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ADAPTIVE NONLINEAR FEEDBACK CONTROL OF CHAOTIC SYSTEMS BASED ON REDUCED PARAMETER QUADRATIC PREDICTORS*

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An adaptive nonlinear feedback-control method is proposed to control continuous-time chaotic dynamical systems, where the adaptive nonlinear controller acts on only one-dimensional error signals between the desired state and the observed chaotic state of a system. The reduced parameter adaptive quadratic predictor used in adaptive feedback cancellation of the nonlinear terms can control the system at any desired state. Computer simulation results on the Lorenz system are shown to demonstrate the effectiveness of this feedback-control method.

Keywords: chaotic control, adaptive nonlinear predictor, adaptive nonlinear feedback control
PACC: 0545

I. INTRODUCTION

Control of chaos is one of the topics which has gained much importance and attention in physics and engineering publications in last few years.[1] Motivated by the Ott–Grebogi–Yorke (OGY) parametric variation-control method,[2,3] various feedback-control approaches and filtering control methods have been developed.[4−10] Though different in control strategy, all the methods basically require the information of the system state (or measurement) to construct the control forces. A typical approach for feedback control is to start with a mathematical model of the underlying system, which can be accurate or approximate. The parametric adaptive control[4−6] is based on the correspondence between the global behaviour (obtained from observations of the system's variables) and a set of parameters manipulated by the observer. One can compare the observed behaviour with the desired one or "goal" behaviour and then correct it through the system parameters. However, there are some important things which have not been clear up to now, such as the criterion of the convergence for selecting the control stiffness and the magnitude of the deviation from the unstable periodic point to be stabilized. The parametric adaptive method may fail in control if the control stiffness and the magnitude of the deviation are not selected properly. The time-series-based identification feedback-control methods have proven effective for stabilizing an unstable system while tracking a reference input or rejecting uncertain disturbances, etc, even for chaotic dynamics.[7−10] Briefly, the attempt of the time-series-based identification feedback control is to find an identification scheme based on a finite set of time series data obtained from a chaotic system, and then to derive a design method capable of controlling the underlying chaotic dynamics. This feedback-control method does not directly employ the standard delay-coordinate technique. Instead, it takes a different approach to reconstruct a predicting model. Owing to the lack of an appropriate nonlinear predictor, a challenge for the conventional feedback-control strategy is whether or not it can also use time series data for controller design, particularly for chaotic systems. It is difficult to identify chaotic systems off-line via nonlinear predictors, because the dynamics of the chaos is very fast. From a practical point of view, the existing results are not satisfactory for the controller design.

More recently, second-order Volterra adaptive filters have been successfully applied to control and predict low-dimensional chaotic time series.[11,12] One main point of this paper is to apply the reduced parameter adaptive quadratic filter to design an adaptive nonlinear feedback controller for controlling the chaotic systems at any desired state on-line. In section Ⅱ, we briefly review the reduced parameter adaptive quadratic filtering predictor, and the adaptive nonlinear feedback-control method is described in section Ⅲ. In section Ⅳ, the effectiveness of this adaptive non-

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linear feedback-control methodology is illustrated by its implementation to the Lorenz system. Section V is devoted to a brief summary and discussion.

II. A REDUCED PARAMETER ADAPTIVE QUADRATIC FILTERING PREDICTOR

The adaptive second-order Volterra filters have been investigated for modelling and prediction of chaotic time series in Refs.[11] and [12]. One of the disadvantages of the second-order Volterra predictive filters is that the number of parameters to be estimated is typically large. A reduced parameter quadratic predictor that can significantly reduce the number of parameters while maintaining approximations of nonlinearity similar to the conventional second-order Volterra filters can be used to predict low-dimensional chaotic time series. To control the speed of convergence and the stable operation of the adaptive filter, the variable step size dependent on predictive error normalized least-mean-squares (LMS) algorithm is proposed to update the coefficients of the proposed predictive filter. The reduced parameter quadratic filtering predictor is defined as follows:

