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DYNAMICAL BEHAVIOUR OF THE SUPERCURRENT IN MESOSCOPIC JOSEPHSON JUNCTIONS

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In this paper we use a numerical method to study the quantum effect of dynamical localization on the supercurrent in a mesoscopic Josephson junction driven by a periodically external electromagnetic field. We find that the charge is localized effectively when the ratio of the field magnitude and the field frequency equal some critical values, and in these cases the amplitude of the oscillation of the supercurrent will remain constant.

Keywords: mesoscopic Josephson junction, dynamical localization, supercurrent
PACC: 7450, 7215R

I. INTRODUCTION

In the past few years, mesoscopic Josephson junctions have been studied extensively.[1] In such systems two effects come into play: the quantization of charge, and the quantum motion of such charges. Many new quantum phenomena have been observed, such as the Coulomb blockade of Cooper pair tunnelling and Bloch oscillations. In order to exploit the fully quantum-mechanical nature of a mesoscopic Josephson junction, an ultra-small capacitance weak link biased with a classical voltage source, and also coupled to a non-classical microwave, has been considered,[2] where some interesting results have been obtained.[2−5]

In this paper we will discuss the quantum effect of the dynamical localization in a mesoscopic Josephson junction system in the presence of a periodically external electromagnetic field. In some approximations the mean-square charge can be solved analytically and some results have been obtained,[6] but even so it is still hard to obtain the supercurrent analytically. Therefore we want to use a numerical method to find the supercurrent and the mean-square charge in a charge representation. According to theory, the dynamical localization should appear, under appropriate conditions, in periodically driven quantum systems. We find that the charge is localized effectively when the ratio of the field magnitude and the field frequency equal some critical values, and in these cases the amplitude of the oscillation of the supercurrent will remain constant.

II. MODEL AND METHOD

Now we consider a circular superconducting ring with a mesoscopic Josephson junction driven by an ideal external periodic field. The Hamiltonian of the system is (in units of $\hbar = k_B = c = 1$)

$$H = (q+Q)^2/(2C) + E_J(1-\cos\theta) + (\phi - \Phi)^2/(2L), \tag{1}$$

where E_J is the Josephson coupling constant. The phase difference across the junction θ (equivalent to ϕ, the total flux through the ring, and $\theta = 2e\phi$), and the charge tunnelling through the junction q are a pair of quantum mechanical conjugate variables, which give the commutation relation $[\theta, q] = 2ei$. Q is the external charge, which depends on the coupling between the junction and the external field, and Φ is the external flux. L is the inductance of the ring and the charge $2e$ comes from the Cooper pairs. We suppose that the mesoscopic Josephson junction has a very small capacitance C, a large L, and operates at a very low temperature.

From Eq.(1) we obtain

$$H = \frac{1}{2C}q^2 + \frac{Q}{C}q + \frac{Q^2}{2C} + E_J - E_J\cos\theta + \frac{\phi^2}{2L} - \frac{\phi\Phi}{L} + \frac{\Phi^2}{2L}. \tag{2}$$

For simplicity, we only consider the voltage across the junction, $V = V_0\sin\omega t$, $\Phi = V_0\cos\omega t/\omega$, $Q =$

$CV_0 \sin \omega t$, and we let $\omega = \sqrt{1/(LC)}$, so $Q^2/(2C) + \Phi^2/(2L) + E_J$ is a constant, which does not affect the dynamic behaviour of the system and can be omitted. Then Eq.(2) can be rewritten, for large L as

$$H = q^2/(2C) + qQ/C - E_J \cos\theta. \tag{3}$$

We assume that the wavefunction in this system is $|\varphi\rangle$, then the Schrödinger equation can be written as ($\hbar = 1$)

$$i\frac{\mathrm{d}|\varphi\rangle}{\mathrm{d}t} = H|\varphi\rangle. \tag{4}$$

When the transport dimension reaches a characteristic dimension, the charge carrier is in a coherence length, we must use quantum mechanics to discuss the problem and we need to consider the charge discreteness. In order to take into account the discreteness of electric charges, we should ensure that the eigenvalues of the electric charge take discrete values

