus under microscopic IBM has been established. We reproduce

successfully the level density and finite-temperature specific heat of nucleus ^{104}Pd and predict theoretically that the shape phase transition in the ground-state band appears at $T \approx 0.230$ MeV and the phase transition of thermal excitation mode takes place at $T \approx 0.630$ MeV for this nucleus. The latter has been already supported by experiments.

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