$$\begin{aligned}\hat{x}_{k+1} =&(h_{1,0}+\sum_{i=1}^{m}h_{1,i}x(k-i))\\ &\times(h_{2,0}+\sum_{i=1}^{m}h_{2,j}x(k-j))+e(k),\end{aligned}\quad(1)$$

where $h_{1,i}$ and $h_{2,j}, i,j=0,1,\cdots,m$, are two branch finite impulse response (FIR) filter coefficients. The total parameters needed in Eq.(1) are equal to $2m+2$. In this case, we have

$$\begin{aligned}\hat{x}_{k+1} =&h_{1,0}h_{2,0}+\sum_{i=1}^{m}[h_{2,0}h_{1,i}+h_{1,0}h_{2,i}]x(k-i)\\ &+\sum_{i=1}^{m}\sum_{j=1}^{m}h_{1,i}h_{2,j}x(k-i)x(k-j).\end{aligned}\quad(2)$$

From Eq.(2), we can see that the model in Eq.(1) is exactly similar to the second-order Volterra filtering predictor under some conditions, although the number of parameters required by each model can be quite different. For instance, consider the case where $m=8$. Up to 45 parameters are needed in Eq.(2), whereas only 18 parameters are needed in Eq.(1). Obviously, the latter situation is much easier to handle for both modelling and forecasting procedures. There is a trade-off between simplicity and generality. The structure of a reduced parameter quadratic predictor is depicted in Fig.1. Each branch of the proposed predictor produces an output that corresponds to one of the linear FIR filters in Eq.(2).

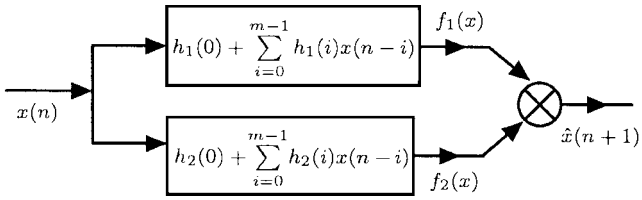


Fig.1. The structure of a reduced parameter quadratic filtering predictor.

Based on the reduced parameter quadratic filter, we may readily get the normalized LMS (NLMS) algorithm for updating the coefficients of each branch FIR filter. The input vector of the filter is defined as $X_k=[1,x(k-1),\cdots,x(k-m)]^{\mathrm{T}}$. The corresponding weights are arranged into the FIR filter coefficient vector $H_1=[h_{1,0},h_{1,1},\cdots,h_{1,m}]^{\mathrm{T}}$ and $H_2=[h_{2,0},h_{2,1},\cdots,h_{2,m}]^{\mathrm{T}}, f_1(x)=H_1^{\mathrm{T}}X_k, f_2(x)=H_2^{\mathrm{T}}X_k$. With the notations defined above, the output of Eq.(2) may be written as

$$\hat{x}_{k+1}=f_1(x)f_2(x)=X_k^{\mathrm{T}}H_1(k)H_2^{\mathrm{T}}(k)X_k.\quad(3)$$

Then, the resulting predictive error is as follows:

$$\begin{aligned}e(k)&=x_{k+1}-X_k^{\mathrm{T}}H_1(k)H_2^{\mathrm{T}}(k)X_k\\ &=x_{k+1}-f_1(x)f_2(x).\end{aligned}\quad(4)$$

Recall that the LMS weight update is

$$\begin{aligned}H_1(k+1)&=H_1(k)-\frac{1}{2}\mu_1\frac{\partial e^2(n)}{\partial H_1(k)}\\ &=H_1(k)+\mu_1e(k)X_kH_2^{\mathrm{T}}(k)X_k\\ &=H_1(k)+\mu_1e(k)Y_{1,k},\end{aligned}\quad(5)$$

$$\begin{aligned}H_2(k+1)&=H_2(k)-\frac{1}{2}\mu_2\frac{\partial e^2(n)}{\partial H_1(k)}\\ &=H_2(k)+\mu_2e(k)X_kH_2^{\mathrm{T}}(k)X_k\\ &=H_2(k)+\mu_2e(k)Y_{2,k},\end{aligned}\quad(6)$$