$$q|n\rangle = 2ne|n\rangle, \tag{5}$$

where $n \in Z$ (set of integers) and $|n\rangle$ is the eigenstate of discrete charge. We expand $|\varphi\rangle$ in the eigenstate $|n\rangle$

$$|\varphi\rangle = \sum_n C_n|n\rangle. \tag{6}$$

Introducing new basis states defined by

$$|\theta\rangle = \sum_{n=-\infty}^{\infty} \mathrm{e}^{im\theta}|m\rangle, \tag{7}$$

we have conversely

$$|m\rangle = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\theta \mathrm{e}^{-im\theta}|\theta\rangle. \tag{8}$$

So we have

$$\mathrm{e}^{i\theta}|m\rangle = |m-1\rangle, \tag{9}$$

$$\mathrm{e}^{-i\theta}|m\rangle = |m+1\rangle, \tag{10}$$

and then with Eqs.(3)–(10) we obtain

$$i\sum_n \dot{C}_n|n\rangle = \sum_n C_n\left(\frac{2n^2e^2}{C} + \frac{2neQ}{C}\right)|n\rangle - \frac{E_J}{2}\sum_n C_n(|n-1\rangle + |n+1\rangle). \tag{11}$$

With the help of $\langle n|m\rangle = \delta_{n,m}$, we obtain from Eq.(11) the following evolution equation for the amplitudes $C_n(t)$

$$i\frac{\mathrm{d}C_n}{\mathrm{d}t} = \left(\frac{2n^2e^2}{C} + \frac{2ne}{C}Q\right)C_n - \frac{E_J}{2}(C_{n+1} + C_{n-1}). \tag{12}$$

III. RESULTS AND DISCUSSION

In this paper we only consider the case that $Q \rangle\rangle e$, and we define the probability propagator $\varphi_n = |C_n|^2$, so the mean-square charge is

$$\langle n^2\rangle = \sum_n \varphi_n n^2 = \sum_n |C_n|^2 n^2. \tag{13}$$

From Eq.(4) we can obtain the supercurrent operator

$$I = -\partial_t q = -i[H, q] = I_{cr}\sin\theta, \tag{14}$$

where $I_{cr} = 2eE_J$.

It is easy to obtain the following expressions from Eqs.(6), (9) and (10)

$$\langle I(t)\rangle = \langle\varphi|I|\varphi\rangle = \langle\varphi|I_{cr}\sin\theta|\varphi\rangle = \frac{I_{cr}}{2i}\sum_n (C_n^* C_{n+1} - C_n^* C_{n-1}). \tag{15}$$

In this paper, we want to obtain the supercurrent $\langle I(t)\rangle$ by means of a numerical method and to discuss the effect of dynamical localization on supercurrent in a mesoscopic Josephson junction. In solving Eq.(12) we use eigenstates $|n\rangle$ (n=0,±1, ±2,$\cdots$,±100), and let C_n=0 when $n < -100$ or $n > 100$. We take the initial state at $n = 0$, i.e. $C_n(0) = \delta_{n,0}$, and let $Q = Ae\sin(\omega t)$ with A=30 (dimensionless). In numerical calculations we have scaled other parameters ω, E_J, etc, with $E_c = 2e^2/C = 1$, and we let $E_J/E_c = 0.12$. Actually, in this way t is also scaled. In numerical calculations, $\sum_n |C_n|^2$ always equals 1. By changing ω we can obtain figures of the mean-square charge $\langle n^2\rangle$ and the supercurrent $\langle I(t)\rangle$ (see Figs.1 and 2).