where $Y_{1,k}=X_kH_2^{\mathrm{T}}(k)X_k=X_kf_2(x), Y_{2,k}=X_kH_1^{\mathrm{T}}(k)X_k=X_kf_1(x), f_1(x)$ and $f_2(x)$ can be viewed as constant as the known output of each branch in Fig.1 at time n. When $|f_i(x)|>1$, the LMS algorithm experiences a gradient noise amplification problem. It has been shown that the NLMS algorithm, exhibiting a rate of convergence that is potentially faster

than that of the standard LMS algorithm for both uncorrelated and correlated input data, can overcome this difficulty. The NLMS weight update is

$$H(k+1) = H(k) + \mu \frac{e(k)}{1+ \parallel X_k \parallel^2} X_k. \qquad (7)$$

Similarly, we can modify the recursion of Eqs.(6) and (7) as follows:

$$\begin{aligned} H_1(k+1) &= H_1(k) + \mu_1 \frac{e(k)}{\mu_1+ \parallel Y_{1,k} \parallel^2} Y_{1,k} \\ &= H_1(k) + \mu_1 \frac{e(k)}{\mu_1 + X_k^{\mathrm{T}} X_k [f_2(x)]^2} X_k f_2(x), \end{aligned} \qquad (8)$$

$$\begin{aligned} H_2(k+1) &= H_2(k) + \mu_2 \frac{e(k)}{\mu_2+ \parallel Y_{2,k} \parallel^2} Y_{2,k} \\ &= H_2(k) + \mu_2 \frac{e(k)}{\mu_2 + X_k^{\mathrm{T}} X_k [f_1(x)]^2} X_k f_1(x), \end{aligned} \qquad (9)$$

where μ_1 and μ_2 are positive step size values for the update equations. We can argue in a heuristic manner that a selection of the step size in a range $0 < \mu_1, \mu_2 < 2$ should result in the stable operation of the system. Simulation results on the Lorenz system identification using the above reduced parameter quadratic filtering predictor show that this model is very effective, as can be seen from the results given in Fig.2.

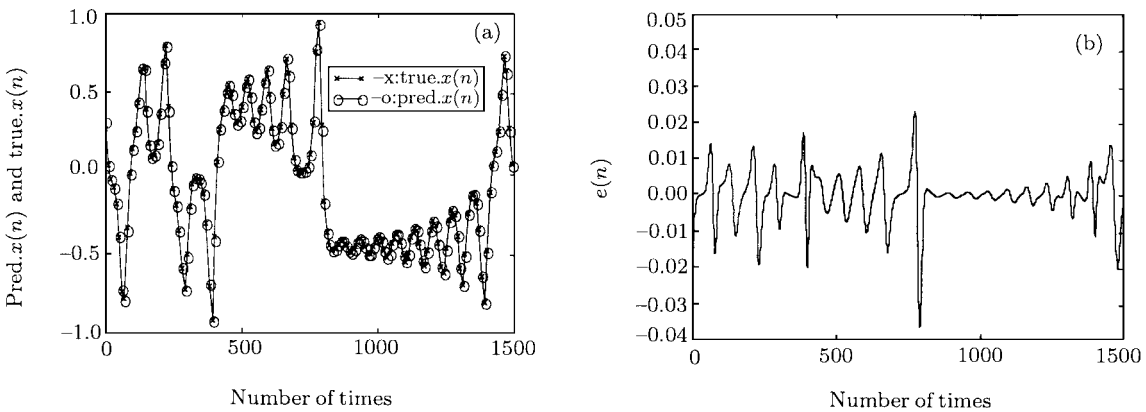


Fig.2. Lorenz x-component time series and its single-step predictions using the reduced parameter quadratic filtering predictor. The accuracy of prediction: MSE=0.0068.

Ⅲ. ADAPTIVE NONLINEAR FEEDBACK CONTROLLER DESIGN

This section discusses how the reduced parameter adaptive quadratic filters may be used to adaptively control a system at any desired state, based only on an identified (quadratic) filtering model in Eq.(1).

Consider a chaotic dynamical system S described by an equation of the time series type

$$\boldsymbol{x}_{k+1} = \boldsymbol{F}(\boldsymbol{x}_k). \qquad (10)$$

Here $\boldsymbol{x}_k$ is an n-dimensional state vector containing the dynamical variables. The vector $\boldsymbol{x}_k$ will be referred to as the state of the system at time k, and the n-dimensional space where $\boldsymbol{x}_k$ lives is called the state or phase space. Based on the adaptive control and feedback-control ideas, the proposed adaptive nonlinear feedback-control method directly uses the reduced parameter adaptive quadratic predictor in Eq.(1) to control the chaotic system at any desired state on-line, rather than the controller based on the reconstructed model off-line via nonlinear predictors. Given a target (unstable) periodic orbit (or equilibrium), $\{\tilde{\boldsymbol{x}}_k\}$, the objective here is to design a feedback controller of the form given by Eq.(1)

$$\boldsymbol{u}_{k+1} = \boldsymbol{A}_k(\boldsymbol{x}_k - \tilde{\boldsymbol{x}}_k) \times \boldsymbol{B}_k(\boldsymbol{x}_k - \tilde{\boldsymbol{x}}_k), \qquad (11)$$

where $\boldsymbol{x}_k - \tilde{\boldsymbol{x}}_k$ will be normalized so that it lies between $+1$ and -1. Both $\boldsymbol{A}_k$ and $\boldsymbol{B}_k$ are quadratic filtering coefficient matrices to be determined such that the trajectory of the controlled system

$$\boldsymbol{x}_{k+1} = \boldsymbol{F}(\boldsymbol{x}_k) + \boldsymbol{u}_k \qquad (12)$$

approaches the target $\tilde{\boldsymbol{x}}_k$ in the sense

$$\lim_{k \to \infty} \parallel \boldsymbol{x}_k - \tilde{\boldsymbol{x}}_k \parallel = 0, \qquad (13)$$

where $\parallel \cdot \parallel$ is the Euclidean norm. To be more general, suppose that the target periodic orbit $\tilde{\boldsymbol{x}}_k$ satisfies

the following dynamical system:

$$\tilde{\boldsymbol{x}}_{k+1} = g(\tilde{\boldsymbol{x}}_k; k), \tag{14}$$

where $g(\cdot)$ is a nonlinear function, not necessarily the same as the identified system function $F(\cdot)$. Thus, the proposed method of control consists of a combination of the set of equations (1), (8), (9) and (11), (12), or

$$\boldsymbol{x}_{k+1} = \boldsymbol{F}(\boldsymbol{x}_k) + u_k, \tag{15}$$

$$\boldsymbol{X}_k = (\boldsymbol{x}_k - \tilde{\boldsymbol{x}}_k)/M, \tag{16}$$

$$\boldsymbol{u}_{k+1} = \boldsymbol{A}_k \boldsymbol{X}_k \times \boldsymbol{B}_k \boldsymbol{X}_k, \tag{17}$$

$$\boldsymbol{e}(k) = \boldsymbol{X}_k - \boldsymbol{x}_k, \tag{18}$$

$$\boldsymbol{A}_{k+1} = \boldsymbol{A}_k + \mu_1 \frac{\boldsymbol{e}(k)\boldsymbol{X}_k}{\mu_1 + \boldsymbol{X}_k^{\mathrm{T}} \boldsymbol{X}_k [f_2 \boldsymbol{X}_k)]^2} f_2(\boldsymbol{X}_k), \tag{19}$$

$$\boldsymbol{B}_{k+1} = \boldsymbol{B}_k + \mu_2 \frac{\boldsymbol{e}(k)\boldsymbol{X}_k}{\mu_2 + \boldsymbol{X}_k^{\mathrm{T}} \boldsymbol{X}_k [f_1 \boldsymbol{X}_k)]^2} f_1(\boldsymbol{X}_k), \tag{20}$$

$$\boldsymbol{u}_{k+1} = \boldsymbol{M} \times \boldsymbol{u}_{k+1}, \tag{21}$$

for the system and the control, respectively, M is the observed value of the maximum system state before control. Finally, it should be noted that, similar to the time-series-based identification feedback-control methods for chaos control, the underlying dynamical system can be of either discrete time or continuous time, for which the above designed controller can be applied in the same way as the feedback-control iteration scheme.