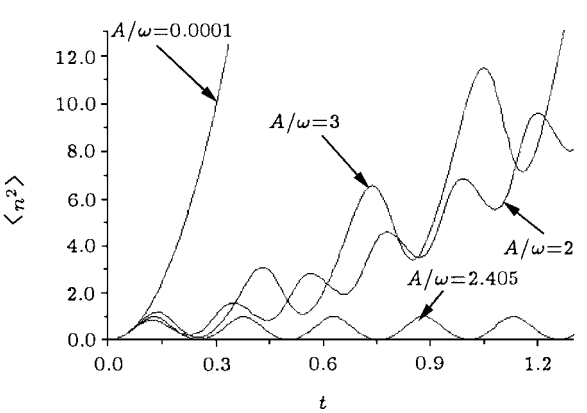


Fig.1. The mean-square charge at different ratios of A/ω.

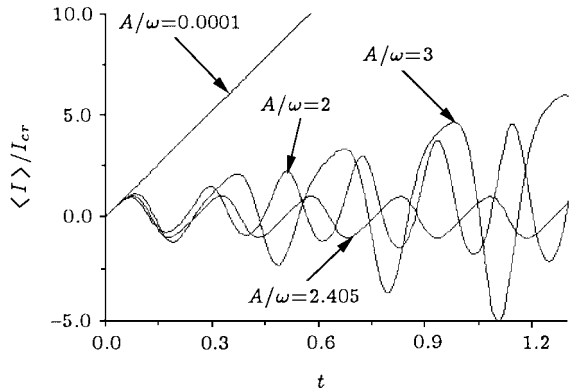


Fig.2. The supercurrent at different ratios of A/ω.

From Figs.1 and 2 we discuss the effect at different ratios of A/ω on the mean-square charge $\langle n^2 \rangle$ and the supercurrent $\langle I(t) \rangle$:

(1) When $A/\omega \to 0$, both the mean-square charge and the amplitude of the oscillation of the supercurrent will escape to infinity quickly. So it will lead to delocalization.

(2) When A/ω=2 or 3, we find that the mean-square charge and the amplitude of the oscillation of the supercurrent will also escape to infinity. But compared with the first case, their speed or the rate of delocalization of the initially localized state is slower.

(3) When A/ω=2.405, we can find that the mean-square charge remains bounded and the amplitude of the oscillation of the supercurrent remains constant, which means that for certain critical values of A/ω, an initially localized state will remain localized.

IV. CONCLUSIONS

In this paper, we have obtained the mean-square charge and the supercurrent by means of a numerical method in a charge representation, and we have discussed their properties. The traditional macroscopic Josephson junctions with dc voltage bias are operated in a phase eigenstate. In this case the phase uncertainty is $\Delta\theta$=0, the charge uncertainty Δq is very large, and tunnelling occurs randomly. The other extreme is the mesoscopic junctions with dc current bias, which are operated in a charge eigenstate. In this case the charge uncertainty is Δq=0, the phase uncertainty is very large and tunnelling occurs regularly. In this paper we have considered a circular superconducting ring with a mesoscopic Josephson junction driven by an ideal external periodic field and found that the charge and the amplitude of the oscillation of the supercurrent have the same properties. The charge is localized effectively when the ratio of the field magnitude and the field frequency equal some critical values, and in these cases the amplitude of the oscillation of the supercurrent remains constant. So we can control the charge uncertainty Δq by the frequency of the external field. This implies that the statistics of the tunnelling electrons should be controllable.

References

[1] Schön G and Zaikin A D 1990 *Phys. Rep.* **198** 237

[2] Vourdas A 1994 *Phys. Rev.* B **49** 12 040

[3] Kuang L M, Wang Y W and Ge M L 1996 *Chin. Phys. Lett.* **13** 359
1996 *Phys. Rev.* B **53** 11 764

[4] Zou J, Shao B and Xing X S 1997 *Phys. Rev.* B **56** 14 116
Shao B, Zou J and Xing X S 1997 *Chin. Phys. Lett.* **14** 905
Zou J and Shao B 1999 *Chin. Phys. Lett.* **16** 449

[5] Shao B 1999 *Chin. Phys.* **8** 368

[6] Wang Z Y, Liao H Y and Zhou S P 2001 *Acta Phys. Sin.* **50** 1996 (in Chinese)