IV. COMPUTER SIMULATIONS

In this section, the Lorenz system is chosen to demonstrate the abilities of the reduced parameter adaptive quadratic predictor based on the adaptive nonlinear feedback controller designed above. The Lorenz system equation is given by

$$\begin{aligned} \dot{x} &= 10(y - x), \\ \dot{y} &= 48x - xz - y, \\ \dot{z} &= xy - 8z/3. \end{aligned}$$

In the simulations, the time series of the observed system states are only generated via the fourth-order Runge–Kutta scheme with a fixed time step of 0.01. The numerical trajectories are integrated from an initial point, (50.1, 10.7, 63.0) and the adaptive nonlinear feedback controller is added at time $n = 500$. The normalized error signal factor M is selected to be $M = 30$, the adaptive step sizes μ_1 and μ_2 for the update equations (19) and (20) are selected to be $\mu_1 = 1.5$ and $\mu_2 = 0.5$. Three typical cases were studied in the control simulations.

Case 1 Controlling the chaotic system from the chaotic attractor to approach an unstable equilibrium point, which is (0,0,0,0) in the simulation. Fig.3 shows the tracking results of the control process on-line when driving the chaotic trajectory to approach the target.

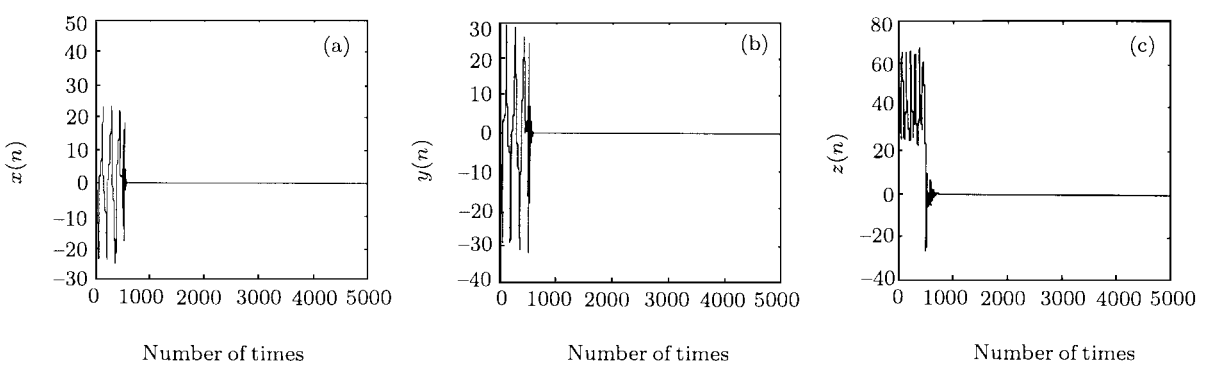


Fig.3. The Lorenz system controlled at unstable equilibrium point (0,0,0) with the adaptive regulating command switched at time n=500: (a) state x; (b) state y; (c) state z.

Case 2 Controlling the chaotic trajectory at a periodic orbit, in which the unstable periodic orbit of the chaotic system is selected to be

$$\dot{x}^* = y^*, \dot{y}^* = \sin(x^*), z^* = 30.$$

The tracking results of the control process on-line are shown in Fig.4.

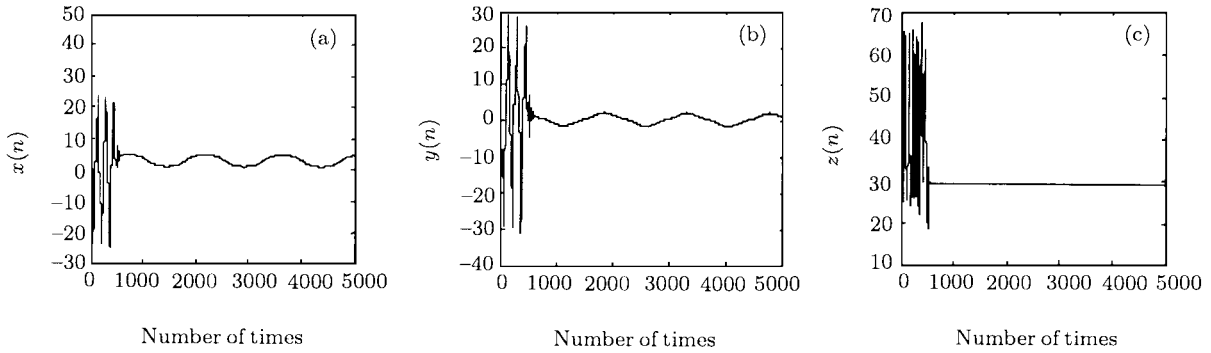


Fig.4. The Lorenz system controlled at an unstable periodic orbit with the adaptive regulating command switched at time n=500: (a) state x; (b) state y; (c) state z.

Case 3 The reference model to be followed is selected to be a circle:

$$x^* = 10\sin(\pi t), y^* = 5 + 5\cos(\pi t), z^* = 50.$$

Figure 5 shows the tracking results of the control process on-line when driving the chaotic trajectory to approach the reference target.

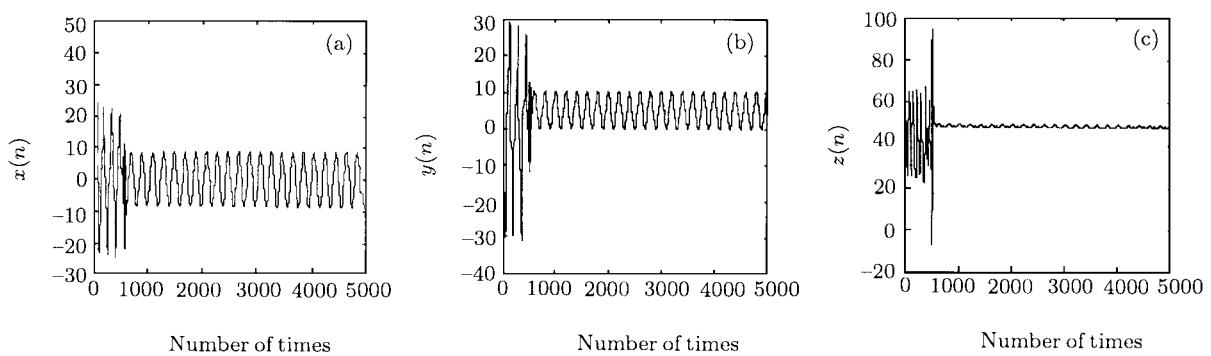


Fig.5. The Lorenz system followed an external reference model with the adaptive regulating command switched at time n=500: (a) state x; (b) state y; (c) state z.

From the results listed above, we note that the adaptive nonlinear controller is independent of the chaotic dynamics, because this controller is based only on the reduced parameter adaptive quadratic predictors, in which the control is implemented by feedback predicting cancellation of tracking error signals. This adaptive controller can quickly control the chaotic dynamic system at any desired state on-line without requiring a precise mathematical model of the underlying dynamical system.

V. CONCLUSIONS

An adaptive nonlinear feedback-control method for the chaotic system has been developed, using only one-dimensional tracking error signals between the desired state and the chaotic systematic state. This successful design has made a point that can be adaptively carried out on-line without requiring a precise mathematical model of the underlying dynamical system. This design methodology indeed combines the merits of the two different, well-accepted approaches, the parametric adaptive control and the time-series-based feedback-control techniques, and can achieve automatic real-time control of chaos on-line with guaranteed controllability, stability and tracking performance. Now we address ourselves to the extension of this work to the adaptive nonlinear feedback control of unknown chaotic systems and coupled chaotic systems.

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